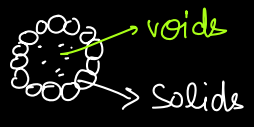
Geotechnical Engineering

[SOIL MECHANICS]

→ Our material soil is not completely solid not completely fluid It shows intermediate property between solid & fluid

→ Father – Karl Terzaghi , Grandfather – Coulomb.

→ Soil is unconsolidated material which is formed due to weathering of rocks.



→ The voids of the soil is either filled with water or air or both.

ORIGIN OF SOIL

→ The formation of the soil is termed as pedogenesis

Pedo → Earth
genesis → birth or origin

→ The soil is formed due to weathering erosion , wear & tear. disintegration of rocks.

→ There are three types of weathering

1) Physical weathering
2) chemical weathering
3) Biological weathering

1. PHYSICAL WEATHERING.

- Following processes are involved in the physical weathering of rocks

a) Exfoliation

b) Slaking
c) Frost wedging
d) Mineral crystallization
e) Pressure release or unloading
f) Action of vegetation
g) Grinding & Impact

a) Exfoliation: In this process the disintegration of rock occurs due to alternate expansion & contraction due to large variation in day & night temperature (e.g. desserts)

b) Slaking: In this process, the disintegration of rock occurs due to alternate drying and wetting process.

c) Frost wedging: The water is entrapped in the fissures of the rock In the night time when the temperature falls it freezes. The volume of ice is more than volume of water for the same weight. As a result widening of fissures causes disintegration of rock.

d) Mineral crystallization: Salts in the form of solution enters in the cracks / fissures of the rock when it undergoes the process of crystallization, the volume of the mineral crystal is more As a result widening of cracks causes the disintegration of rocks

e) Pressure release / unloading: Due to tectonic movement and mining process the disintegration of rock is observed.

f) Action of vegetation: Growth & development of roots of the plants or trees is responsible for weathering of rocks

g) Grinding & Impact: Grinding and impact of water, wind, snow and gravity can cause the disintegration of rock

Note: Sand and Gravel are formed by physical weathering of rock.

Note:- If your soil is formed by physical weathering then the mineral constituents will remain same as that of its parent rock.

2. CHEMICAL WEATHERING: Following processes are involved in the chemical weathering of rock

a) Hydration
b) Carbonation
c) leaching
d) Oxidation
e) Reduction

a) Hydration: Hydration means adsorption of water over the surface of rock from the atmosphere. This process of hydration causes the swelling or increase in volume of minerals as a result disintegration of rock occurs

b) Carbonation: In the carbonation the disintegration of the rock occurs due to carbonic Acid and other acids.

$$CO_2 + H_2O \longrightarrow H_2CO_3 \quad [\text{Carbonic Acid}]$$

$$H_2CO_3 + CaCO_3 \longrightarrow Ca(HCO_3)_2 \quad [\text{Calcium bi-carbonate}]$$
(water soluble)

- As a result rock is no longer rigid or solid which causes disintegration of rock.

c) leaching: When rock is exposed to water over in long duration then water soluble substances are removed from the rock. As a result rock no longer remains solid or rigid which causes disintegration of rock.

d) Oxidation: In the process of oxidation the oxygen ions combined with minerals of the rock and remove one or more electrons which makes the rock weaker. It leads to disintegration of rock.

e) Reduction: In the process of reduction, the oxygen is removed. It is the reverse process of the oxidation. This reduction process can be observed in water logged areas where no/ little oxygen is present.

Note:- Clay & Silt (upto some extent) are formed by chemical weathering.

Note:- If the soil is formed by chemical weathering then the mineral constituents will be different or changed in the weathered soil from its parent rock.

3. BIOLOGICAL WEATHERING: The biological weathering of rock occurs due to chemical organic acids released by the living organisms. Some other processes are also involved in the biological weathering of rock.

(i) Due to the growth / extension of the roots of plants & trees in the weaker plane of the rock can cause the disintegration of rock or widening of fissures / cracks.

(ii) For the growth of the plants extract some important nutrients from the rock which make favourable condition of disintegration of rock.

(iii) Due to the movement of living organisms (humans & animals) disintegration of rock may occur.

The general cycle involved can be given as

→ Erosion → Transportation → Deposition → upheaval

- After weathering of rock two types of soils are obtained.

1) TRANSPORTED SOIL: After weathering if soil is transported away from parent rock, then it is termed as transported soil.

2) RESIDUAL SOIL: After weathering if weathered soil is retained over the surface of the parent rock then it is called residual soil.

Based on the mode of transportation following type of soils:

a) Alluvial soil: It is a transported soil which is deposited from the suspension of the running water of the river. It is found along the rivers banks in the North of India.

b) Lacustrine Soil: It is transported soil which is deposited from the suspension of the still fresh water in the lakes.

c) Aeolian Soil: The soil which is transported by blowing wind is termed as Aeolian soils.

Note:- Aeolian soils have:

(i) low density & higher permeability
(ii) loose in nature.
(iii) It has less specific surface area.
(iv) It shows more compressibility

d) Marine Soil: It is a transported soil which is deposited from the suspension in the marine (sea) water.

e) Glacial Soil: The soil which is transported by following snow is called as glacial soil.

f) Colluvial Soil: It is also known as "Talus" soil. The soil which is transported by gravity is called as colluvial soil. The colluvial soil is generally found in the valley areas near the hilly areas.

The above six types of soils were discussed based on mode of transportation. Now some more types of soil can be discussed.

g) Loess Soil: It is wind blown uniformly graded silt which is slightly / weakly cemented due to the presence of calcium carbonate and montmorillonite

h) Loam soil: It is the mixture of clay silt sand in different proportions also known as Garden Soil. It also contains organic matter.

i) Marl Soil: It is finely grained calcium carbonated marine originated soil which is formed due to chemical decomposition of aquatic animals & plants.
(Marine = living)

j) Black cotton soil: It is a residual soil formed due to chemical weathering of basalt rock, also known as Expansive Soil. It is mostly found in the central region of India. It contains high percentage of montmorillonite clay mineral which is responsible for high swelling and shrinkage characteristics in the soil.

k) Bentonite Soil: It is also a residual soil. It is chemically weathered volcanic ash. It also contains high % of montmorillonite mineral which is responsible for high swelling & shrinkage. It has high plasticity and compressibility. It has low shear strength. It is used as a drilling mud for lubrication in drilling operations.

l) Peat soil: It is highly organic soil which consists of entirely decomposed vegetative compound in the different stages.

- Peat soil is highly fibrous and compressible
- It is found black or dark brown in color.
- It gives bad organic odour (smell)
- Symbol - {pt}

m) Muck Soil: mixture of finely grained inorganic substances and decomposed organic matter. It is generally found in marshy areas and the areas where no proper drainage facility is available. It is generally found after the overflooding of the rivers.

Note: the mixture of peat & muck is called cumulose.

n) Fissured Soil: The soil is warmed or heated at the depth and the seperation is observed at the surface due to the developement of crack/ fissures is called fissured soil.

o) Laterite soil: Laterite soil is formed due to the leaching out of the silica compounds and deposition of the iron oxides & alumina oxide (Fe_2O_3 & Al_2O_3)

p) Tuff soil: It is finely grained slightly cemented voleanic ash which is transported either by wind or water.

q) Gumbo Soil: It is black/ dark color sticky soil which is plastic in nature. It can be either physically or chemically withered.

r) Varved soil: Varved soil is composed of periodically repeated layers of silt & clay. Alternate layers of silt & clay are deposited in the glacial lakes from the washant of the glaciers Silts are generally deposited in the warm or summer seasons during heavy runoff and clay is deposited in winter season during small runoff. That means 1. layer of silt & 1-layer of clay are deposited in 1-year.

s) Humus Soil: Black/ dark brown color organic soil which is composed of partially decomposed vegetative compounds.

t) Indurated clay: It is the soil hardened by heating and pressure such that it didn't get soften even in long time in water

The properties of soil like shape, size, texture, sorting depends upon the mode of transportation as discussed below.

S. No.	Properties	Water	Air	ICE	Gravity	Organism
(i)	Size	Reduction	Considerable reduction (Comparatively more reduction)	Considerable grinding and impact	Considerable impact	Minor Abrasion (Minor reduction)
(ii)	Shape (Roundness)	Rounding off sand and gravel	High degree of rounding	Angular solid particles	Angular (Non-spherical particles)	-
(iii)	Surface Texture	Smooth, Shine polished	Impact produces frosted surfaces	Striated surface	Striated surface	-
(iv)	Degree of sorting	Considerable sorting	Considerable sorting	Very little sorting	No sorting	Limited sorting

PHASE SYSTEM

The soil is composed of solids liquid & gas mixed properly with each other.

Solids → Inorganic
Solids → Organic
Liquid → water
Gas → Air

$\rho_{solids} \Rightarrow 2500 kg/m^3 - 2900 kg/m^3$
$\rho_{water} \Rightarrow 1000 kg/m^3$
$\rho_{air} \Rightarrow 1.204 kg/m^3$

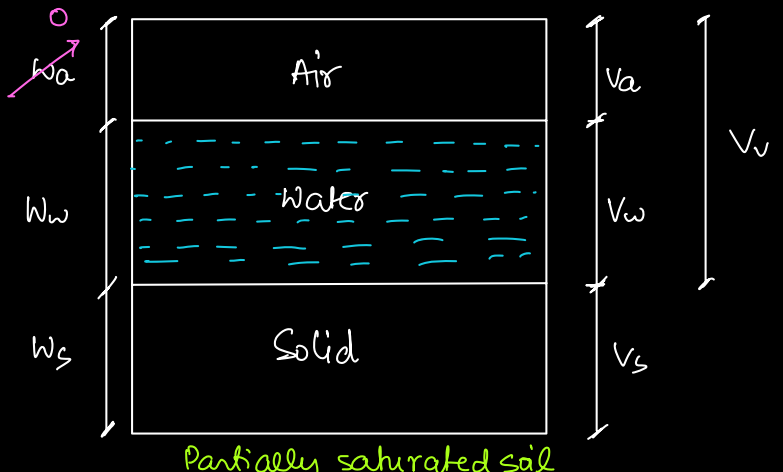
$V_{soil} \Rightarrow V_{air} + V_{water} + V_{solid}$

$W_{soil} \Rightarrow W_{solid} + W_{water}$

$\therefore \rho_{air} <<< \rho_{water} <<< \rho_{solids}$

$\therefore$ In soil mechanics, we ignore the weight of air.

THREE PHASE SYSTEM



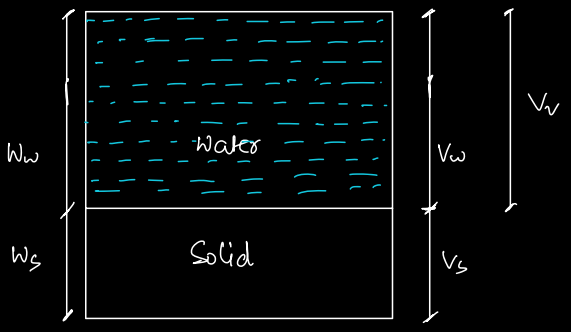
$W = W_a + W_w + W_s$

$\boxed{W = W_w + W_s}$

$V = V_v + V_s$

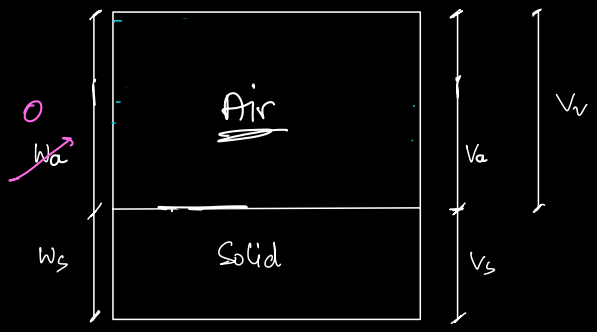
$\boxed{V = V_a + V_w + V_s}$

TWO PHASE SYSTEM



Fully saturated water.

- $W = W_w + W_s$
- $V = V_v + V_s$
- $V = V_w + V_s$



For Dry soil

- $W = W_s$
- $V = V_v + V_s$
- $V = V_a + V_s$

Note: Minimum number of phases → 2

Note: Maximum number of phases → 3

Note: Compulsary phase :- Solids

Note: voids are always present in the soil they are either filled with air / water or both. Therfore soil never exists in single phase system.

PROPERTIES OF SOIL

1 WATER CONTENT (w): defined as the ration of weight of water to the weight of solids.

$$w = \frac{W_w}{W_s} \times 100$$

→ if $w = 0$; Dry soil

→ water content can be greater than / equal to 100%.

→ For some typically marine soil w can reach upto 400%.

→ that means that there is no upper limit for water content

$\because W = W_w + W_s$

$W = w \cdot W_s + W_s$

$W = (1+w) W_s$

$\Rightarrow W_s = \frac{W}{(1+w)}$ → total weight; water content; weight of solids.

→ Water content can be expressed in the terms of total weight of soil also.

$$w' = \frac{W_w}{W} \times 100$$

* w is more accurate.
* Physical significance is more too.

$$w' = \frac{W_w}{W_w + W_s} \times 100$$

$$w' = \frac{\frac{W_w}{W_s}}{1 + \frac{W_w}{W_s}} \times 100 = \frac{w}{1+w} \times 100$$

$$w' = \frac{w}{1+w}$$

Note: w and w' both represents the weight of water present in the given soil sample. The theoretical significance of w is more than the w' because w is being measured with respect to a constant or stable parameter that is weight of solids

2 VOID RATIO (e):

Defined as the ratio of volume of voids to the volume of solids

$$e = \frac{V_v}{V_s}$$

→ Void ratio cannot be equal to zero ($e \neq 0$).

→ Void ratio can be greater than or equal to 1

→ There is no upper limit of void ratio.

→ $e \neq 0$, because $V_v \neq 0$, because voids are always present in the soil either filled with water or air or both.

$\because V = V_v + V_s$ $\quad\therefore V = eV_s + V_s$

$\because e = \frac{V_v}{V_s}$ $\quad V = (1+e)V_s$

$V_s = V/1+e$

$V_{solids} = \frac{V}{1+e}$

V_{solids} ← Volume of solids

V → Volume Total

e → void ratio.

Note: For the same volume of fine grained / coarse grained soil void ratio of fine grained soil is greater than the void ratio of coarse grained soil.

$$e_{FGS} > e_{CGS}$$

$$(\text{Volume of single void})_{CGS} > (\text{Volume of single void})_{FGS}.$$

$$(\text{No. of voids})_{CGS} < (\text{No of voids})_{FGS}$$

$$(\text{Total volume of voids})_{CGS} < (\text{Total volume of voids})_{FGS}.$$

$\because$ V_v (fine grained soil) > V_v (coarse grained solid)

$\therefore$ $e_{FGS} > e_{CGS}$.

Note: Volume of voids represents the water storage capacity of soil.

$\because$ $(V_v)_{FGS} > (V_v)_{CGS}$

$\therefore$ (Water storage capacity)$_{FGS}$ > (Water storage capacity)$_{CGS}$

Note: The void ratio is also used to represent the denseness of soil.

if $e \uparrow$ → looseness ↑ → denseness ↓

if $e \downarrow$ → looseness ↓ → denseness ↑

Note: Void ratio is not a constant value of the given soil. it can be reduced by method of compaction.

Note: Let soil (A) and soil (B) are mixed to form a new soil soil (C) then,

$$V_v(A) + V_v(B) \neq V_v(C)$$

$$V_A + V_B \neq V_C$$

$$e_A + e_B \neq e_C$$

But,

$$(W_s)_A + (W_s)_B = (W_s)_C$$

$$(V_s)_A + (V_s)_B = (V_s)_C$$

Note: In the given soil W_s and V_s are the most stable or constant parameters which never change on changing the time, location temperature and water content.

3. POROSITY (η): It is defined as the ratio of volume of voids to the total volume of soil.

$$\eta \neq 0$$
$$\eta \neq 100\%$$

$$\eta = \frac{V_v}{V} \times 100$$

$$\eta = \frac{V_v}{V_v + V_s} = \frac{V_v/V_s}{V_v/V_s + 1} = \frac{e}{1+e}$$

$$\eta = \frac{e}{1+e}$$

$$\left\{ 1 - \eta = \frac{1}{1+e} \right\}$$

Note: 'e' and 'η' both represents the volume of voids present in the given soil. The significance of void ratio 'e' is more than the significance of 'η' because void ratio 'e' is measured with respect to volume of solids.

4. DEGREE OF SATURATION (S): It is defined as the ratio of volume of water to the volume of voids

$$0 \leq S \leq 100\%$$

$$S = \frac{V_w}{V_v} \times 100$$

$S = 0$

$S = 1$ or 100%

$S \neq 1$

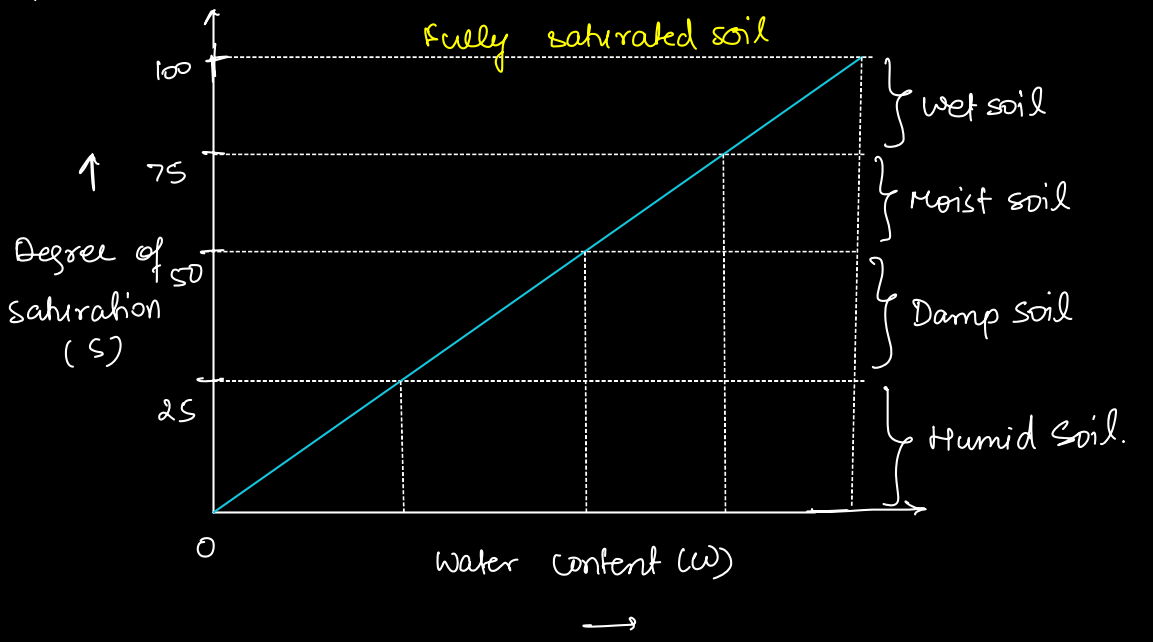
if $S=0$; Dry soil

if $S=100\%$; fully saturated soil. } TWO PHASE SYSTEM.

if $0 < S < 100\%$, three phase system. (partially saturated

Note: Degree of saturation can be correlated with the water content.

if $w\uparrow \rightarrow S\uparrow$



5 AIR CONTENT (a_c): defined as the ratio of volume of air to the volume of voids.

$$a_c = \frac{V_a}{V} \times 100. \qquad 0 \leq a_c \leq 100\%$$

$\{a_c + S = 100\%\}$

$a_c = 0$; fully saturated

$a_c = 100\%$: Dry soil

6 PERCENTAGE AIR VOIDS (n_a): It is defined as the ratio of volume of air to the total volume of soil.

$$n_a = \frac{V_a}{V} \times 100 \text{ ,} \qquad n_a = 0 \text{ : fully saturated}$$

$n \neq 100\%$

$$\because \quad n_a = \frac{V_a}{V}$$

$$n_a = \frac{V_a}{V} \times \frac{V_v}{V_v} = \frac{V_a}{V_v} \times \frac{V_v}{V} = a_c \times n$$

$n_a = a_e \times n$ → porosity

a_e → air content

n_a → pair content

7 UNIT WEIGHT OR DENSITY OF SOIL:

1. Bulk unit weight or Bulk density:

It is defined as the ratio of total wt of soil to the total volume of soil.

$$\gamma = \frac{\text{Total weight of soil}}{\text{Total volume of soil}} = \frac{W}{V} \quad KN/m^3$$

$$\rho = \frac{\text{Total mass of soil}}{\text{Total volume of soil}} = \frac{M}{V} \quad Kg/m^3$$

2. Dry unit weight (γ_d):

It is defined as the raho of dry weight of soil (W_s) to the total volume of soil.

$$\gamma_d = \frac{W_s}{V} \quad KN/m^3$$

Note: Bulk unit weight > Dry unit weight.

Note: γ_d is used to express denseness of soil.

Note: If $\gamma_d \uparrow$, $V_v \downarrow$. → $e \downarrow$ → denseness ↑

3. Saturated unit weight (γ_{sat}):

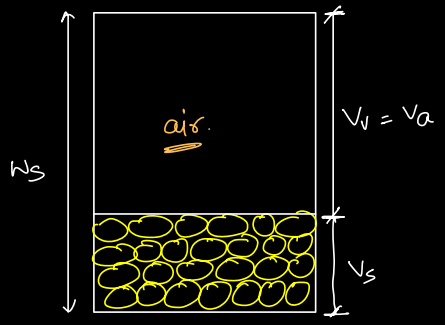
It is defined as the ratio of saturated wt of soil to the total volume of the soil

$$\gamma_{sat} = \frac{W_{sat}}{V} \quad KN/m^3$$

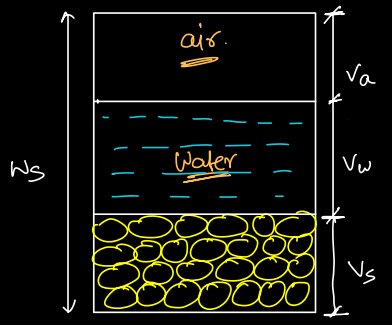
- saturated unit weight is calculated for fully saturated soil.

[$S = 1$, $V_a = 0$, $V_v = V_w$]

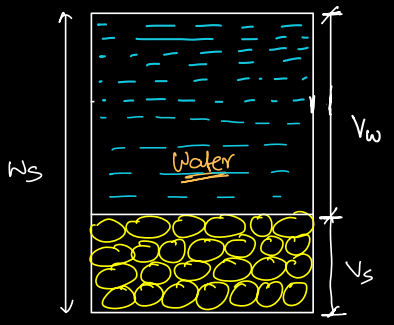
PTO



Dry soil (γ_d)



Partially Saturated Soil. (γ)



Fully Saturated Soil. (γ)

$$\gamma_d = \frac{W_s}{V} \qquad \gamma = \frac{W}{V} \qquad \gamma_{sat} = \frac{W_{sat}}{V}$$

$$\{\gamma_{sat} > \gamma > \gamma_d.\}$$

4. Submerged unit weight / Buoyant unit weight : (γ_{sub} or γ')

It is defined as the ratio of submerged weight of the soil to the total volume of the soil.

$$\gamma_{sub} = \frac{W_{sub}}{V} \quad KN/m^3$$

→ γ_{sub} is calculated when the soil is completely submerged below the water level.

→ The net weight of the soil when it is completely submerged below water level is termed as submerged wt (W_{sub}).

→ When the soil is completely submerged it experiences a upward force called as buoyancy force. the value of buoyancy force is equal to the wt of water displaced by the soil.

$$\text{Buoyancy force} = \gamma_w V = F_B \text{ (say)}$$

$$W_{sub} = W_{sat} - F_B$$

$$\gamma_{sub} = \frac{W_{sat} - \gamma_w V}{V} = \frac{(\gamma_{sat} - \gamma_w)V}{V}$$

$$\{\gamma_{sub} = \gamma_{sat} - \gamma_w\}$$

* Based on the Archimedes' principle
* Smallest unit weight
* $\gamma_{sub} \approx \frac{\gamma_{sat}}{2}$

5. Unit weight of solids (γ_s): Defined as the ratio of total weight of solids to the total volume of solids

$$\gamma_s = \frac{W_s}{V_s} \quad KN/m^3$$

$\rightarrow \gamma_s > \gamma_{sat} > \gamma > \gamma_d > \gamma_{sub}$.

$\rightarrow$ γ_s is very stable parameter which is independent of location, time temperature and location.

8 SPECIFIC GRAVITY OF SOIL

1. True sp. gravity or Absolute sp. gravity or sp. gravity of solids: defined as the ratio of unit weight of solid to the unit weight of standard fluid (fresh water) of same volume

$$G \text{ or } G_s = \frac{\gamma_s}{\gamma_w} = \frac{\frac{W_s}{V_s}}{\frac{W_w}{V_w}} = \frac{W_s}{W_s \mp W_w}$$

∴ because water (W_w) used in we is present inside the soil whereas this W_w is hypothetically taken having same volume as solids

2. Mass specific gravity or Apparent specific gravity or Bulk sp. gravity: It is defined as the ratio of bulk unit weight of soil to the unit weight of the standard fluid (water) of the same volume.

$$G_m = \frac{\gamma}{\gamma_w} \quad \text{(No unit)}$$

- If soil is in dry state then $\gamma \rightarrow \gamma_d$
- If soil is in saturated state then $\gamma \rightarrow \gamma_{sat}$
- $G > G_m$.

True Sp. gravity or sp. gravity of solids for different types of soils are given below:

Types of Soils	G or Gs
clean Sand & Gravel	2.65 – 2.68
Silt & Silty Sand	2.66 – 2.70
Inorganic clay soil	2.70 – 2.80
Soils with high percentage of Mica & Fe	2.75 – 2.85
organic soil	1.0 – 2.0

→ The sp. gravity of fine grained soil is more than the sp. gravity of coarse grained soil.

→ The sp. gravity of Inorganic soil is more than the sp. gravity of organic soil.

→ On increasing the % of minerals in the soil the sp. gravity increases

→ As per IS code the sp. gravity of solids shall be calculated at 27°C.

$$G = \frac{\gamma_s}{\gamma_w}$$

Independent of temperature

depends upon the temperature

If also depends upon the temperature

$$(\gamma_s)_{27^\circ C} = (\gamma_s)_{T^\circ C}$$

$$G_{(27^\circ C)}\,\gamma_{w(27^\circ C)} = G_{(T^\circ C)} \cdot \gamma_{w(T^\circ C)}$$

$$G_{(T^\circ C)} = G_{27^\circ C} \cdot \frac{\gamma_{w(27^\circ C)}}{\gamma_{w(T^\circ C)}}$$

γ_w → unit weight of water

γ_w → 1 gm/cm³ (@ 4°C)

$\gamma_w = 0.9965$ gm/cm³ (@ 27°C)

→ If Standard fluid is changed:

Then specific gravity of solids can be given as,

$$G' = \frac{\gamma_s}{\gamma_f} = \frac{\gamma_s}{G_f \cdot \gamma_w} = \frac{G}{G_f} \Rightarrow G = G' \cdot G_F$$

γ_s : unit wt of solid

γ_f : unit wt of fluid

G' : unit wt of solids w.r.t to given fluid.

G_f : Sp. gravity of given fluid wrt water.

G : Sp. gravity w.r.t to water as standard liquid.

MAXIMUM AND MINIMUM VOID RATIO IN SPHERICALLY SHAPED UNIFORM PARTICLES IN LOOSEST AND DENSEST STATE

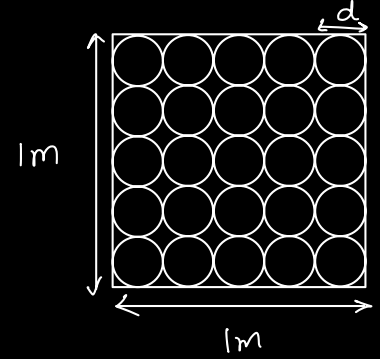
Note: this study of estimation of e_{max} & e_{min} is purely theoretical because practically all the particles cannot have equal sizes and perfectly spherical shape in the field.

In this case, soil can be assumed to attain

(i) loosest state in cubide array

(ii) densest state in rhombohedral array

1. CUBICAL ARRAY: In cubical array, each particle is in contact with maximum 6 other particles.



Total volume of soil = $1m^3$

Volume of a single solid = $\frac{\pi}{6} d^3$

total number of solids = $\frac{1}{d^3}$

total volume of solids (V_s) = $\frac{\pi}{6} d^3 \cdot \frac{1}{d^3} = \frac{\pi}{6} m^3$

$V_s = 0.523 m^3$

$V_v = V - V_s$

$V_v = 1 - 0.523$

$V_v = 0.477 m^3$

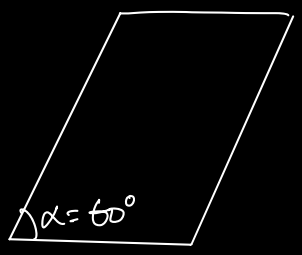
$e_{max} = \frac{V_v}{V_e} = \frac{0.477}{0.523} = 0.91$

$e_{max} = 0.91$

$\eta_{max} = 0.477$

→ In cubical array our soil attains loosest state with $e_{max} = 0.91$ and $n_{max} = 0.477$.

2. RHOMBOHEDRAL ARRAY: In rhombohedral array each particle is in the contact with maximum 12 other particles.



Volume of soil in rhombohedral array

$$V = \sqrt{1+2\cos\alpha}\,(1-\cos\alpha)$$

$$V = 0.707\ m^3$$

$V_s = 0.523\ m^3$ (since it does not change)

does $V_v = 0.707 - 0.523 = 0.184$

$$e_{min} = \frac{0.184}{0.523} = 0.35$$

$$n_{min} = \frac{0.184}{0.707} = 0.26.$$

Note: In rhombohedral array soil attains densest state with $e_{min} = 0.35$ and $n_{min} = 0.26$.

RELATIONSHIP BETWEEN DIFFERENT PROPERTIES OF SOIL

1. e, n.

$$n = \frac{V_v}{V}$$

$$n = \frac{V_v}{V_v + V_s}$$

$$n = \frac{\frac{V_v}{V_s}}{\frac{V_v}{V_s} + \frac{V_s}{V_s}}$$

$$n = \frac{e}{1+e} \quad \text{or} \quad 1-n = \frac{1}{1+e}$$

2. a_c, S

$$V_v = V_a + V_w$$

$$\frac{V_v}{V_v} = \frac{V_a}{V_v} + \frac{V_w}{V_v} \Rightarrow S + a_c = 1$$

3. n_a, n_e, a_c

$$n_a = \frac{V_a}{V}$$

$$n_a = \frac{V_a}{V} \times \frac{V_v}{V_v}$$

$$n_a = \frac{V_a}{V_v} \times \frac{V_v}{V}$$

$$n_a = a_c \times n$$

4. V_s, V, e

$$e = \frac{V_v}{V_s}$$

$$V_v = e \cdot V_s$$

$$V = V_v + V_s$$

$$V = e \cdot V_s + V_s$$

$$V_s = \frac{V}{1+e}$$

5. w_s, w, w

$$w = \frac{W_w}{W_s}$$

$$W_w = w \cdot W_s$$

$$W = W_w + W_s$$

$$W = W_s(1+w)$$

$$W_s = \frac{W}{1+w}$$

6. γ_s, γ, w, e

$$W_s = \frac{W}{1+w}$$

$$V_s = \frac{V}{1+e}$$

$$\gamma_s = \frac{W_s}{V_s}$$

$$\gamma_s = \frac{W}{V}\left(\frac{1+e}{1+w}\right)$$

$$\gamma_s = \gamma\left(\frac{1+e}{1+w}\right)$$

7. $\gamma_d, \gamma, w.$

$$\gamma_d = \frac{W_s}{V}$$

$$\gamma_d = \frac{W}{(1+w)V}$$

$$\gamma_d = \frac{\gamma}{1+w}$$

8. $e, S, w, G.$

$$e = \frac{V_v}{V_s} = \frac{V_v \times V_w}{V_s \times V_w}$$

$$e = \frac{V_v}{V_w} \times \frac{V_w}{V_s}$$

$$e = \frac{1}{S} \times \frac{V_w}{V_s}$$

$$e = \frac{W_w}{\gamma_w} \times \frac{1}{V_s} \times \frac{1}{S}$$

$$e = \frac{W_w}{\gamma_w} \times \frac{G\gamma_w}{W_s} \times \frac{1}{S}$$

$$se = wG$$

9. $\gamma, G, e, S, \gamma_w$

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_v}$$

$$\gamma = \frac{GV_s\gamma_w + \gamma_w \times V_w}{V_s + V_v}$$

$$\gamma = \frac{G\gamma_w + \gamma_w \times V_w/V_s}{1 + \frac{V_v}{V_s}}$$

$$\gamma = \frac{\gamma_w[G + V_w/V_s]}{1+e}$$

$$\gamma = \frac{\gamma_w\left[G + \frac{V_w \cdot V_v}{V_s \cdot V_v}\right]}{1+e}$$

$$\gamma = \frac{\gamma_w[G + es]}{1+e}$$

γ → Bulk unit weight

10. $G_m, G, e, S.$

$$\therefore \gamma = \left(\frac{G+es}{1+e}\right)\gamma_w$$

$$\frac{\gamma}{\gamma_w} = \frac{G+es}{1+e}$$

$$G_m = \frac{G+es}{1+e}$$

G_m → Apparent specific gr.

11. $\gamma_{sat}, G, e, \gamma_w$

$\because \quad \gamma = \left(\frac{G+es}{1+e}\right)\cdot\gamma_w$

if $S=1, \quad \gamma_{sat} = \gamma$

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w$$

12. γ_d, G, e, γ_w

$\because \quad \gamma = \frac{G+es}{1+e}\cdot\gamma_w$

if $S=0, \quad \gamma = \gamma_d$

$$\gamma_d = \left(\frac{G}{1+e}\right)\cdot\gamma_w$$

13. $\gamma_{sub}, G, e, \gamma_w$

$\because \gamma_{sub} = \gamma_{sat} - \gamma_w$

$$\gamma_{sub} = \left(\frac{G+e}{1+e} - 1\right)\gamma_w$$

$$\gamma_{sub} = \left(\frac{G-1}{1+e}\right)\gamma_w.$$

14. $n_a, \gamma_d, G, w, \gamma_w$.

$$V = V_a + V_w + V_s$$

$$\frac{V}{V} = \frac{V_a}{V} + \frac{V_w}{V} + \frac{V_s}{V}$$

$$1 = n_a + \frac{W_w}{\gamma_w \cdot V} + \frac{W_s}{G\gamma_w \cdot V}$$

$$1 - n_a = \frac{W_s \cdot w}{\gamma_w \cdot V} + \frac{W_s}{G\gamma_w \cdot V}$$

$$1 - n_a = \frac{W_s}{\gamma_w \cdot}\left[\frac{w}{V} + \frac{1}{GV}\right]$$

$$1 - n_a = \frac{W_s}{V\cdot\gamma_w}\left[w + \frac{1}{G}\right]$$

$$1 - n_a = \frac{\gamma_d}{\gamma_w}\left[w + \frac{1}{G}\right]$$

$$\gamma_d = \frac{G\gamma_w(1-n_a)}{(Gw+1)}$$ → SHALL BE USED IN COMPACTION.

15. $\gamma, \gamma_d, \gamma_{sat}, S$.

$$\gamma = \left(\frac{G+es}{1+e}\right)\gamma_w$$

$$\gamma = \frac{G\gamma_w}{1+e} + \frac{es\gamma_w}{1+e} \quad -(A)$$

$$\gamma_{sat} - \gamma_d = \frac{(G+e)\gamma_w - G\gamma_w}{1+e}$$

$$\gamma_{sat} - \gamma_d = \frac{(G+e-G)\gamma_w}{1+e}$$

$$\gamma_{sat} - \gamma_d = \left(\frac{e}{1+e}\right)\gamma_w$$

↳ put in eqn (A)

$$\gamma = \frac{G\gamma_w}{1+e} + (\gamma_{sat} - \gamma_d)S$$

$$\gamma = \gamma_d + (\gamma_{sat} - \gamma_d)S$$ → final relation.

16. $\gamma_{sat}, \gamma_d, n$.

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w$$

$$\gamma_{sat} = \frac{G\gamma_w}{1+e} + \frac{e\gamma_w}{1+e}$$

$$\{\gamma_{sat} = \gamma_d + n\gamma_w\}$$

17. $\gamma_{sub}, \gamma_d, n$

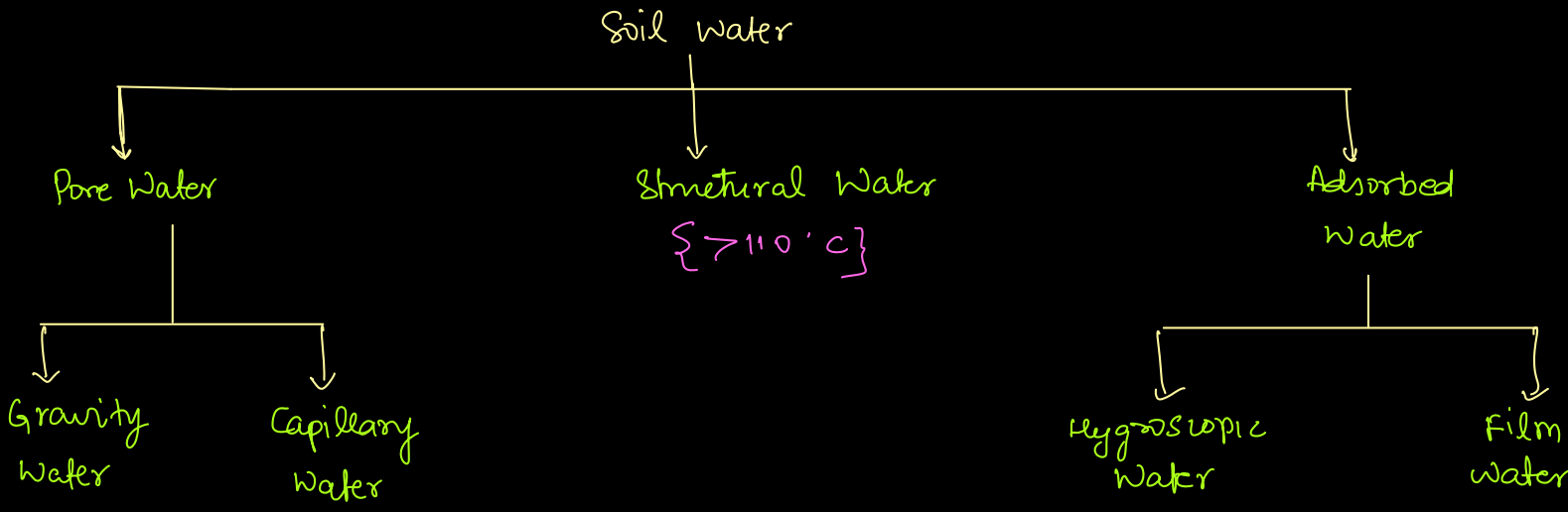
$$\gamma_{sub} = \left(\frac{G-1}{1+e}\right)\gamma_w$$

$$\gamma_{sub} = \gamma_d - \frac{\gamma_w}{1+e}$$

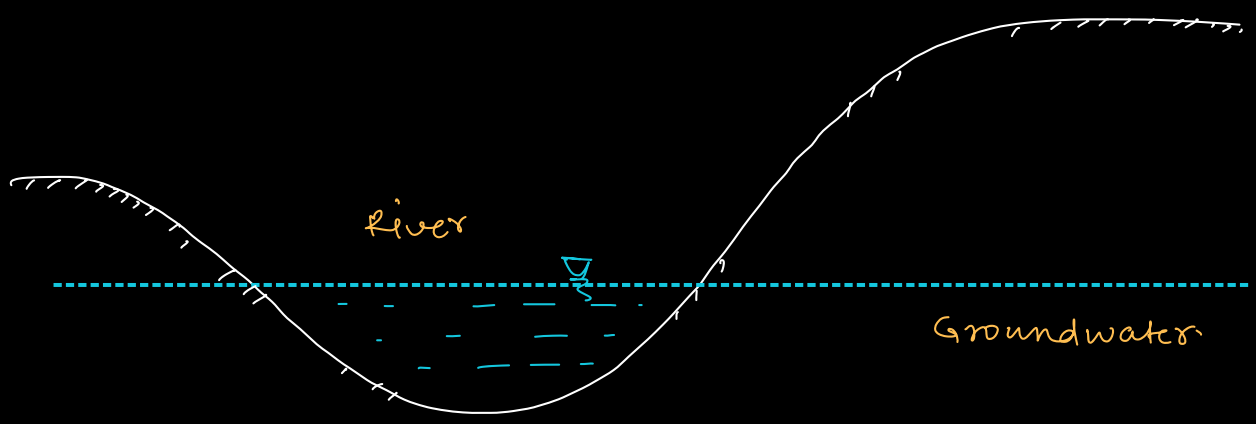
$$\gamma_{sub} = \gamma_d - (1-n)\gamma_w.$$

SOIL WATER

There can be following types of soil water



1. GROUNDWATER / FREEWATER / GRAVITY WATER:



→ It is the subsurface water which fills the voids / pore of the soil continuously upto the groundwater table. Therefore the soil below water table is considered to be completely fully saturated. It is also subjected to any kind of external pressure or force except its own self weight, Therefore it is also known as gravity water.

→ It obeys all the laws of hydraulics and capable of moving under the hydrodynamic conditions

2. CAPILLARY WATER: It is the water which is lifted above the groundwater table and held in suspension by the force of surface tension.

- Capillary water is in tension.
- It also obeys all the laws of hydraulics
- It can not be observed below Ground water table
- Gravity water & capillary water also termed as Pore Water. because they fill the pores and voids of the soil.

3. STRUCTURAL WATER: Structural water is the water which is chemically combined with crystal structural of the soil solid

→ since it is chemically combined with crystal structure of solid therefore under the normal loading conditions structural water can not be removed or drained out in the process of consolidation

→ if soil is heated upto the 110°C, then also this water cannot be removed.

→ If temperature is raised beyond the 110°C then permanent destruction of solid occurs which may lead to the loss of structural water.

ADSORBED WATER: Also termed as Pellicular water

4. HYGROSCOPIC WATER: Water which is adsorbed from the atmosphere by the soil solid over it's surface by the force of attraction and held tightly by force of adhesion.

→ The water adsorption depends upon the specific surface area of the solids

→ if particle size ↑ → SSA ↓ → water adsorption ↓

→ if particle size ↓ → SSA ↑ → water adsorption ↑

→ Increasing order of soils based on their water adsorption capacity can be given as,

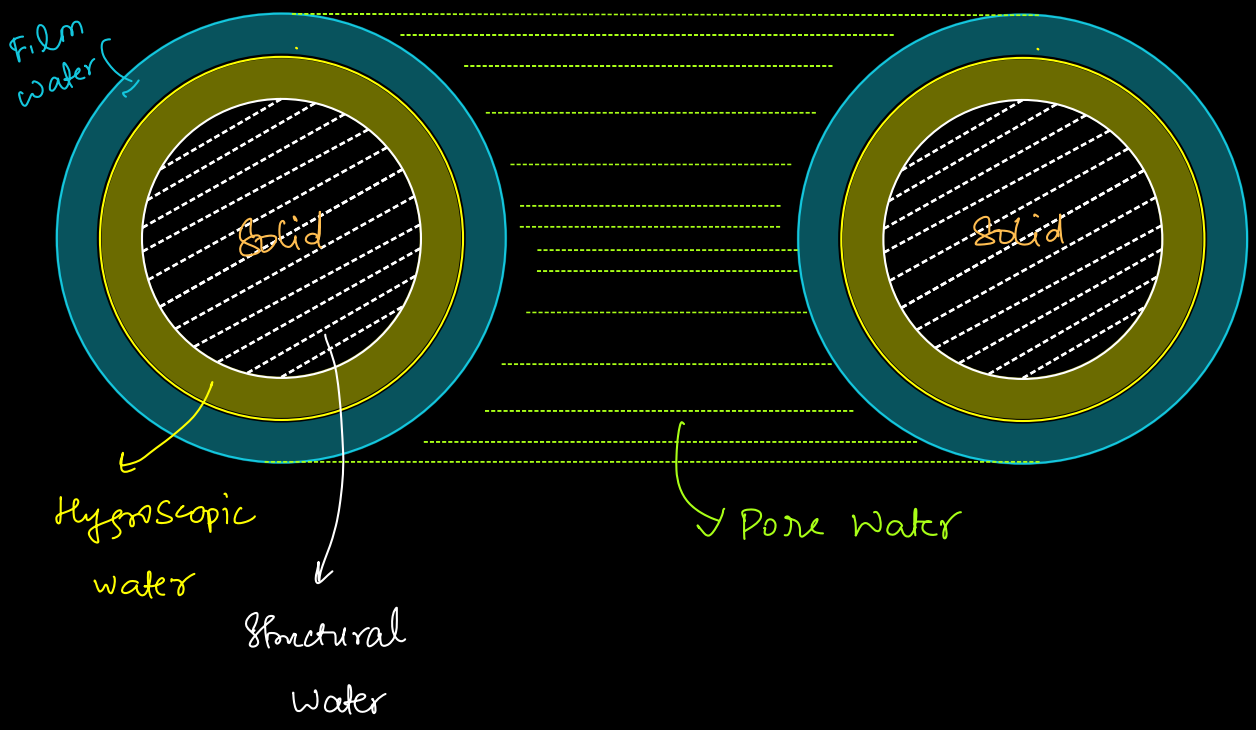
Sand < Silt < Clay

→ The properties of hygroscopic water are different from the normal water. For ex. hygroscopic water has high viscosity,

high boiling point & low freezing point.

→ It doesn't follow the laws of hydraulics and not capable in moving under the normal hydrodynamic forces.

5. FILM WATER: A thin layer of water is formed over the layer of hygroscopic water is termed as film water. The layer of film water is formed by the process of condensation The properties of film water are intermediate between normal water & hygroscopic water.



DETERMINATION OF INDEX PROPERTIES

Index properties are the properties which are used to identify & classify the given soil sample. For ex. water content, unit weight. Sp. Gravity, particle size distribution, consistency of soil, sensitivity thixotrophy, Activity, density index, Relative compaction etc.

1. Determination of WATER CONTENT:

1. Oven dry method
2. Sand bath method
3. Alcohol bath method
4. Calcium Carbide method
5. Pycnometer method
6. Radiation method

7. Torsion balance method.

a. Oven dry method:

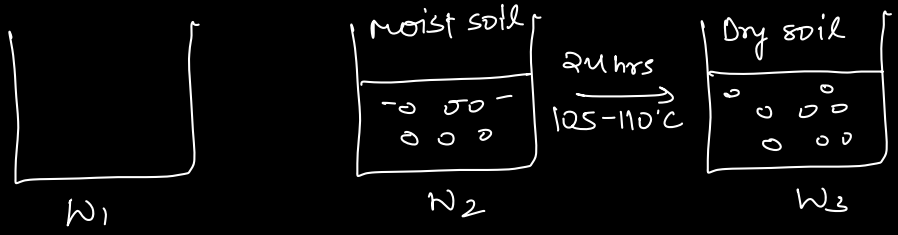
- Most accurate method of determination of water content in the laboratory.
- In this method the container with the moist soil is kept in the oven drying machine for 24 hours at a temperature of 105 - 110°C.

Let

W_1 : wt of empty container

W_2 : wt of container with moist soil

W_3 : wt of container with dry soil.



$$w = \frac{W_w}{W_s} = \frac{W_2 - W_3}{W_3 - W_1}$$

- For inorganic soil like sand the drying is required for 4 hours and for inorganic soil like clay and silt drying is required for 12 - 16 hours. Then also to ensure the complete removal of water, the drying process is continued upto 24 hours
- In case of organic soils temperature is not raised beyond 60°C to prevent oxidation of organic matter.
- If soil contains gypsum [$CaSO_4 \cdot 2H_2O$] in high percentage then the temperature is not raised beyond 80°C to avoid the loss of water of crystallization
- In case, temperature is not raised beyond 110°C to avoid the permanent destruction of soil solids which may lead

to the loss of structural water.

- In this method, except the structural water and the water of crystallization, all other types of water can be removed.

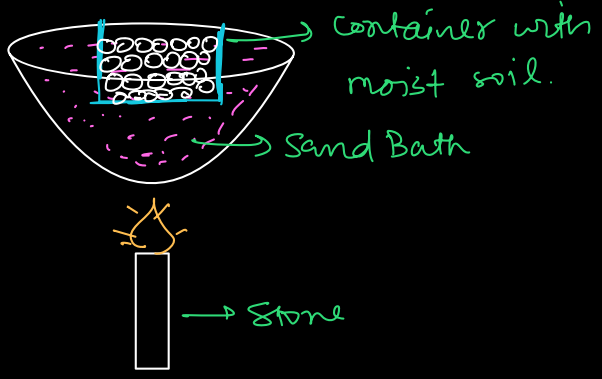
b. Sand bath method: It is a quick field method to determine the water content.

→ The container with moist soil is placed over the sand bath and heated with stove for 30 minutes to 1 hour.

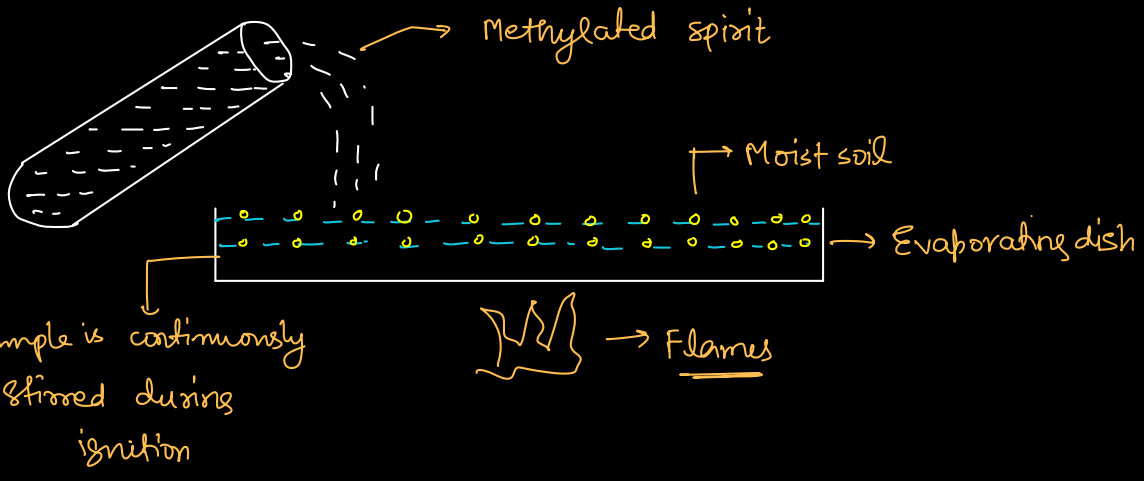
→ The procedure of measurement of water content is same as that of previous method.

$$w = \frac{W_s}{W_s} = \frac{W_2 - W_3}{W_3 - W_1}$$

→ Since there is no control on temperature, therefore not suitable for organic soils & soils with high gypsum content



c. Alcohol method:



→ A quick field method to determine water content

→ Methylated spirit is added to the given moist sample to increase rate of evaporation.

→ The observation & measurement is same as over drying method

$$w = \frac{W_w}{W_s} = \frac{W_2 - W_3}{W_3 - W_1}$$

→ In this method, the time required is less than the time required in sand bath method.

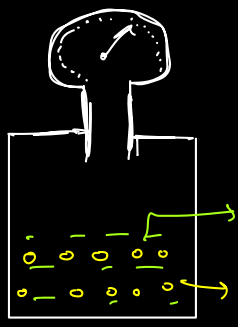
→ This method is not suitable for organic soil or soils having high % of gypsum because there is no control on the temperature.

Note: The results obtained from sand bath method and alcohol method are not accurate

d. Carbide Method or Rapid Moisture Meter method: (FASTEST)

$$CaC_2 + H_2O \longrightarrow C_2H_2\uparrow + Ca(OH)_2$$

(Acetylene)



Rapid Moisture meter

→ It is the quickest method for the determination of water content. This test can be completed in 5 - 10 minutes

→ In this method a fixed amount of moist soil sample (4 - 6gm) is taken in the container of the rapid moisture meter and sufficient amount of CaC_2 is added

→ The moisturemeter is shaked continuously for 5 - 10 minutes

→ The CaC_2 comes in contact with moisture present in the soil and creates acetylene gas.

→ This acetylene gas exerts pressure on the gauge which is

calliberated in the terms of water content with respect to the total weight of the soil.

$$w' = \frac{W_w}{W} = \frac{w}{1+w}$$

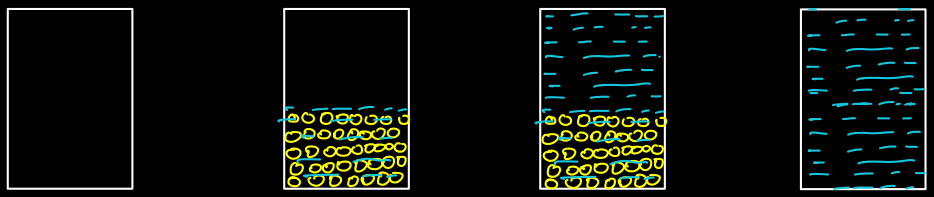
Note: If size of the container is larger then moist soil sample can be taken upto 26 gm

Soil + $CaCl_2$ + Steel balls (→ इस्पात के अण्डकोश)

↳ to break lumps while shaking.

e. Pycnometer bottle: Pycnometer is a 900 ml density bottle having conical brass top with 6mm dia opening.

- using pycnometer the water content can be computed for those soils whose specific gravity is always known.
- The dried and cleaned pycnometer is weighed as W_1
- The moist soil is added in the pycnometer and it is weighed as W_2.
- The remaining space is filled with fresh water and it is weighed as W_3
- Now the pycnometer is cleaned and refilled with the fresh water and weighed as W_4.



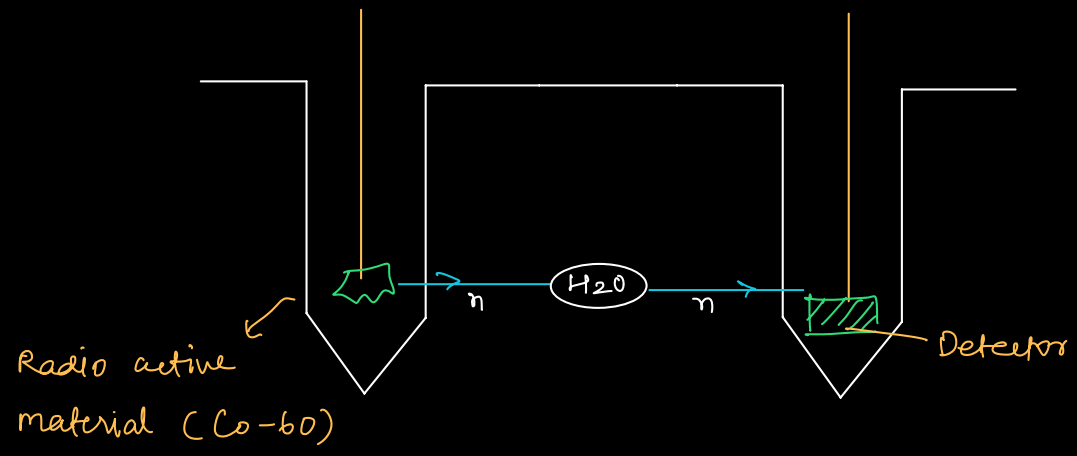
$$w = \frac{W_2 - W_1}{W_3 - W_4}\left(\frac{G-1}{G}\right) - 1$$

Note: In the 3rd stage of this test where the remaining space of pycnometer is completely filled with water, the air present in

the soil shall be removed by continuous stirring during addition of water in the pycnometer.

Note: if air is not completely removed then w_3 should come out to be less than result the water content is overestimated thus the air is needed to be removed.

f. Radiation Method:



→ In radiation method in-situ water content is measured at the field

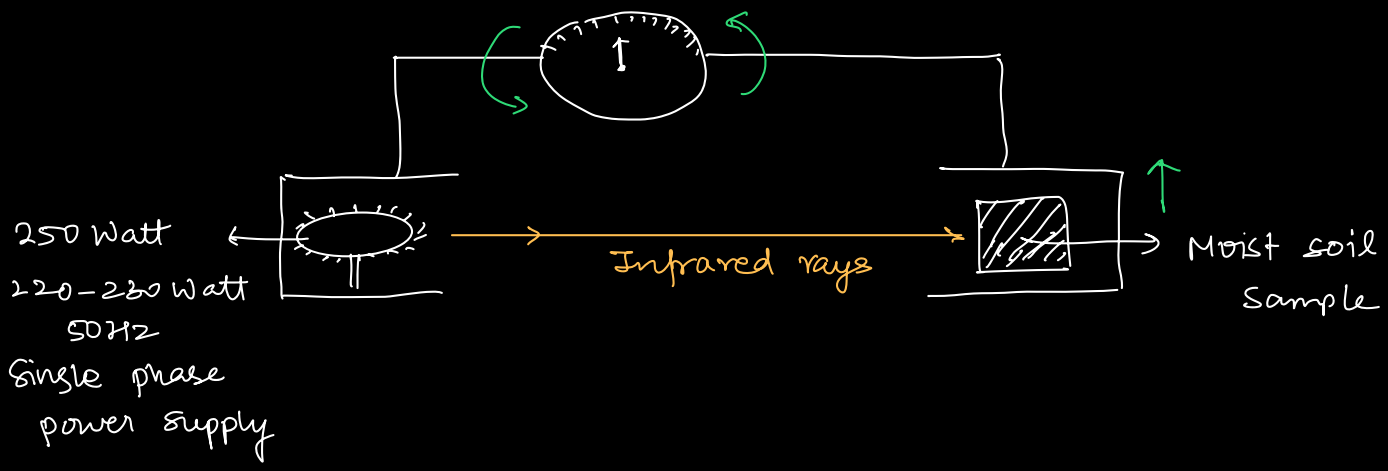
→ In this method two casings are lowered below the ground surface. In the first casing radio active material [Cobalt-60] and in the second casing detector is placed

→ when radio active material is activated it emits the neutrons will strike the hydrogen atoms in the moisture, as a result loss of energy is observed which is detected by the detector in other casing

→ this loss of energy of neutron is calibrated in the terms of water content.

higher the loss of energy of neutrons ⟶ higher the water content in the soil.

g. Torsion balance method:



→ In this method, the given moist soil is dried with the help of Infrared radiation which can be produced by a 250 watt bulb, operated at 220-230V, 50Hz & single phase power supply. The torque is produced which is calibrated in the terms of water content w.r.t total weight of soil (w').

→ In this method drying and weighing is done simultaneously therefore this test is very much suitable for the soils which quickly reabsorbs the water from atmosphere.

$$w' = \frac{W_w}{W} \quad , \quad w' = \frac{w}{W}$$

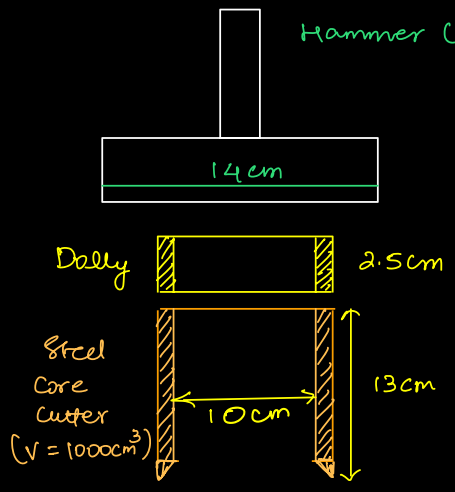
→ Higher the torque, higher the moisture content.

Determination of unit weight of the soil:

1. Core cutter method
2. Water displacement method
3. Submerged Mass density method
4. Sand replacement method
5. Water balloon method

PTO

a. Core Cutter Method:



- In this method, a steel core cutter of 10cm diameter, 13cm height and specific volume of $1000\ cm^3$ is used
- A steel dolly of 2.5cm height is placed over the steel core cutter
- A hammer of 14cm diameter of weight 9kg is used for the driver of the core cutter & dolly.
- With the help of hammer no of blows are given and entire assembly is forced below the ground surface, the entire assembly is penetrated in such a way that only 1cm height of dolly is visible above ground.
- Now the entire assembly is withdrawn & the wt of core cutter is noted as W_2.

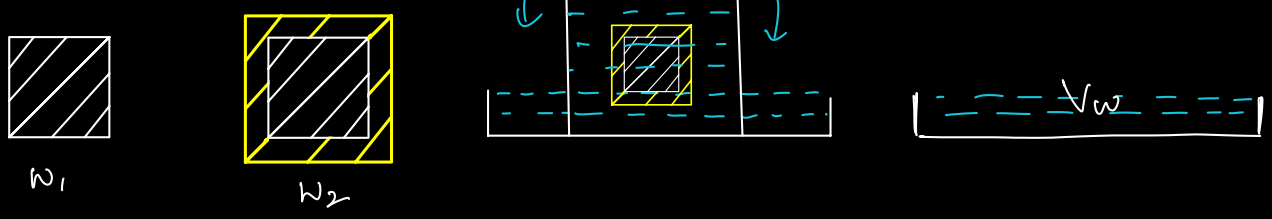
- Let W_1 be the empty weight of core cutter.
- wt of collected soil + wt of core cutter is W_2
- wt of collected soil : $W_2 - W_1$
- total volume of soil (V) = Vol of the core cutter

$$V = 1000cm^3$$

unit weight of soil, $\gamma = \frac{W}{V}$

$$\gamma = \frac{W_2 - W_1}{V} \quad gm/cm^3$$

b. Water displacement method:



V_w: Volume of water displaced by coated soil specimen

$V_w = V + V_p$

$V_p = \frac{W_p}{\gamma_p} \rightarrow W_2 - W_1$

$V = V_w - V_p \rightarrow$ Volume of paraffic wax.

$V = V_w - \frac{W_2 - W_1}{\gamma_p}$ → unit weight / density of paraffin wax.

Volume of soil

Volume of water displaced.

∴ The unit weight of paraffin wax can be written as.

$$\gamma = \frac{\text{weight of soil}}{\text{Volume of soil.}}$$

$$\gamma = \frac{W_1}{V}$$

→ Suitable for cohesive soils that are highly sticky in nature so that they can be trimmed into the uniform regular shape.

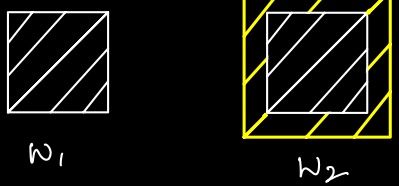
→ The given soil is trimmed in the uniform regular shape and weighed as W_1.

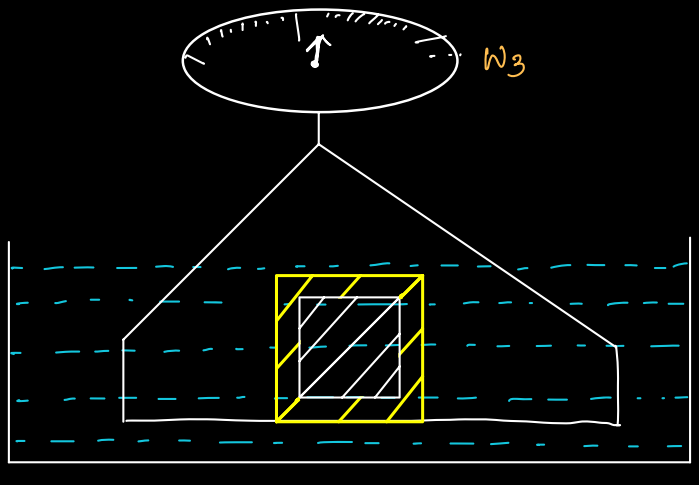
→ The trimmed specimen is now coated by Paraffin wax and weighed as W_2.

→ This coated specimen is now immersed in the container which is completely filled with water

→ The volume of water displaced is called V_w.

C. Submerged mass density method:





W_3: Submerged weight of coated soil specimen

$$W_3 = W_2 - F_B$$

↳ Buoyancy force which is equal to the weight of water displaced by the coated specimen

let V_w is the volume of water displaced by coated soil specimen. Therefore

$$W_3 = W_2 - F_B$$

$$W_3 = W_2 - V_w \gamma_w$$

$$V_w = \frac{W_2 - W_3}{\gamma_w}$$

↳ This volume of water is equal to the volume of coated soil specimen.

$$V_w = V_p + V_{soil}$$

$$V_{soil} = V_w - V_p$$

$$V_{soil} = \frac{W_2 - W_3}{\gamma_w} - \frac{W_2 - W_1}{\gamma_p}$$

γ_w: 1 gm/cm³

γ_p: 0.908 gm/cm³

∴ The unit of soil can be given as

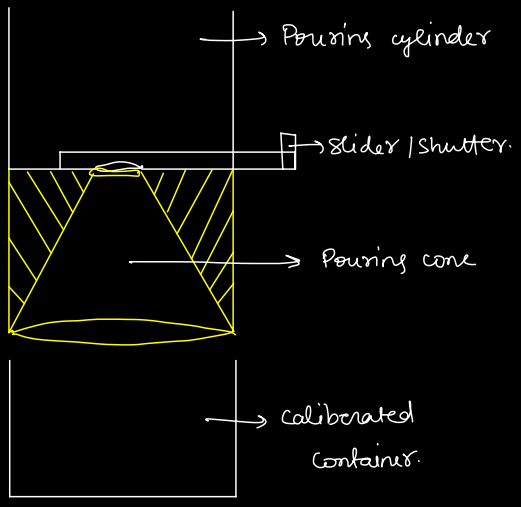
$$\gamma = \frac{W_1}{V_{soil}} \quad KN/m^3.$$

→ It is also suitable for cohesive & highly sticky soils.

→ The given soil is trimmed in regular shape & coated with paraffin wax is weighed as W_1 and W_2 respectively

→ Now this coated specimen is immersed in the water to measured the submersed weight W_3.

d. Sand Replacement method: In this method, pouring cylinder & pouring cone & caliberated containers of known value are used.



→ In this method the entire procedure is carried out in two stages

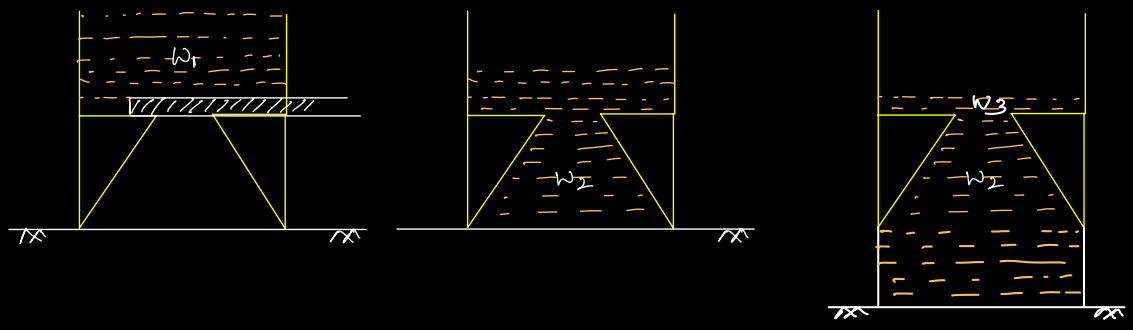
→ In the 1st stage the unit weight of sand (passing through 600μ sieve but retained over the 300μ sieve) is determined.

→ In the 2nd stage the unit wt of the given soil sample is determined.

→ Let W_1 is the total weight of sand filling the pouring cylinder.

→ Let W_2 is the wt of sand filling the pouring cone

1st Stage: Estimation of unit weight of sand



W : weight present in the container

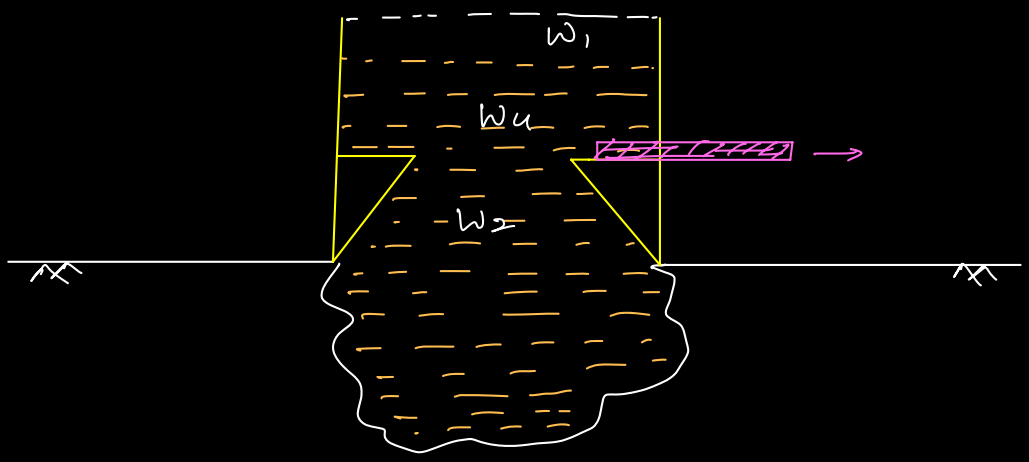
$$W = W_1 - W_2 - W_3$$

$$\gamma_{sand} = \frac{W_{sand}}{V_{sand}}$$

V_{sand} → Volume of caliberated Container.

$$= \frac{W_1 - W_2 - W_3}{\text{Total volume.}}$$

2nd Stage: Determination of unit wt of given soil.



$$W_{trench} = W_1 - W_2 - W_4$$

$$V_{trench} = \frac{W_{trench}}{\gamma_{sand}} = V_{soil}.$$

W_{soil}: Weight of excavated soil.

$$\gamma_{soil} = \frac{W_{soil}}{V_{soil}}$$

e. Rubber balloon method:

- In this method the ground is excavated and weight of soil 'W' is collected.
- In this method, the volume of excavation is determined with the help of inflated rubber balloon.

$$\gamma_{soil} = \frac{W_{soil}}{V_{soil}}$$

NOTE: In case of Hard, Granular, Gravelly soils sand replacement method can be used where core cutter method is not suitable.

Determination of SPECIFIC GRAVITY:

(i) 50 ml density bottle → All the soils (Kerosene)

(ii) 500 ml Flask } COARSE GRAIN SOILS (Water)

(iii) 900 ml Pycnometer. }

↳ The true specific gravity (G or Gs) is determined for oven dried soil sample.

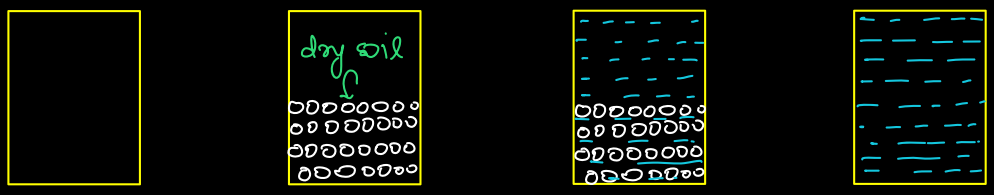
↳ 50 ml density bottle can be used for all the types of soil. The fluid used in 50ml density bottle shall be kerosene (Non-polar)

↳ 500ml flask and pycnometer are used for coarse grained soil with water as a standard fluid in the test

↳ that means for fine grained soil 50ml density bottle should be used with kerosene.

↳ if 50ml density bottle is not available, in that case flask or pycnometer can be used as standard fluid as kerosene for fine grained soil.

↳ In case of fine grained soil, kerosene is used because it has good wetting characteristics and since it is non-polar thus it can't be adsorbed on surface of fine grained soil.



$$G_s = \frac{(W_2 - W_1)}{(W_2 - W_1) + (W_4 - W_3)}$$

↳ This is the specific gravity of the soil solids with respect to the standard fluid as a water used in the test

Note: $G' \rightarrow G$

(Kerosene)

$$G = G' \times G_{Kerosene}$$

$G_{Kerosene}$ → 0.733 @ 27°C

Note: If in the 3rd stage air is not removed completely then 'w_3' comes out to be less. As a result sp. gravity will be less.

air ↑ → w_3 ↓ → G ↓

Note: Pyenometer is also used in the determination of the water content. If air is present in the pycnometer in the 3rd stage.

air ↑ → w_3 ↓ → w ↑

Question

Q.1 **Specific gravity of soil solids was determined in the lab with the following observations**

$M_s = M_2 - M_1 = 200$ gm

$M_3 = 1605$ gm

$M_4 = 1480$ gm

Determine the specific gravity of soil Grains.

In order to confirm the value, a second observation in made for M_3 after repeatedly evacuating the Pycnometer using the vaccum pump. The value of specific Gravity thus obtained was 2.72. Determine the volume of air entrapped, in the soil suspension when the first measurement was done.

(Note :- In both observations temperature is same)

Solution

M_S = 200 gram

M_3 = 1605 gm

M_4 = 1480 gm

$$G = \frac{200}{200 + (1480 - 1605)} = 2.66.$$

let the volume of voids be V.

Value of M_3,

$$2.72 = \frac{200}{200 + (1480 - M_3)}$$

$$2.72 \times 200 + 2.72 \times (1480 - M_3) = 200$$

$$M_3 = 1606.470 \text{ gm}$$

difference in mass of water = 1606.470 − 1605

Amount of water extra added after evacuating the air. = 1.470 gm

Volume of air entrapped = $\frac{1.470}{1}$ ≈ 1.470 cm^3

unit wt of water.

CONSISTENCY OF SOIL:

↳ Consistency of soil is directly related to the strength of soil based on available water content of the soil.

Consistency is term which is used to describe the

degree of firmness in the qualitative manner which is expressed as soft, very soft etc.

↳ when water is more then the degree of firmness comes out to be less

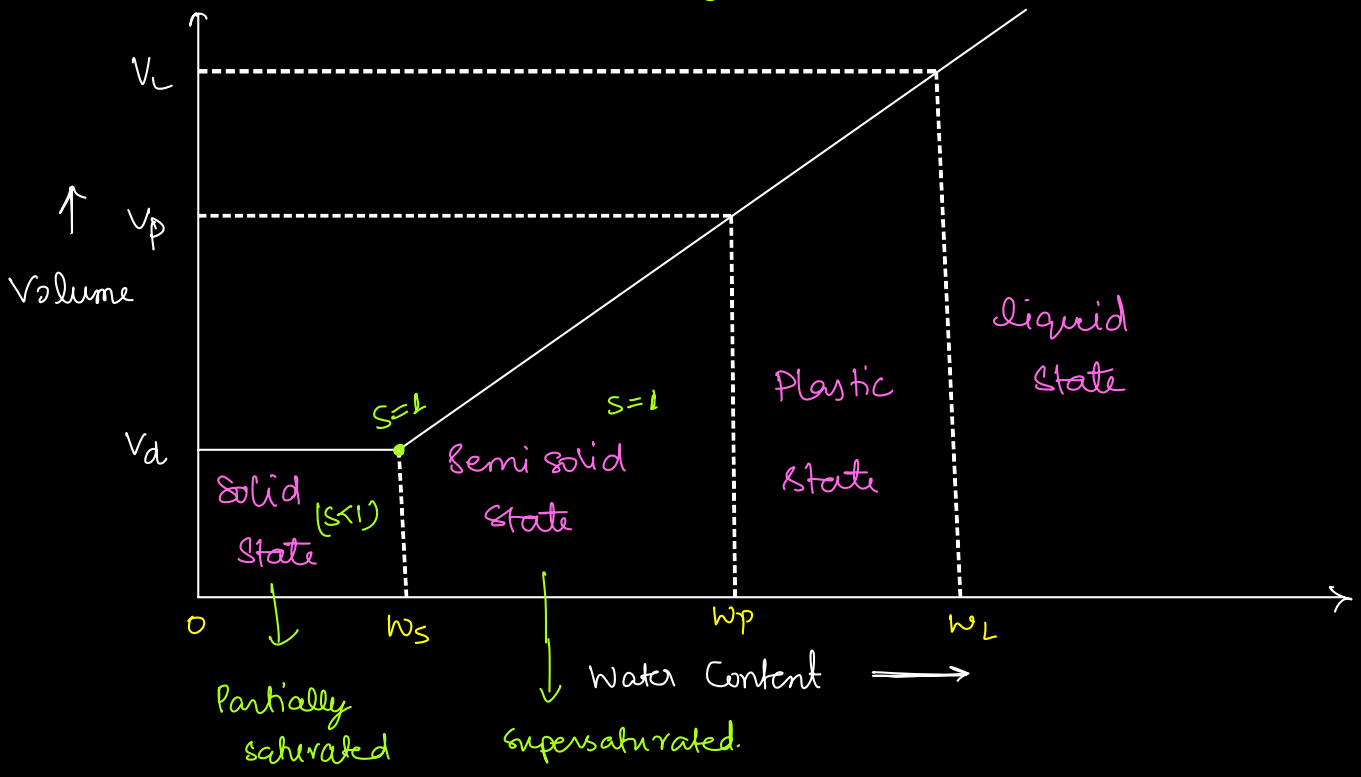
↳ Consistency of soil shall be described for fine grained soil. [Clay & Silt].

↳ On changing the water content in the soil, soil passes from one state of consistency that limit of water content is termed as consistency limits or Atterbergs limits.

↳ There are 3- limits & 4 stages of consistency.

3 consistency limits	4 - stages of consistency
(i) liquid limit (w_L)	(i) Liquid state
(ii) Plastic limit (w_p)	(ii) Plastic state
(iii) Shrinkage limit (w_s)	(iii) Semi - Solid State
	(iv) Solid state

• Consistency chart



→ If water is added to the given dry soil or partially saturated soil upto the shrinkage limit, then no change in the

volume of soil is absorbed. But beyond the shrinkage limit if water is added then volume of soil starts increasing that means at shrinkage limit, soil has become fully saturated. therefore at shrinkage limit, $S = 1$.

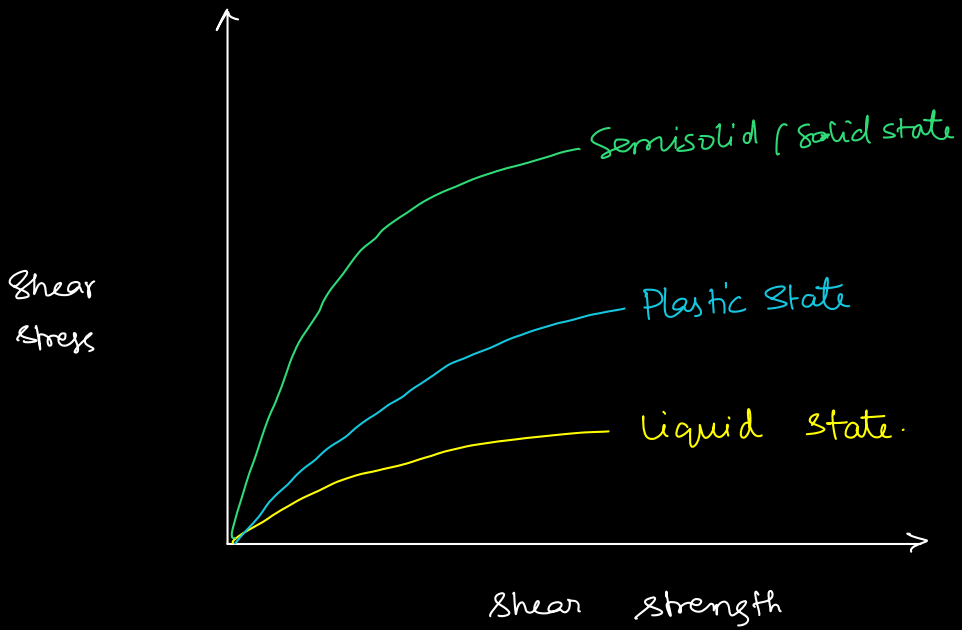
→ Since, at shrinkage limit soil is fully saturated then also if water is added then volume of soil starts increasing as volume of voids starts increasing.

→ In this state soil is termed as supersaturated. For supersat. soil, $S = 1$.

→ Beyond shrinkage limit if water content is increased then Volume of soil, void ratio, Porosity, vol. of voids increases

→ Beyond shrinkage limit if water content is reduced then Volume of soil, void ratio, Porosity, vol. of voids decreases

→ Below the shrinkage limit, if the water content is reduced it does not lead any change in the volume of soil, volume of voids, void ratio, porosity because water is being replaced by air.



✱ Determination of CONSISTENCY LIMITS:

a. Liquid limit (w_L): It is defined as the minimum water content at which soil has a tendency to flow or it can also be defined as the minimum water content at which soil

passes from plastic to liquid state.

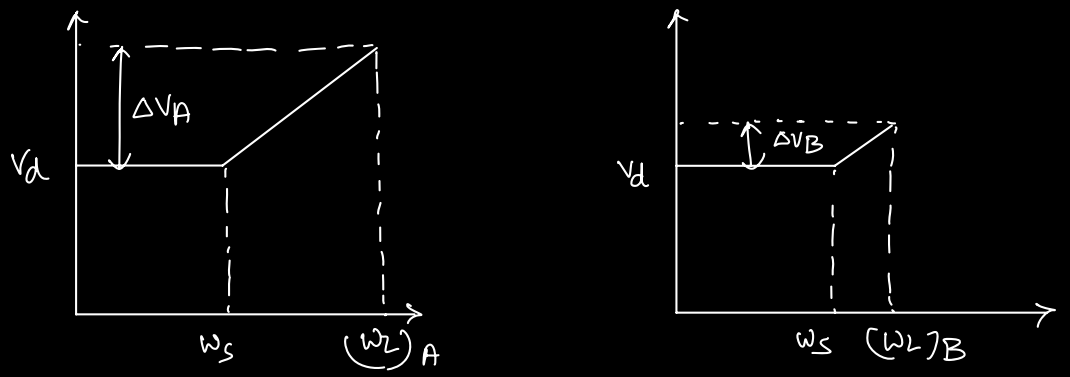
Note: At liquid limit, all the soil possesses same shear strength approximately equals to 2.7 KN/m² or 27 gm/cm²

- If liquid limit of a soil is more then compressibility will be more.

if $w_L \uparrow \rightarrow$ Volume change $\uparrow \longrightarrow$ Compressibility $\uparrow$

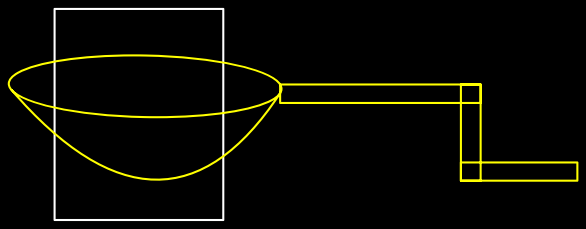
- Comparing the volume change behaviours of two soils, Soil A and Soil B with same dry volume & shrinkage limit but different liquid limits

let $(w_L)_A > (w_L)_B$



Type of Soil	w_L(%)
Gravel	Non-Plastic
Sand	"
Silt	30 - 40
Clay (Alluvial)	40 - 150
clay (Black cotton)	400 - 500
Clay (bentonite)	500 - 800

→ from the determination point of view the liquid limit can be defined as the minimum water content at which two parts of the soil cut by a groove of half inch (12.5 mm) flow together and meet under the impact of 25 Number of blows.



→ The tool which is used to cut a soil can be casagrande tool or ASTM Tool. Only there is a difference in the dimension of the groove.

Tool	Bottom Width	Top Width	height
Casagrand	2mm	11mm	8m
ASTM	2mm	13.5m	10m

→ ASTM tool is used for low plastic soils like silt and casagrand tool is used for highly plastic soil like clay

→ To determine the liquid limit take about 120gm of soil sample passing through 425 μ sieve.

→ Add sufficient amount of water to form a uniform soil paste.

→ The soil cup is filled with this soil paste and a groove of $\frac{1}{2}$ inch is made

→ Now number of blows are given at a rate of 2 revolution per second.

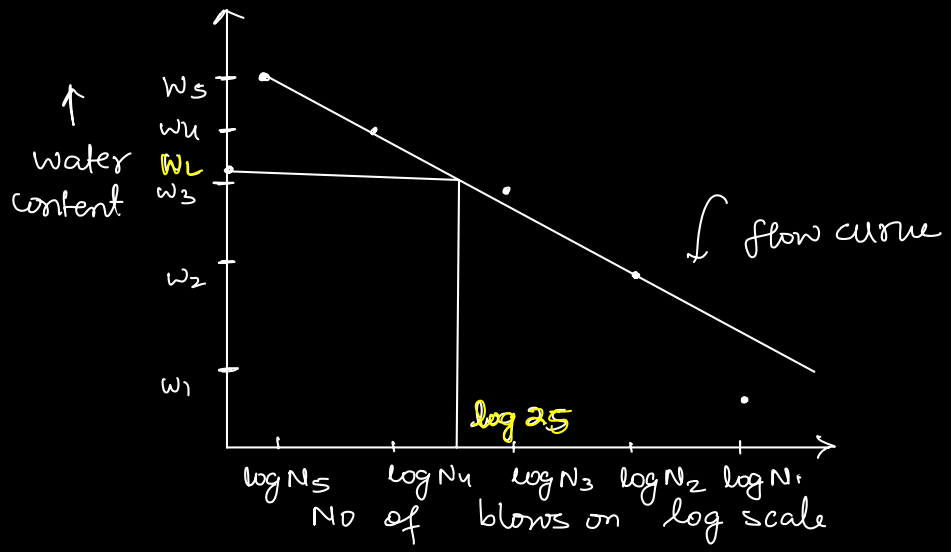
→ The height of fall is about 1 cm.

→ The number of blows required to flow the soil by $\frac{1}{2}$ inches is noted

→ Minimum 10 g of moist sample is taken from soil cup (After completing the test) closed to groove to determine the water content. from the oven drying machine.

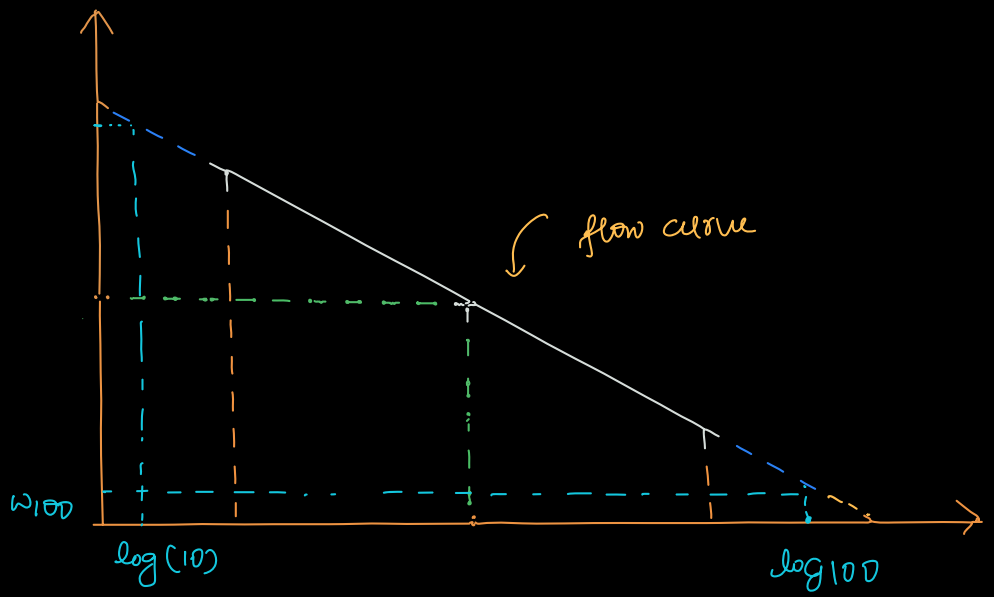
→ The test is repeated for different values of water content for which number of blows range from 20 to 50.

→ Different values of water content & corresponding number of blows a curve is drawn on semilog scale termed as flow curve.



→ The slope of the flow curve is called flow index (I_f). This flow index shows the rate of loss of shear strength on losing the water content.

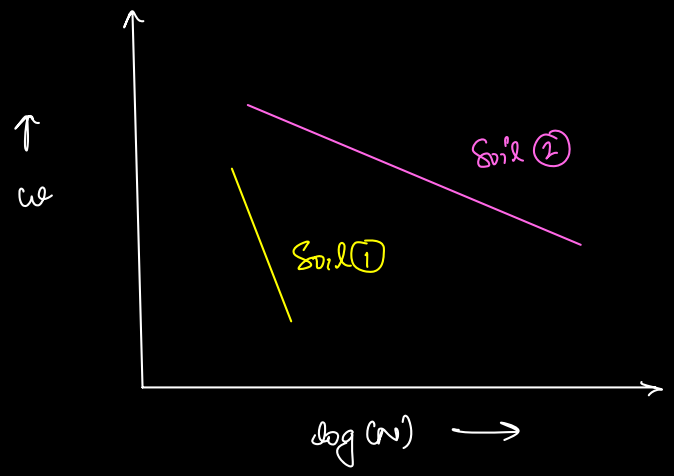
$$\tan\theta = I_f = \frac{w_2 - w_1}{\log N_1 - N_2} = \frac{w_2 - w_1}{\log\left(\frac{N_1}{N_2}\right)}$$



→ By the method of extrapolation we have estimated the water content w_{10} and w_{100} required for 10 and 100 No of blows respectively.

$$\therefore \quad I_f = \frac{w_{10} - w_{100}}{\log_{10}\left(\frac{100}{10}\right)} = w_{10} - w_{100}$$

$$\{ I_f = w_{10} - w_{100} \}$$

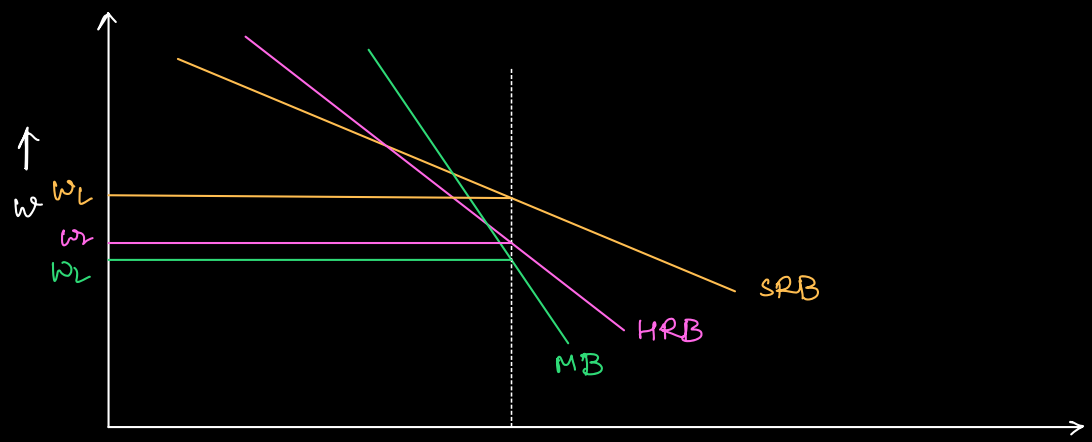


Note: $(\text{slope})_① > (\text{slope})_②$

$(I_f)_① > (I_f)_②$

$(\text{Shear strength})_① < (\text{Shear strength})_②$

If different bases are used in casagrande apparatus to determine liquid limit [soft Rubber base, Hard Rubber Base, Metallic base]:



↳ Considering constant number of blows as 25, the required water content to make the flow of two parts of the soil by $\frac{1}{2}$ inch will be more in case of soft rubber base because soft rubber base absorb more energy.

↳ considering constant value or same value of water content the requirement of the number of blows will be less in case of Metallic base and more in case of soft rubber base.

∴ As a result:

↳ (Slope) SRB < (Slope) HRB < (Slope) MB

↳ $(I_s)_{SRB} < (I_f)_{HRB} < (I_f)_{MB}$

↳ $(I_f)_{SRB} > (I_f)_{HRB} > (I_f)_{MB}$.

↳ $(w_L)_{SRB} > (w_L)_{HRB} > (w_L)_{MB}$

→ Liquid limit can be determined by one point method using cassagrande apparatus:

$$w_L = w_n \left(\frac{n}{25}\right)^x \qquad \boxed{w_n \rightarrow n.}$$

∵ empirical equation proposed by Mohan.

w_L : liquid limit

n : No of blows corresponding to w_n water content.

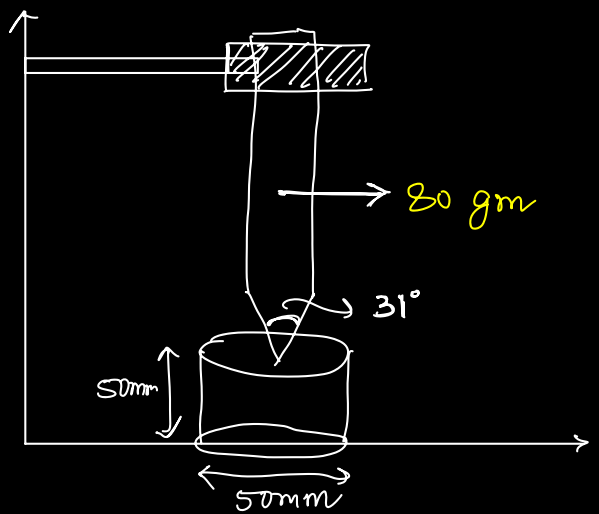
x : 0.0682 – 0.121

If $n = 20$ to 30 then $x = 0.1$

As per IS 2720 (Part 5) the w_L can also be determined by cone penetration test.

→ In this method a sliding mass of 80 gm cone penetrometer is allowed to penetrate the given moist sample to determine the liquid limit

→ The liquid limit can be defined as the water content at which penetrometer penetrates the soil sample upto depth 20mm in the cylindrical mould of D = 50 mm & H = 50 mm



→ Liquid limit can also be determined by one point method (given by IS code) using cone penetration Apparatus.

$$w_L = \frac{w_n}{0.77 \log_{10} D}$$

w_L: Liquid limit

D: Depth of penetration (in mm) corresponding to w_n water content

Note: Trial water content can be accepted for D = 15–26mm

b. Plastic limit (w_p): Plastic limit can be defined as the minimum water content at which soil reaches to plastic stage of consistency.

from determination point of view plastic limit can be defined as the minimum water content at which soil begins to crumble when rolled into threads of 3mm diameter.

→ To determine the plastic limit take 20 gm of soil from its natural state, passing through 425 μ sieve. Add sufficient amount of water to form a uniform paste.

→ Make a ball of about 8 gm and roll into a thread of 3mm diameter above the glass plates.

→ Rolling of ball is required at the rate of 80–90 strokes per min.

→ Number of observations are required with different values of water content.

→ Small amount of soil is taken from the crumbled soil and water content is determined in oven drying machine.

→ The water content at which soil begins to crumble when rolled into the 3mm diameter wire then that water content is termed as plastic limit.

→ The final result can be taken as the average of the last 3 results

Types of soils	wp (%)
Gravel	Non- Plastic
Sand	Non- Plastic
Silt	20 - 25
Clay (Alluvial)	25 - 50
Clay (Black cotton)	150 - 200
Clay (bentonite)	250 - 350

→ When natural water content in the soil is greater than plastic limit but less than liquid limit then soil exist in plastic state of consistency.

Note: In case of fine grained soil the surface area is more therefore surface electrical forces are predominating in the fine grained soil.

But in case of CGS, SSA is less therefore surface electrical forces have less significance as compared to gravity force due to self weight of solids.

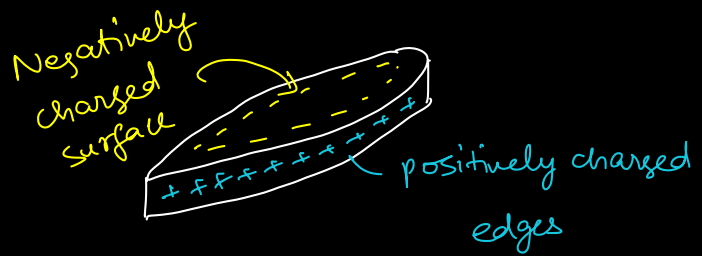
In coarse grained soils gravitational forces are predominating.

Note: In case of FGS the surface of the solid is negatively charged surface attract the cations present in the water. This force of attraction is termed as surface electrical force. As the distance increases from the surface of the solids, the conc^{n} of the cation decreases and the concentration of Anion increases. The distance upto which these cations and Anions are diffused is termed as diffused double layer.

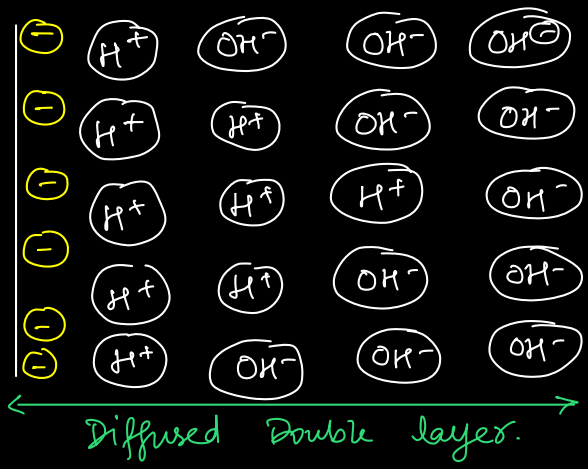
The water which is under the influence of surface electrical force will show different

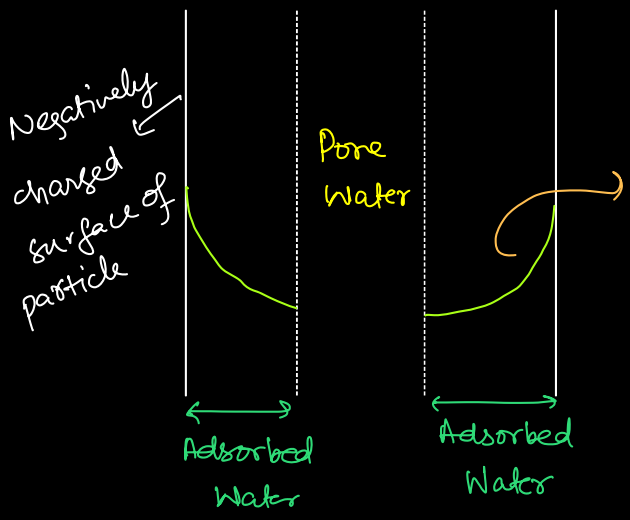
properties from the pore water pressure.

→ the adsorbed water will be highly viscous, highly dense and has boiling point and low freezing points.



clay particle which has flaky shape.





→ Influence of surface electrical force is decreasing as the distance increases from the surface of the solid.

→ the water molecules which are away from the surface of solid. can be reoriented in the adsorbed layer when the soil is deformed. that means it is providing plasticity of soil.

→ The plasticity can be defined as the property of soil by virtue of which soil can be moulded or remoulded (or deformed) without volume change or cracks or rupture.

→ If kerosene or Nepthalene are used in the place of water in the soil then soil doesn't show the plasticity because kerosene & napthalene are the Non-polar materials whereas water is a polar material.

Note: For all the soils plastic limit (w_p) is always smaller than liquid limit (w_L).

only for coarse grained soil, plastic limit is equal to liquid limit, since $w_p = w_L$ in CGS, therefore plastic state doesn't exist in CGS.

∴ Coarse grained soil are called as Non-plastic soils

C. Determination of shrinkage limit:

→ It is defined as the minimum water content at which soil becomes fully saturated when water is being added

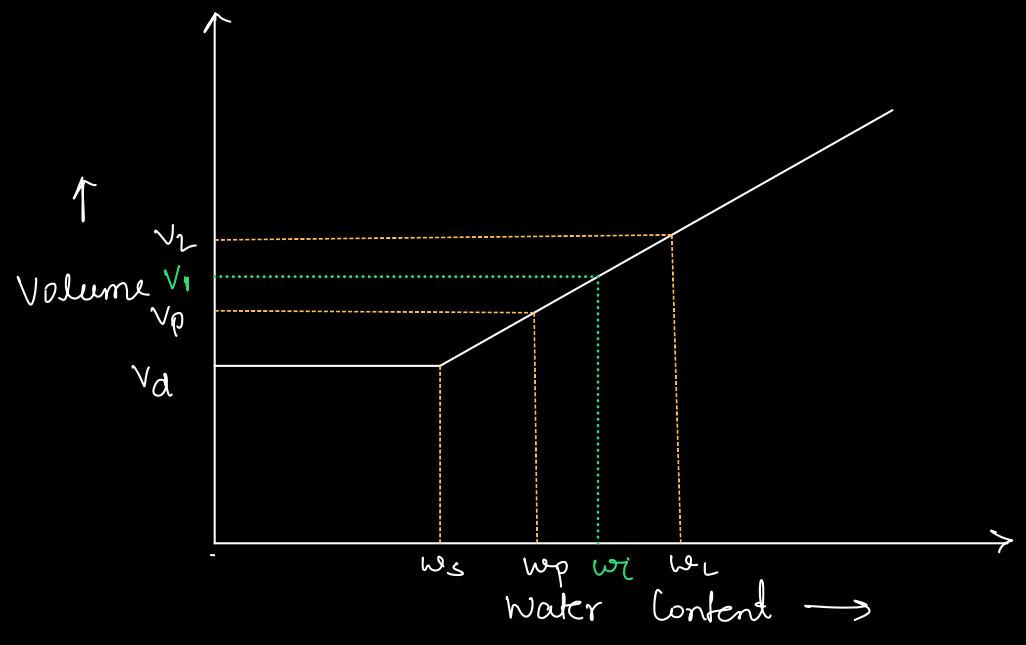
→ It can also be defined as the maximum water content below which if soil is dried then it doesn't lead to any change in the volume because water is being replaced by air.

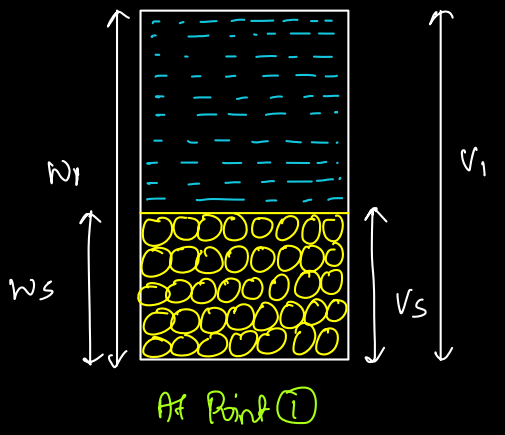
→ Below the shrinkage limit when the soil is dried no change in the volume of soil, volume of voids, void ratio and porosity is observed.

→ Shrinkage limit can also be used to express the compressibility [volume change] of soil. If the shrinkage limit is more compressibility will be less.

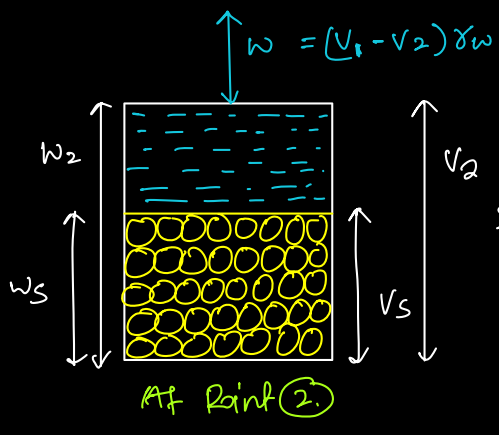
if $w_s \downarrow$ → Compressibility ↑ → Vol change ↑

if $w_s \uparrow$ → Compressibility ↓ → Vol change ↓



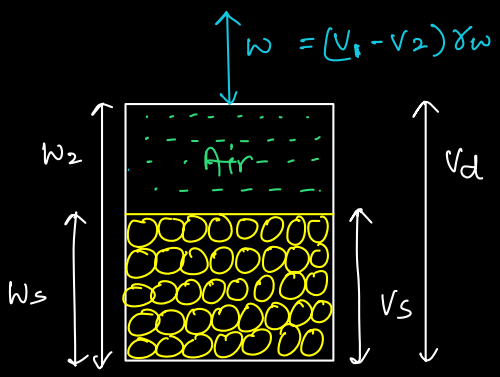


At Point (1)



At Point (2)

$\{W_2 = W_1 - (V_1 - V_2)\gamma_w\}$



At point 3

1st Approach:

$$W_s = \frac{W_w}{W_s} = \frac{(W_1 - W_s) - (V_1 - V_2)\gamma_w}{W_s}$$

$$= \left(\frac{W_1 - W_s}{W_s}\right) - \frac{(V_1 - V_2)\gamma_w}{W_s}$$

$$W_s = W_i - \frac{(V_1 - V_d)\gamma_w}{W_s}$$

→ Take 30 gram of soil sample from its natural state passing through 425 μ sieve. Add sufficient amount of water content (greater than W_s).

→ Take this sample of super saturated soil in shrinkage dish.

x_1: empty wt of shrinkage dish

x_2: shrinkage dish with supersaturated soil

$x_2 - x_1$: wt of super saturated soil (W_1)

Volume of shrinkage dish = Vol. of super saturated soil.

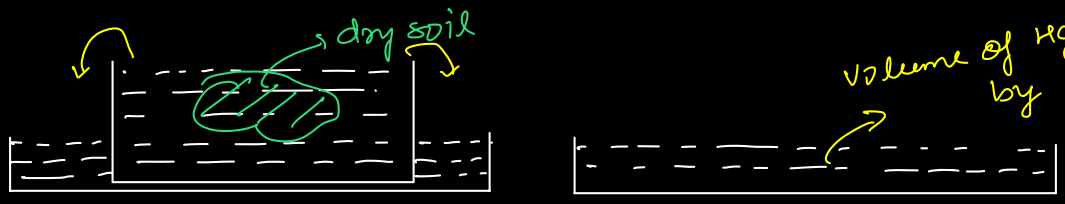
→ Now keep the shrinkage dish in the oven drying machine to determine water content

→ wt of shrinkage dish and dry soil : x_3

→ Therefore, $W_i = \frac{W_w}{W_s} = \frac{x_2 - x_3}{x_3 - x_1}$

∴ wt of solids (W_s) = $x_3 - x_1$

→ To calculated the dry volume put the oven dried sample in the shrinkage dish which is completely filled with mercury. The volume of Hg displaced by dried soil can be measured.



Vol. of dry soil (V_d) = Vol. of mercury displaced.

Note: All the required values, we have calculated, now estimate the shrinkage limit using:-

$$w_s = w_i - \left(\frac{V_1 - V_d}{W_s}\right)\gamma_w$$

2nd Approach:

$$w_s = \frac{w_w}{W_s} = \frac{(V_2 - V_s)\gamma_w}{W_s}$$

$$w_s = \frac{(V_d - V_s)\gamma_w}{W_s}$$

$$w_s = \frac{V_d\gamma_w}{W_s} - \frac{V_s\gamma_w}{W_s}$$

$$w_s = \frac{\gamma_w}{\left(\frac{W_s}{V_d}\right)} - \frac{\gamma_w}{\left(\frac{W_s}{V_s}\right)} = \frac{\gamma_w}{\gamma_d} - \frac{\gamma_w}{\gamma_s}$$

$$w_s = \frac{1}{G_d} - \frac{1}{G}$$

← Shrinkage limit

→ True specific gravity

↓ Mass specific gravity in dry state

$$G_m = \left(\frac{G+es}{1+e}\right), \quad \text{put } S=0, \quad G_d = \frac{G}{1+e}$$

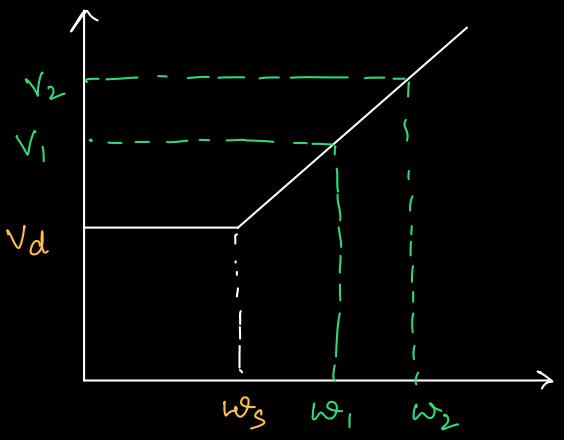
$$w_s = \frac{1+e}{G} - \frac{1}{G}$$

$w_s = \frac{e}{G}$ → e → void ratio

→ G → true specific gravity

Nothing but $es = wG$ at $S = 1$.

SHRINKAGE RATIO: It is defined as the decrease in the volume of soil expressed in the percentage of it's dry volume on reducing the water content beyond the shrinkage limit



$$R = \frac{\left(\frac{V_2 - V_1}{V_d}\right)}{(w_2 - w_1)}$$

$$w_2 - w_1 = \frac{\Delta w_w}{w_s}$$

$$w_2 - w_1 = \frac{(V_2 - V_1)\gamma_w}{w_s}$$

$$R = \frac{\left(\frac{V_2 - V_1}{V_d}\right)}{\frac{(V_2 - V_1)\gamma_w}{w_s}} = \frac{w_s}{V_d \gamma_w} = \frac{\gamma_d}{\gamma_w} = G_M$$

$R = G_m$ → Mass sp. gravity corresponding to dry soil volume.

Shrinkage Ratio

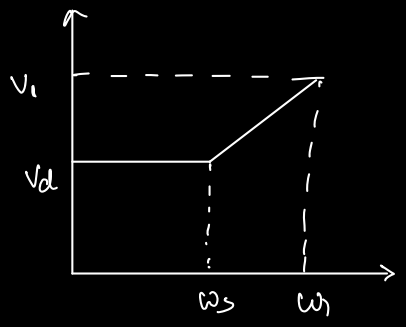
$$\therefore \quad w_s = \frac{1}{G_m} - \frac{1}{G}$$

$$\therefore \quad w_s = \frac{1}{R} - \frac{1}{G}$$

R → Shrinkage Ratio.

Volumetric shrinkage, linear shrinkage, Degree of shrinkage.

i. Volumetric shrinkage:

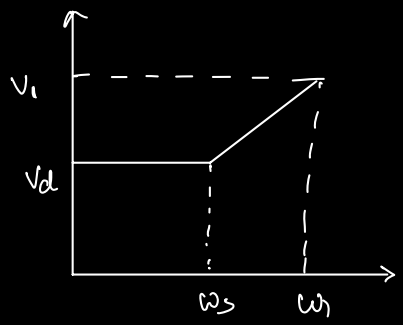


$$V_s = \left(\frac{V_1 - V_d}{V_d}\right)$$

$$V_s = R\,(w_1 - w_s)$$

→ It is defined as the decrease in volume of soil expressed in the percentage of its dry soil when water is reduced to the shrinkage limit.

(ii) Degree of shrinkage: It is defined as the decrease in volume of soil expressed in the percentage of its initial volume when water content is reduced upto the shrinkage limit.



$$DOS = \frac{V_1 - V_d}{V_1}$$

(iii) Linear Shrinkage: It is defined as the decrease is the one dimension of the soil specimen expressed in the percentage of its average dimension when water is reduced upto shrinkage limit.

$$LS = \frac{\Delta L}{L_{avg}} = \frac{L_o - L_{avg}}{L_{avg}}$$

$$LS = 100\left[1 - \left\{\frac{100}{100 + VS}\right\}^{\frac{1}{3}}\right]$$

VS: Volumetric shrinkage

L_o: initial length of specimen

L_{avg}: Avg reduced length of specimen.

Question

Q.1 A soil sample has a shrinkage limit of 10% and specific gravity of soil solids as 2.7. The porosity of the soil at shrinkage limit is [ESE : 1995]

(A) 21.2% (B) 27%
(C) 73% (D) 78.8%

Solution

$w_s = 10\%$

$G = 2.7$

$$w_s = \frac{e}{G} \Rightarrow 0.1 = \frac{e}{2.7} \Rightarrow e = .27$$

$$\eta = \frac{e}{1+e} = \frac{.27}{1.27} = .2125 \text{ or } 21.25\%$$

Question

3 The liquid limit of a soil is 40% and plastic limit is 28%. When the soil is dried from the plastic limit, the volume change is 20% of the volume at plastic limit. The corresponding volume change from the liquid limit to dry state is 30% of its volume at liquid limit.

→ (i) The shrinkage limit of the soil is
(A) 11.15% (C) 8.92%
(B) 6.74% (D) 7.29%

→ (ii) The shrinkage ratio is
(A) 2.73 (C) 1.49
(B) 1.67 (D) 2.33

Solution

$w_L = 40\%$

$w_p = 28\%$

V_p: Volume at Plastic limit

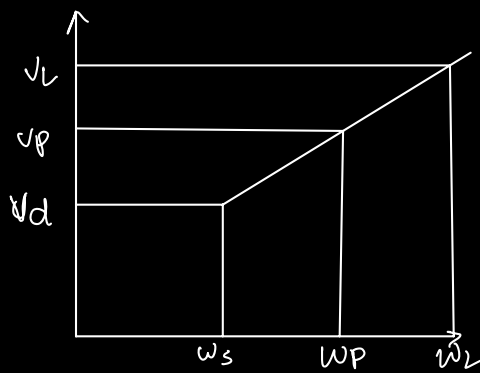
V_L: Volume at liquid limit

$$V_p - V_d = 0.2V_p$$

$$V_L - V_d = 0.3V_L$$

$$V_d = 0.8V_p \Rightarrow V_p = 1.25V_d$$

$$V_d = 0.7V_L \Rightarrow V_L = 1.43V_d$$



1st observation:

$$w_s = w_i - \left(\frac{V_i - V_d}{w_s}\right)\gamma_w$$

$$w_s = w_p - \left[\frac{V_p - V_d}{w_s}\right]\gamma_w$$

$$w_s = w_p - \left(\frac{.25 V_d}{w_s}\right)\gamma_w \quad \text{— (II)}$$

$$w_s = w_L - \left(\frac{.43 V_d}{w_s}\right)\gamma_w \quad \text{— (I)}$$

equating (I) & (II):

$$w_p - \left(\frac{.25 V_d \gamma_w}{w_s}\right) = w_L - \left(\frac{.43 V_d}{w_s}\right)\gamma_w$$

$$w_L - w_p = \frac{.18 V_d \gamma_w}{w_s}$$

$$.12 = .18 \frac{V_d \gamma_w}{w_s}$$

$$\left\{\frac{V_d \gamma_w}{w_s} = \frac{2}{3}\right\} \quad \text{— (III)}$$

Put in (I):-

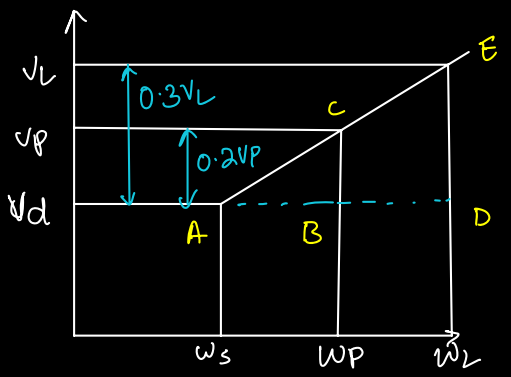
$$w_s = .4 - 0.43 \times \frac{2}{3} = 0.1133$$

or 11.33%. Ans

(ii) Shrinkage Ratio:

$$R = \frac{0.2 \times 1.25 V_d}{\frac{V_d}{.28 - .1133}} = 7.45 \times 0.2 = 1.5$$

2nd Method:



ΔABC & ΔADE:

$$\frac{CB}{BA} = \frac{ED}{DA}$$

$$\frac{0.2 V_p}{w_p - w_s} = \frac{0.3 V_L}{w_L - w_s}$$

$$\frac{0.2 \times 1.25 V_d}{w_p - w_s} = \frac{0.3 \times 1.43 V_L}{w_L - w_s}$$

$$0.2 \times 1.25 (0.4 - w_s) = 0.3 \times 1.43 (0.28 - w_s)$$

$$\Rightarrow \quad -0.179 w_s = -0.02 \Rightarrow w_s = 0.1117$$ Ans

Consistency Indices:

① Plastic index (I_p): Plastic Index is defined as the difference between liquid limit and plastic limit

↳ Plastic limit is used to express the range of consistency when the soil exists in the plastic state.

$$I_p = w_L - w_p$$

↳ The plasticity of soil depends upon the type & amount of clay minerals

↳ Following minerals have been arranged in increasing order of plasticity

Kaolinite < Illite < Montmorillonite

I_p (%)	Description
0	Non-plastic soil
<7	low plastic soil
7-17	Medium Plastic soil
>17	Highly plastic soil.

↳ For all the soil plastic limit is less than liquid limit w_L. But in case of CGS only w_p & w_L equal therefore there I_p is reported as zero and they are termed as Non-plastic soils.

↳ If in any case, $I_p < 0$, I_p reported as zero.

→ When a non plastic soil, sand is mixed with a plastic soil clay, then the plasticity of the plastic soil decreases. Because w_L & w_p both decreases the decrease in liquid limit is more than the decrease in plastic limit, thus plastic limit decreases

→ If two soils of different plasticity Indices are mixed then the plasticity index of the New soil (mixture) is estimated as:

$$I_{pmix} = \frac{(I_p)_1 x_1 + (I_p)_2 x_2}{(x_1 + x_2)}$$

→ liquid limit > 100% is rarely to be seen in case of inorganic soils

→ The plasticity index (or plasticity) of organic soil is generally less because for organic soil liquid limit and plastic limit both are very high. Generally $w_L > 50\%$ for organic soil as a result they are highly compressible

$$(I_p)_{inorganic} > (I_p)_{organic}$$

→ If two soils of same plasticity index are compared then that soil will be considered as more plastic which has less value of liquid limit.

for ex:

Soil (A)	Soil (B)
$(w_L)_A = 80\%$	$(w_L)_B = 60\%$
$(w_P)_A = 60\%$	$(w_P)_B = 40\%$
$(I_p)_A = 20\%$	$(I_p)_B = 20\%$

Soil (B) will be more plastic due to less w_L thus having less compressibility

Note: The plasticity (or plasticity index) is the measure of dry strength of the soil. If I_p is more dry strength will be more. In this case liquid limit can be used to express the dry strength and compressibility of soil.

∴ for the soil (B)

$w_L \downarrow \rightarrow$ Compressibility $\downarrow \rightarrow$ dry strength $\uparrow \rightarrow$ Plasticity $\uparrow$

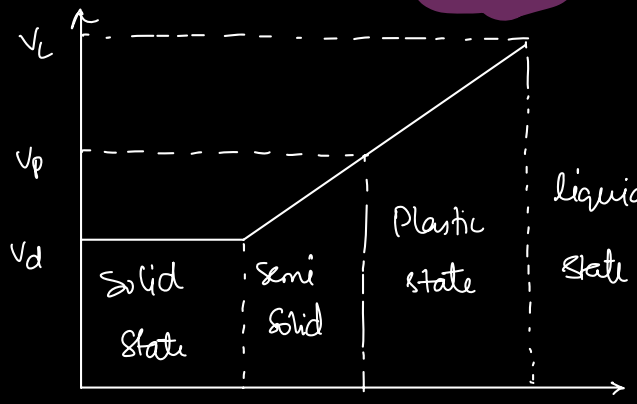
(2) Shrinkage Index (I_s): It can be defined as the difference between the plastic limit and shrinkage limit.

$I_s = w_p - w_s$

→ Shrinkage index is used to express, the consistency of the soil when the soil exists in semi-solid state

(3) Consistency Index (I_c): It is used to express the degree of firmness of the fine grained soil. It is also termed as relative consistency. It can be given as

$$I_c = \frac{w_L - w}{w_L - w_p} \quad \text{or} \quad I_c = \frac{w_L - w}{I_p}$$



$w_L \Rightarrow 60\%$

$w_p \Rightarrow 40\%$

$w \Rightarrow 20\%$

$$I_c = \frac{60-20}{60-40} = 2$$

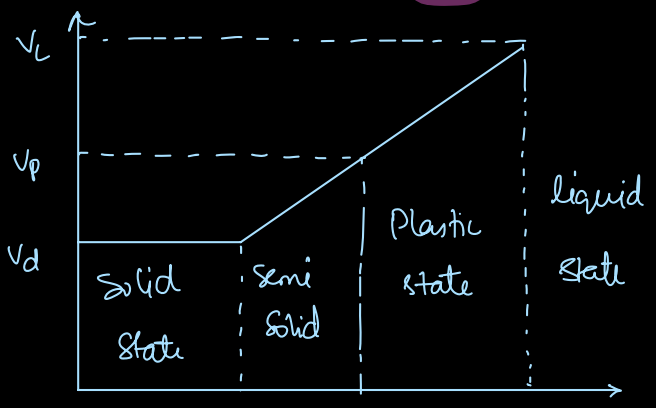
If $w > w_L$: liquid state ; $I_c < 0$: LIQUID STATE

If $w_p \le w \le w_L$: plastic state ; $0 \le I_c \le 1$: PLASTIC STATE

if $w < w_p$; semisolid or solid ; $I_c > 1$: SOLID STATE

(4) Liquidity Index: Also used to express the degree of firmness of soil

$$I_L = \frac{w - w_p}{w_L - w_p} \quad \text{or} \quad I_L = \frac{w - w_p}{I_p}$$



if $w > w_L$: liquid state : $I_L > 1$

if $w_p \le w \le w_L$: Plastic state : $0 \le I_L \le 1$

if $w < w_p$: semisolid / solid : $I_L < 0$.

↳ we want higher value of consistency index and lower value of liquidity index.

$\{ I_c + I_L = 1 \}$

- State of consistency:

State of consistency	Description	I_c	I_L	UCS (KN/m²)
Liquid	fluid (liquid)	< 0	> 1	—
Plastic	very soft	0 - 0.25	0.75 - 1.0	< 25
	soft	0.25 - 0.50	0.50 - 0.75	25 - 50
	Medium stiff	0.50 - 0.75	0.25 - 0.50	50 - 100
	stiff	0.75 - 1.0	0 - 0.25	100 - 200
Semi solid	Very stiff to Hard	> 1	< 0	200 - 400
Solid	Hard to very Hard	> 1	< 0	> 400.

→ Unconfined compressive strength (UCS): It is defined as load as the load per unit area at which unconfined soil sample fails under the simple compression test.

---

(5) Toughness Index (I_T): It can be defined as the ratio of plastic index to the flow index.

$$I_T = \frac{I_P}{I_f}$$ where I_f → flow index.

→ The toughness index is used to express the strength of the soil at the plastic limit.

→ The value of Toughness Index varies from 0 to 3 ($0 \le I_T \le 3$)

→ If I_T is less that 1 for the given soil, then that soil can be easily crumbled upon remoulding

→ If $I_f \downarrow$ → $I_T \uparrow$ → shear strength at plastic limit.

General relationship between Atterberg limits and different engineering properties:

Eng properties	Comparing soils at same w_L but increasing I_P	Comparing soils at same I_P but increasing w_L.
* Dry strength	INCREASES	DECREASES
* Toughness at plastic limit	INCREASES	DECREASES
* Compressibility	CONSTANT	INCREASES
* Permeability	DECREASES	INCREASES
* Rate of volume change.	DECREASES	INCREASES.

SENSITIVITY & THIXOTROPHY:

The consistency of the soil can be altered without changing the water content by remoulding. This change in the state of consistency causes the loss of of strength of the soil. This loss of strength is due to:

1. Permanent destruction of soil solids during remoulding
2. Reorientation of water molecules in the absorbed layer of the soil solids.

- This loss of strength can be expressed in the terms of sensitivity which can be defined as the ratio of unconfined compressive strength of undisturbed soil to the unconfined comp. strength of remoulded soil.

$$\text{Sensitivity} = \frac{(\text{Unconfined compressive strength})_{\text{undisturbed}}}{(\text{Unconfined compressive strength})_{\text{disturbed}}}$$

$$S = \frac{[UCS]_{\text{undisturbed}}}{[UCS]_{\text{Remolded}}}$$

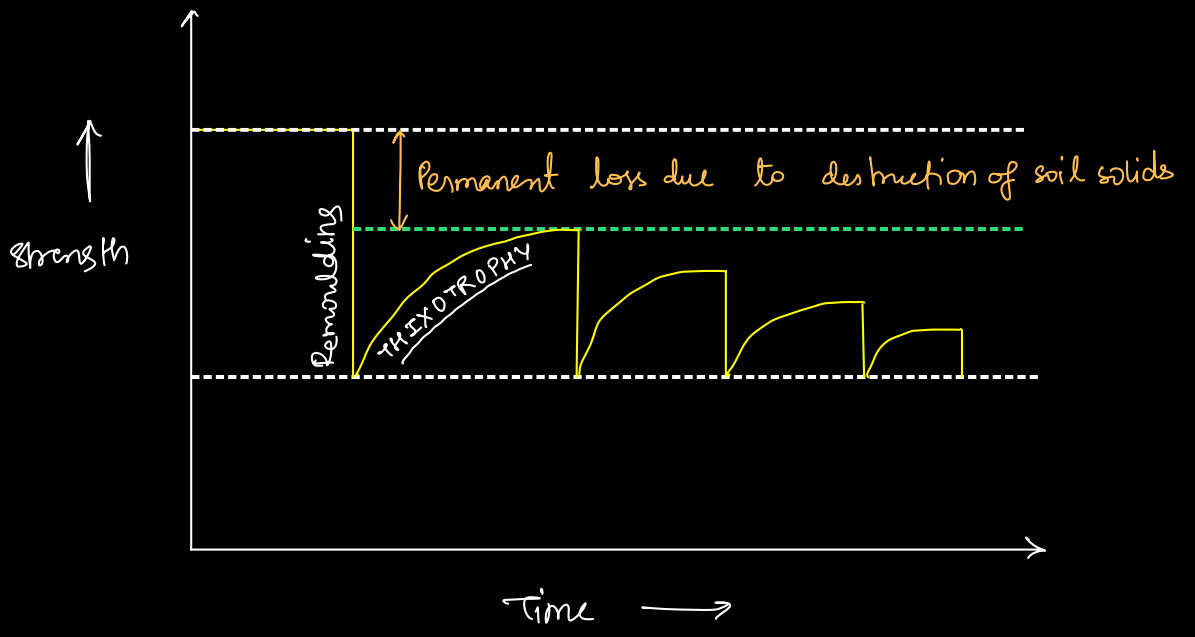
SENSIVITY	DESCRIPTION
1	No loss of strength
1 - 2	low sensitive soil (Honeycomb)
2 - 4	Medium sensitive soil (Honeycomb)
4 - 8	sensitive soil (Flocculated / Honeycomb)
8 - 16	Extrasensitive soil (Flocculated)
>16	Quick soil or unstable soil.

Note: In general sensitivity greater than or equal to 1. But in such case sensitivity can be less than 1 in the stiff clay soil having cracks or fissures. During remoulding these weakness are removed from the given soil as a result sensitivity comes out to be less than 1.

Thixotrophy: For a long period of time if this remoulded soil is not disturbed then over this long period some part of lost strength is regained due to rehabilitation of water molecules in the absorbed layer of the soil solids. But the loss of strength occured due to the permanent distruction of solids cannot be regained.

This property of regaining a part of lost strength over a period of time is termed as "thixotrophy".

→ Higher the sensitivity, higher will be the thixotrophy.
The soil having flocculated / dispersed structure can have higher sensitivity whereas soils having coarse grained structure can have less sensitivity.

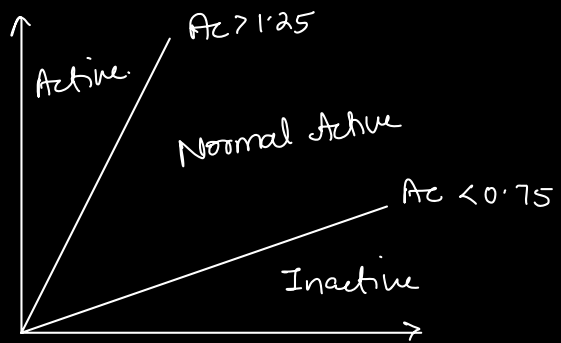


ACTIVITY:

Skempton have analysed the volume change behaviour of the soil on changing the water content in the soil. This volume change behaviour of the soil is expressed in terms of a parameter known as Activity. This activity of the soil depends upon the plasticity index and % of clay sized particles. The plasticity of the soil depends upon the type of minerals, amount of minerals and amount of water content present in the soil. The activity can be defined as the ratio of I_p to the % of clay fraction.

$$A_c = \frac{(I_p)\ (\%)}{\%\ \text{clay}}$$

A_c	Description
<0.75	Inactive
$0.75 - 1.25$	Normal Active
>1.25	Active



TYPES OF CLAY MINERALS	ACTIVITY (Ac)
Kaolinite	0.4 - 0.5
Illite	0.5 - 1.0
Montmorillonite	1 - 7
Na - Montmorillonite	4 - 7
Ca - Montmorillonite	1.5
Quartz	0

Note: Quartz mineral is found in sands which is Non-plastic in nature. Therfore its activity is zero

Note: The property "Activity" is typically defined for clay soils only (Not even for silts).

Note: Kaolinite: least Active

Illite: Moderately Active

Montmorillonite: Most Active.

Note: if Ac ↑ → volume change ↑ → swelling & shrinkage ↑

Note: when water is added → swelling is observed

when water is removed → shrinkage is observed

Note: Swelling: increase in volume of soil

Shrinkage: decrease in volume of soil.

Collapsibility: The soil which shows large decrease in volume on increasing the water content is called collapsible soil & this property knowns as collapsibility.

The collapsibility is measured in terms of collapse potential CP

These collapse potential can be defined as the ratio of change in volume of soil to the initial volume of soil

$$Cp = \frac{\Delta V}{V_o}$$

$\Delta V \rightarrow$ change in volume [decrease in volume]

$V_0 \rightarrow$ initial volume of soil.

∵ Change in volume of soil = change in volume of voids

$\therefore \quad \Delta V = \Delta V_v = \Delta e \, V_s$

$\Rightarrow \quad \Delta V = \Delta e \cdot \dfrac{V_0}{(1+e_0)}$

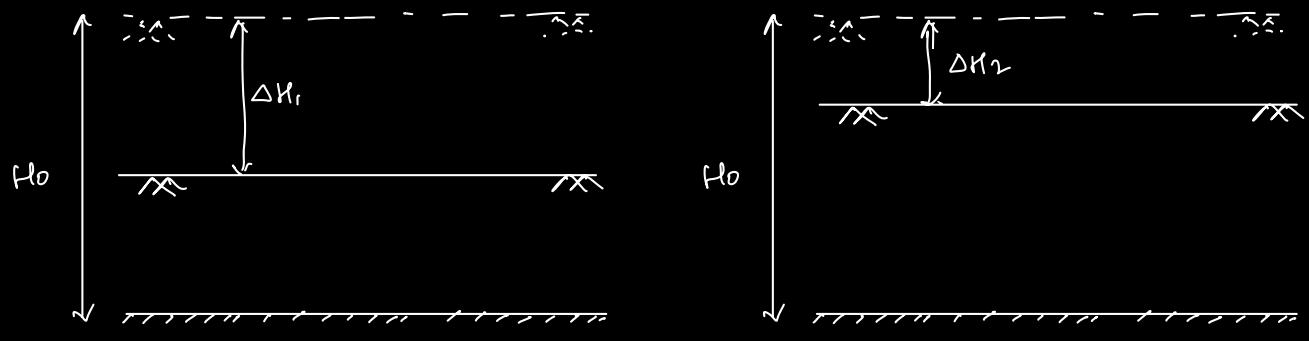
$\Rightarrow \quad \dfrac{\Delta V}{V_0} = \dfrac{\Delta e}{1+e_0} = \dfrac{\Delta H}{H_0}$

$$\left.\begin{aligned} C_p = \frac{\Delta V}{V_0} = \frac{\Delta H}{H_0} = \frac{\Delta e}{1+e_0} \end{aligned}\right\}$$ THREE FORMULAS FOR COLLAPSE POTENTIAL.

C_p	Effect on structure
<1	No trouble
1-5	Moderate trouble
5-10	Trouble
10-15	Severe trouble
15-20	Very Severe trouble.

Note: Loess soil is the best example of collapsible soil.

Soil (1) ✓ Soil (2).



$C_{p_1} = \dfrac{\Delta H_1}{H_0} > C_{p_2} = \dfrac{\Delta H_2}{H_0}$

∵ $C_{p_1} > C_{p_2}$

$\therefore$ (Collapsibility)$_1$ > (Collapsibility)$_2$

DENSITY INDEX / RELATIVE DENSITY: It can be defined as the ratio of difference between maximum void ratio & natural void ratio to the difference between maximum and minimum void ratio.

→ Density index is used to express the degree of denseness or compactness or packing of solids

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}}$$

e_{max} : max. void ratio in loosest state

e_{min} : min. void ratio in densest state

e : void ratio in natural state

If $e = e_{max}$; then soil will be in loosest state, $I_D = 0\%$

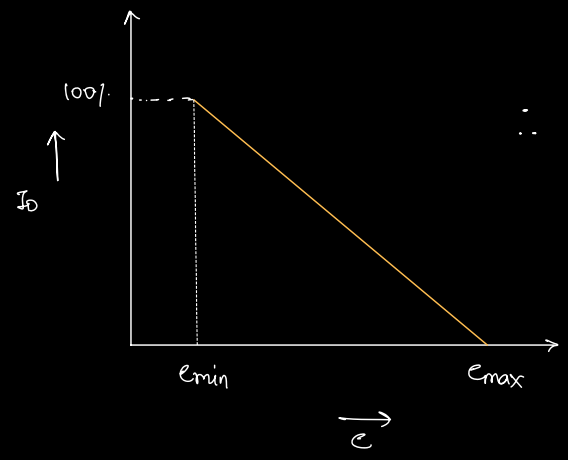
if $e = e_{min}$; then soil will be in densest state, $I_D = 100\%$

$$\{\text{Range} \rightarrow 0 \leq I_D \leq 100\%\}$$

Note: the formula of density index can be expressed in the terms of dry density also,

$$\because \quad \gamma_d = \frac{G\gamma_w}{1+e}$$

$$\therefore \quad e = \frac{G\gamma_w}{\gamma_d} - 1$$



$$e_{max} = \frac{G\gamma_w}{\gamma_{dmin}} - 1 \qquad \{e \rightarrow max,\ \gamma_d \rightarrow min\}$$

$$e_{min} = \frac{G\gamma_w}{\gamma_{dmax}} - 1$$

Thus,

$$\frac{\frac{G\gamma_w}{\gamma_{dmin}} - \frac{G\gamma_w}{\gamma_d}}{\frac{G\gamma_w}{\gamma_{dmin}} - \frac{G\gamma_w}{\gamma_{dmax}}} = \frac{\frac{1}{\gamma_{dmin}} - \frac{1}{\gamma_d}}{\frac{1}{\gamma_{dmin}} - \frac{1}{\gamma_{dmax}}}$$

$$\left\{ I_D = \frac{\frac{1}{\gamma_{dmin}} - \frac{1}{\gamma_d}}{\frac{1}{\gamma_{dmin}} - \frac{1}{\gamma_{dmax}}} \times 100. \right\}$$

I_D	Description
<15	very loose
15-35	Medium
35-65	loose
65-85	dense
>85	very dense.

→ The importance of density index I_D for CGS is same as that of consistency index in case of FGS

→ Void ratio (e) and dry density (γ_d) both are used to express the degree of denseness of the given soil in the qualitative manner whereas density index (I_D) express the degree of denseness in absolute manner

→ I_D is more significant than e and γ_d.

→ Two diff. soils of same void ratio or dry density are compared then they can show entirely different behaviour / packing / denseness and Engg. properties.

whereas two different soils with same (or equal) density index can show almost same properties thus I_D is more significant parameter.

→ I_D is calculated to show degree of denseness of coarse grain [cohesionless] soils. It is not estimated for fine grain [cohesive soils]. Because in the estimation of I_D, e_{max} is required where large uncertainities are present in it's estimation In FGS in its loosest state.

RELATIVE COMPACTION : It can be defined as the ratio of dry density of soil in natural state to the maximum dry density of soil in the densest state.

$$R_e = \frac{\gamma_d}{\gamma_{dmax}} \quad \text{or} \quad R_c = \frac{\frac{G\gamma_w}{1+e}}{\frac{G\gamma_w}{1+e_{min}}} = \frac{1+e_{min}}{1+e}.$$

$$\left\{ R_c = \frac{1+e_{min}}{1+e} \right\}$$

→ Relative compaction is used to express the degree of compactness, packing of particles & denseness of soils for both cohesion and cohesionless soil

→ That means it can be estimated for a coarse grain & fine grain soils both.

FGS → cohesive (clay & silt)

CGS → cohesionless (sand & Gravel)

→ Relative compaction can be correlated as density index empirically

$$R_c = 80 + 0.2\, I_D$$

In loosest state, $I_D = 0$, $R_c = 80\%$

In densest state, $I_D = 100\%$, $R_c = 100\%$.

Note: Relative compaction can be estimated for FGS because void ratio of loosest state cannot be determined (e_{max}) which is required in determination of I_D, which have a lot of uncertainities in case of FGS.

Question

Q.1 A given cohesionless soil has $e_{max} = 0.85$ and $e_{min} = 0.50$. In the field, the soil is compacted to a mass density of 1800 kg/ m³ at a water content of 8%. Take the mass density of water as 1000 kg/m³ and G_s as 2.7. The relative density (in %) of the soil is

(A) 56.43 (B) 60.25
(C) 62.87 (D) 65.71 ✓

[GATE 2014 : IIT Kharagpur]

Solution

$e_{max} = 0.85$

$e_{min} = 0.50$

$\gamma = 1800 \text{ Kg/m}^3, \quad w = .08$

$$\gamma_d = \frac{\gamma}{1+w} = \frac{1800}{1.08} = 1666.67 \text{ Kg/m}^3$$

$$\gamma_d = \frac{G\gamma_w}{1+e}$$

$$e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.7 \times 1000}{1666.67} - 1 = 0.62.$$

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}} = \frac{0.85 - 0.62}{0.85 - 0.50} \times 100 = 65.71\%.$$

(DENSE SOIL)

Question

A fine-grained soil has 60% (by weight) silt content. The soil behaves as semi-solid when water content is between 15% and 28%. The soil behaves fluid-like when the water content is more than 40%. The 'Activity' of the soil is

(A) 3.33 (B) 0.42
(C) 0.30 ✓ (D) 0.20

[GATE 2015 : IIT Kanpur]

Solution

60% Silt, 40% clay

$w_s = 15\%, \quad w_p = 28\%$

$w_L = 40\%$

$I_p = 40 - 28 = 12\%$

$$\text{Activity} = \frac{12}{40} = 0.3$$

PARTICLE SIZE DISTRIBUTION

In the particle size distribution the range of particle sizes present in the soil are analysed which heeps in the identification or classification of soil. The particle size distribution is carried out in two stages :

(i) Sieve Analysis [For CGS]

(ii) Sedimentation Analysis [For FGS]
- Pipette Analysis
- Hydrometer Analysis

Type of Soil	Particle Sizes	
Clay	$d < 2\mu$	FGS
Silt	$2\mu < d < 75\mu$	FGS
Sand	$75\mu < d < 4.75mm$	CGS.
Gravel	$4.75mm < d < 80mm$	CGS.

SEIVE ANALYSIS: Sieve Analysis is carried out for CGS for which particle size varies from 75μ to 80 mm.

There are two types of sieve Analysis:

(i) Coarse Sieving [Gravel]
$4.75mm < d < 80mm$

(ii) Fine Sieving. [Sand]
$75\mu < d < 4.75mm$

IS 460: 1962

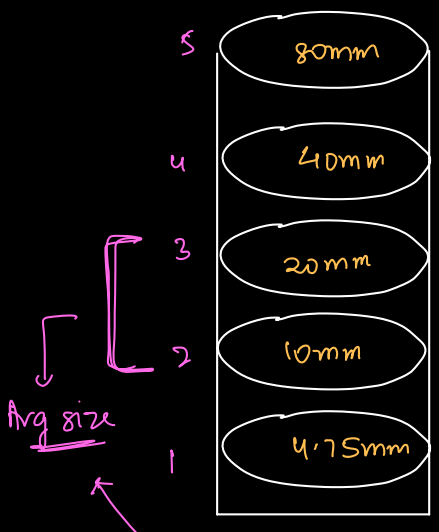
→ As per IS Code 460: 1962 the sieves are designated by their sieve size of square openings

→ In the sieve Analysis, the sieves are arranged in descending order in vertical plane from top to bottom.

→ For sieve Analysis oven dried soil sample shall be used

→ The number of sieves are shaked together either manually or with the help of sieve shaker.

(i) Coarse Sieving: It is done for gravel [$4.75mm < d < 80mm$]



Sieve Sizes	Weight retained	% weight retained	Cumulative wt retained	% finer
80mm	50 gm	5%	5%	95%
40mm	250 gm	25%	30%	70%
20mm	350 gm	35%	65%	35%
10 mm	150 gm	15%	80%	20%
4.75mm	200 gm	20%	100%	0%

% finer = 100% − (cumulative wt retained)

$$\text{fineness modulus} = \frac{\text{Sum of cumulative wt retained}}{100}$$

$$= 2.8.$$

From the table;

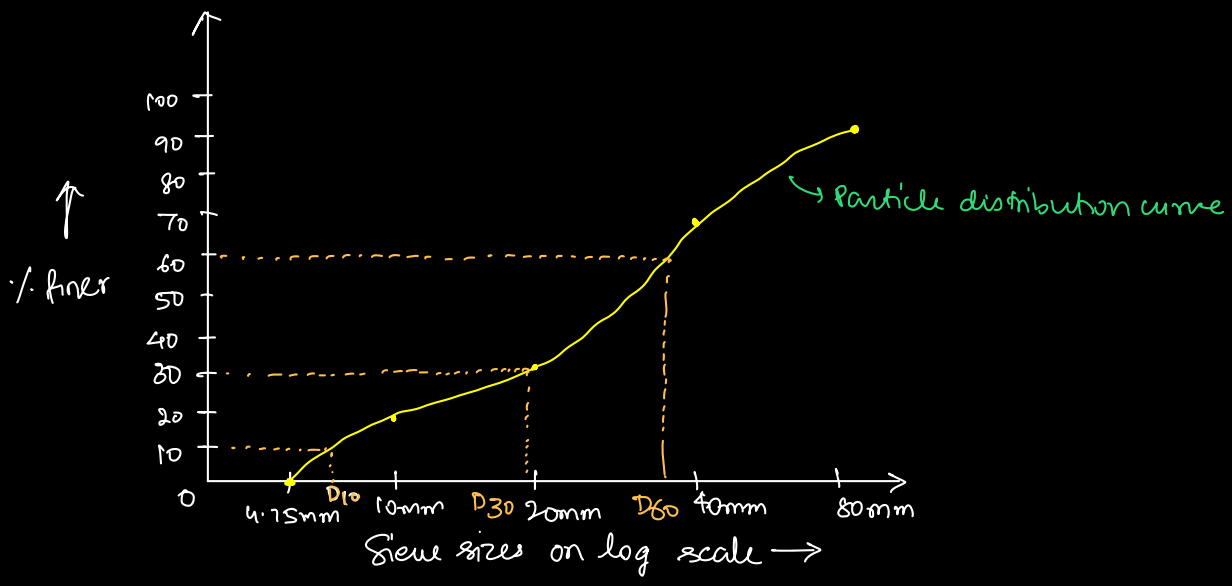
$D_{70} = 40mm$ $D_{60} = ?$

$D_{35} = 20mm$ $D_{30} = ?$

$D_{20} = 10mm$ $D_{10} = ?$

$D_{70} = 40mm$; means in the given soil sample 70% particles have sizes less than 40mm.

- Particle size distribution graph:



Note: In sieve analysis sieve are arranged or taken in Geometric progression.

for ex: 4.75mm, 10mm, 20mm, 40mm, 80 mm

(ii) Fine Sieving: It is used for sandy soils of particle size varying from 75μ to 4.75mm. It is also termed as wet sieving.

→ In the fine sieving, following sieves are arranged in descending order from top to bottom

→ Before sieving, the given sample of sand is washed to remove the clay and silt particles present over the surface of the sand particles

→ To wash the sandy soil 2gm of sodium hexametaphosphate $(NaPO_3)_6$ is added in 1 litre of water.

Sodium hexametaphosphate - Deflocculant Reagent.

↳ Trade Name - CALGON.

→ Now the sieving of sand can be started after oven drying process and particle size distribution curve can be drawn.

→ from the particle distribution curve we can determine the D_{10}, D_{30}, and D_{60} same as that of coarse sieving

D_{10} : It is the sieve size or particle size through which 10% material is finer. It is also termed as effective size, mean size or Average size of the particle.

D_{30} : It is the sieve size or particle size through which 30% material is finer.

D_{60} : It is the sieve size or particle size through which 60% material is finer.

Note: $D_{10} < D_{30} < D_{60}$.

→ D_{10}, D_{30}, D_{60} are the highly significant sizes in the soil sample because they are used to estimate two important shape factors which represents the characteristics of particle size distribution curve. On the basis of these shape factors gradation of soil sample can be done

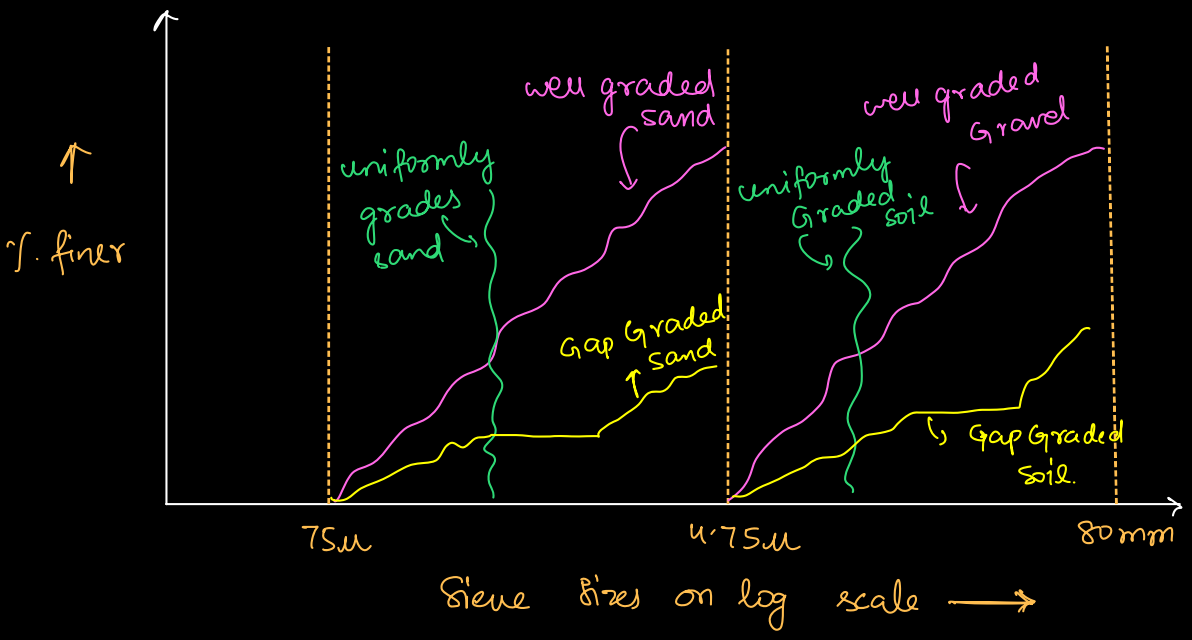
GRADATION OF SOIL:

1. Well Graded: when wide range of particle sizes are present then it is termed as well graded soil.
2. Poorly graded: when small range of particle sizes are present then it is termed as poorly graded soil.
3. Uniformly graded: When all the particles in a soil is having same size.

4. **Gap Graded**: when in the given wide range of particles some particular sizes of particles are missing from the soil then it is termed as gap graded soil.

Note: These gradations are given for coarse grained soils.

Note: The gradation of soil is decided based on the sheape factors & particle size distribution curves.



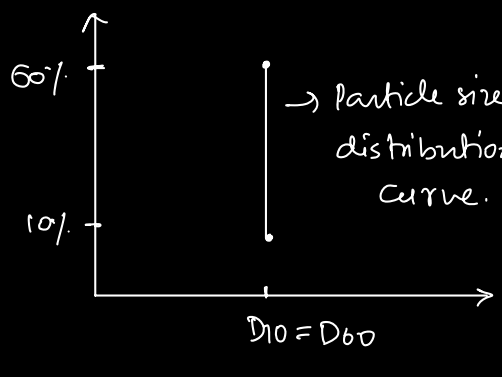
- There are two types of shape factors:

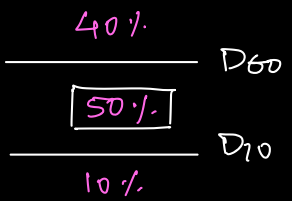
(i) **Coefficient of uniformicity [C_u]**: It is used to express the range of particle sizes in the particle size distribution curve. It is defined as the ratio of D_{60} to D_{10}.

$$C_u = \frac{D_{60}}{D_{10}} \quad \text{[unitless]}$$

D_{10} → effective / Mean / Average size of particles

→ if $C_u = 1$; $D_{60} = D_{10}$; uniformly graded soil.





→ if $D_{60} = D_{10}$ that means 50% particles in the given soil have equal size. Therefore it can be treated as uniformly graded soil. The particle size distribution curve will be vertical.

For Gravel:

$C_u \geq 4$ GW [well graded gravel]

$C_u < 4$ GP [Poorly graded gravel]

For Sand:

$C_u \geq 6$ SW [well graded sand]

$C_u < 6$ SP [Poorly graded sand]

(iii) Coefficient of curvature (C_c): It represents the shape of the particle size distribution curve, it can be defined as the ratio of square of D_{30} to the product of D_{60} & D_{10}.

$$C_c = \frac{D_{30}^2}{D_{60} \times D_{10}}$$ (unitless)

(i) For Gravel:

if $1 \leq C_c \leq 3$; well graded Gravel [GW]

(ii) For sand:

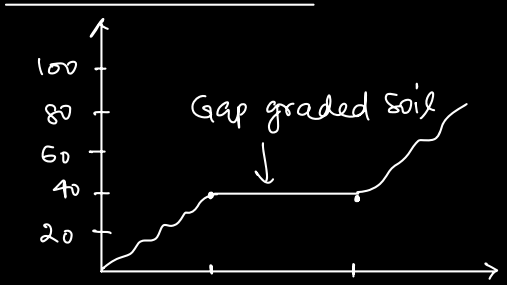
if $1 \leq C_c \leq 3$; well graded sand [SW]

if $1 > C_c > 3$

then such soils are called as Gap Graded Soils

Note: For gap graded soil particle size distribution curve is horizontal.

For example:



60% — 4mm

0%

40% — 2mm

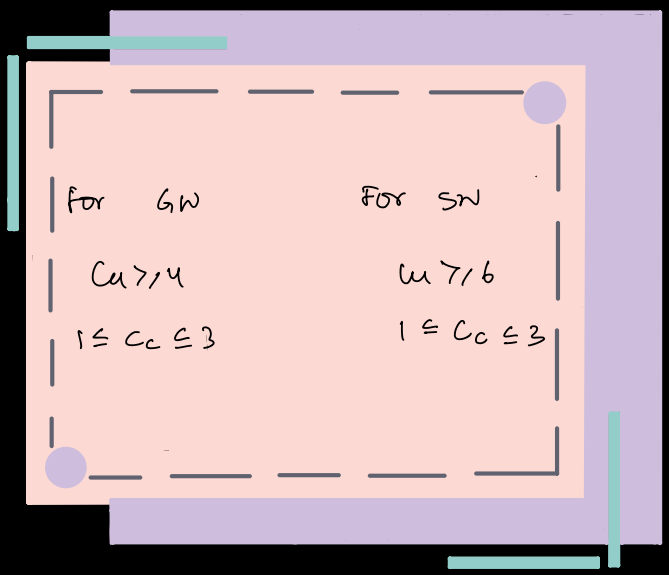
In this example 2mm to 4mm sizes are missing in the soil.

2mm 4mm

Note → when the soils are uniformly / gap graded symbolically they will be represented as poorly graded.

Note: For example in coarse sieving if $C_u = 1$ then it will be represented as GP

Note: For given soil to be well graded both C_u and C_c should be in the suggested range.



2nd STAGE OF PARTICLE SIZE DISTRIBUTION: Sedimentation Analysis

→ It is used for fine grained soil [particle size < 75μ] for which sieve Analysis was not suitable.

→ The particle size < 0.2μ, can not be even analysed by sedimentation method. For particle size less than 0.2μ, X-Ray diffraction technique shall be used.

→ In the sedimentation Analysis, the discrete settling velocity of spherically shaped particle in an infinite medium depends upon the size, shape, mass of the solid particles. It is based on stokes law.

Based on the stoke's law settling velocity can be given as

$$V_s = \frac{(G-1)\gamma_w d^2}{18\mu}$$

→ Dynamic viscosity.

V_s → settling velocity

d → diameter of solid particle

G → sp. gravity of solids

μ → dynamic viscosity

at 20°C : , $\mu = 0.01$ poise , 1 poise $= \frac{1}{10} \frac{N\text{-}s}{m^2}$ (Pa-s)

$$V_s = \frac{(G-1)\gamma_w d^2}{18\mu} = \frac{(2.65-1)\cdot 9810 \cdot d^2 \cdot 10^{-6}}{18 \times 0.001}$$

$V_s = 0.9\, d^2$ m/s (at 20°C)

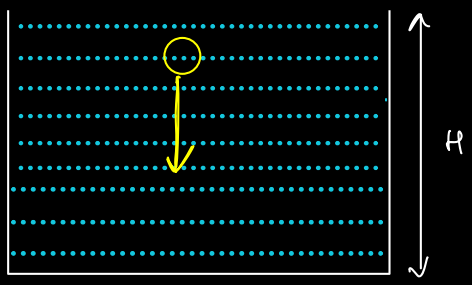
put d in mm

at 27°C : $\mu = 0.00855$ poise (Keeping same value G and γ_w)

$$V_s = \frac{1.65 \times 9810 \times 10^{-6} d^2}{18 \times 0.000855}$$

$V_s = 1.05\, d^2$ m/s @ 27°C

d → mm



$V_s = \frac{H}{t}$ → time required to travel the distance H.

→ Limitations of Stoke's law:

- The particle size is assumed to be spherical but infact the clay particles have flakey shape.
- Settling is assumed to be discrete but actually particle aggregate during settling.
- The particles are assumed to settle in infinite medium but actually sedimentation tank has finite dimensions
- The particles of size $< 0.2\mu$ and $> 200\mu$ can not be analysed by sedimentation method.

. In case of 200μ size of particles gravitational acceleration gets set as a result they don't settle with constant velocity
. whereas in case of 0.2μ size of particles brownian motion gets set as a result they don't settle.

→ The sedimentation Analysis can be carried out by any of the following methods.

i) Pipette Analysis

ii) Hydrometer Analysis.

→ Before starting Analysis the homogeneous soil suspension is required to be produced:

The homogenous soil suspension can be produced in three steps:

1st step: First of all if the organic compound are present in the soil that are removed to avoid the flocculation /aggregation / agglomeration of the particles.

→ to remove organic matter add oxidising agent like hydrogen peroxide H_2O_2.

→ to remove calcium compounds add 0.2N HCl in the soil.

2nd step: In this step, soil suspension is produced by adding the water to the given soil.

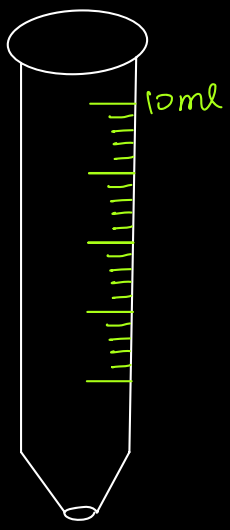
a) In case of pipette Analysis 12-30 gm of dry soil sample is taken and mixed with the water to produce the homogeneous soil suspension of volume 500 ml.

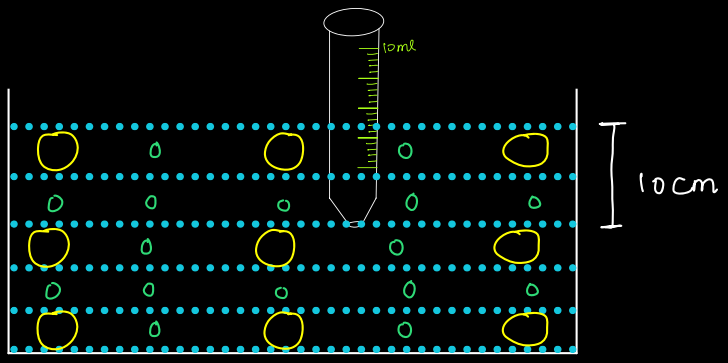
b) In case of hydrometer analysis 24-60 gm of dry soil sample is mixed with the water to produce the homogeneous soil suspension of volume 1000 mL.

3rd step: In this step Deflocculant / Dispersion Agent is added to the soil suspension to avoid the aggregation of particles during settlement. The dispersion reagent like sodium silicate, sodium carbonate, sodium oxilate, Sodium hexametaphosphate can be used.

Note: The homogeneous soil suspension is ready for the test. Either we can perform pipette method or Hydrometer Method.

Pipette Analysis:





Sedimentation jar with Homogeneous Soil Suspension.

→ In the pipette analysis, fixed amount of soil suspension is collected of 10ml in the pipette from the fixed sampling depth of 10cm.

→ Sampling depth (He) can also be termed as effective depth

→ Sampling is done at different time intervals such as $\frac{1}{2}$ min, 1 min, 2 min, 4 min, 8 min, 15 min, 30 min, 1 hour, 2 hours, 4 hours.

→ Every time the sample is taken, it is sent for oven drying to determine the weight of solids collected in the pipette.

→ The diameter of particle that travels the distance of He, i.e 10cm at anytime t can be calculated as,

$$V_s = \frac{(G-1)\, d^2 \gamma_w}{18\mu} = \frac{H_e}{t}$$

$$d = \sqrt{\frac{18\mu}{(G-1)\gamma_w} \cdot \frac{H_e}{t}}$$

$$d = \sqrt{\frac{18\mu}{(G-1)\gamma_w}} \cdot \sqrt{\frac{H_e}{t}}$$

$$d = Q \cdot \sqrt{\frac{H_e}{t}} \quad \text{where} \quad Q = \sqrt{\frac{18\mu}{(G-1)\gamma_w}}$$

→ Percentage finer for this particle of diameter 'd' can be computed as

$$\% \text{ finer} = \frac{\text{Concentration of soil suspension in pipette collected from the depth } H_e \text{ at anytime } t}{\text{Concentration of soil suspension in the sedimentation tank initially at } t = 0}$$

$$\% \text{ finer} = \frac{\frac{W_s'}{V_p}}{\frac{W_{soil}}{V_{suspension}}}$$

V_p: Volume of soil suspension collected in the pipette. [fixed value, $V_p = 10ml$]

W_s': weight of solids collected in the pipette at any time t from the fixed sampling depth of H_e = 10cm.

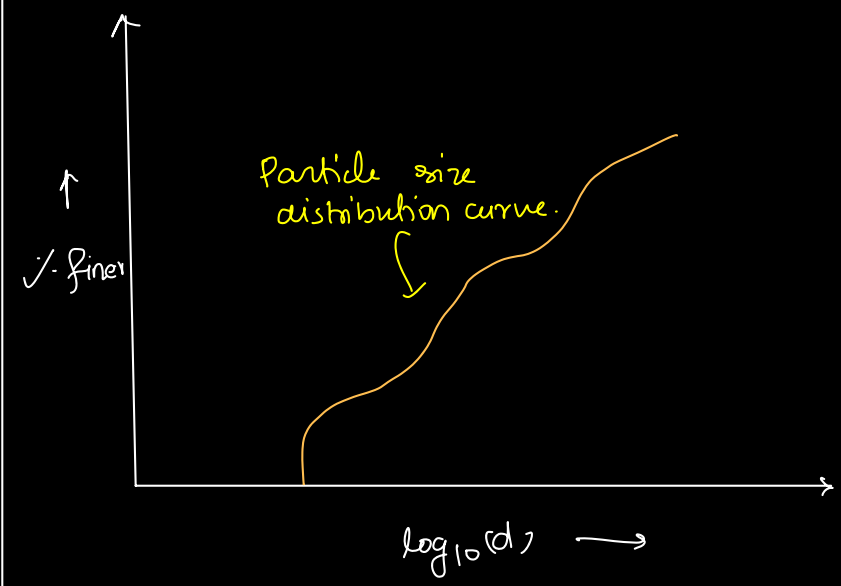
W_{soil}: it is the weight of dry soil added to the water to prepare soil suspension initially (W_{soil} → 12-30 gm)

$V_{suspension}$: Total volume of suspension produced in the pipette analysis

($V_{suspension}$ = 500ml = 500 cm^3)

t	d	% finer
0.5 min		
1		
2		
4		
8		
15		
30		
1 hr		
2 hr		
4 hr		
8 hr		
16 hr		
24 hr		

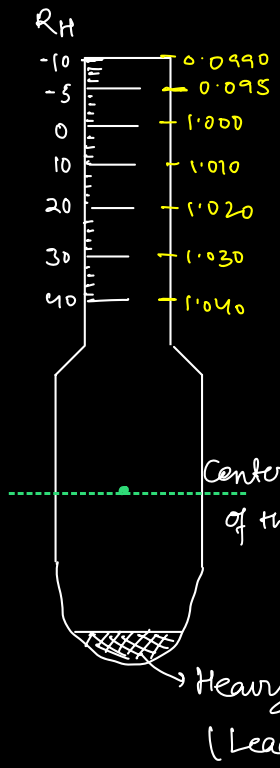
$t \uparrow \rightarrow d \downarrow \rightarrow$ % finer $\downarrow$

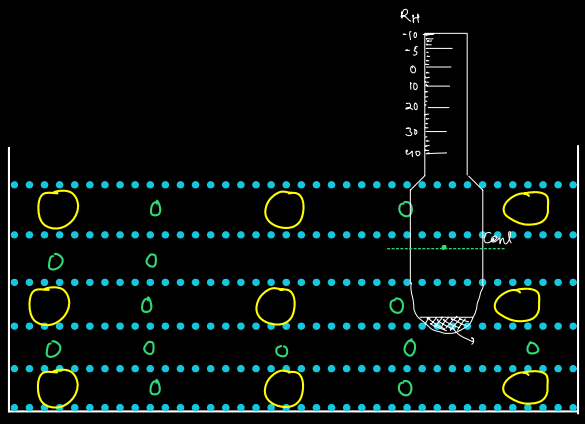


→ Considering the conc^n of deflocculating agent, % finer can be calculated as:

$$\% \text{ finer} = \frac{\left(\frac{W_s'}{V_p}\right) - \left(\frac{W_{DF}}{V_{suspension}}\right)}{\left(\frac{W_{soil}}{V_{suspension}}\right)}$$

HYDROMETER ANALYSIS:





R_H = Reduced hydrometer reading

$R_H = (G_{suspension} - 1) \times 1000.$

→ Hydrometer is a devile which is used to measure the 'specific gravity' or 'density' of the soil suspension

→ Reading of hydrometer unlike other scales increases in downward direction

→ For simplification, the sp. gravity of suspension is represented as hydrometer readings on the scale,

$$R_H = (G_{suspension} - 1) \times 1000$$

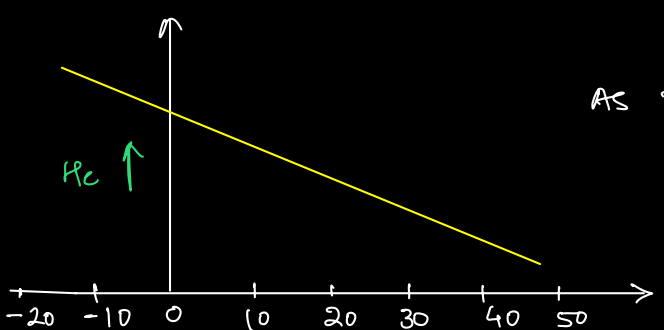
→ The volume of hydrometer below the centre line and above centerline is same

→ The principle of pipette & hydrometer analysis is almost same only there is difference in the procedure of test. For ex-

- In the pipette analysis sampling depth (He) is constant i.e 10 cm. whereas in Hydrometer analysis sampling depth increases with the time
- In pipette analysis the weight of solids in the collected sample is calculated by oven dry method. whereas in hydrometer Analysis sample is not collected means without computing Ws at any depth and time is calculated mathematically
- As the time passes, the hydrometer moves downwards as result of which reading decreases and the sampling depth increases

$$R_H \propto \frac{1}{He}$$

- He & R_H can be calliberated graphically



As time passes → R_H ↓

↓

He ↑

$R_H \longrightarrow$

→ At different time intervals R_H is measured and then from the value of R_H, sampling depth H_e is determined from the $R_H - H_e$ plot.

→ R_H is measured at different time intervals like 0.5min, 1, 2, 4, 8, 15, 30, 1hr, 2hr, 4hr, 8hr, 16hr, 24hr.

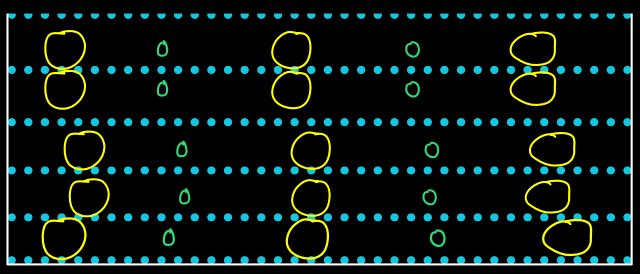
→ R_H is measured at the top of the soil suspension whereas the sp. gravity of suspension is obtained at the sampling depth (H_e).

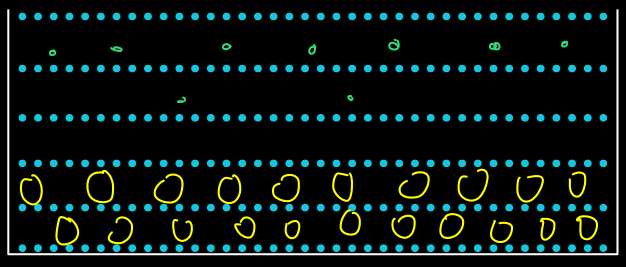
→ Now we have different values of H_e at different time intervals t. we can estimate the max diameter of particles as follows:

$$v_s = \frac{(G-1)\, d^2 \gamma_w}{18\mu}$$

$$d = Q\sqrt{\frac{H_e}{t}} \quad \text{where} \quad Q = \sqrt{\frac{18\mu}{(G-1)\gamma_w}}$$

t	R_H	$G_{suspension}$	H_e	d^2	% finer
					?





$$\frac{W_{solid\ in\ layer}}{V_{layer}} = \frac{R_H}{1000}\left(\frac{G}{G-1}\right)\gamma_w$$

Lecture 32

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$$\% \text{ finer} = \frac{\frac{W_{solid\ in\ layer}}{V_{layer}}}{\frac{W_{solids}}{V_{suspension}}} \times 100$$

$$= \frac{\frac{R_H}{1000}\left(\frac{G}{G-1}\right)\gamma_w}{\frac{W_{soil}}{V_{suspension}}} \times 100$$

Question.

$W_{soil\ sample} = 25\,gm$

$V_{suspension} = 1\ litre$

$\mu = 0.01\ poise$

$G_1 = 2.65$

$He = 5\,cm$

$t = 4\ hours$

After 4 hours the concentration at the sampling depth reduces to 2g/litre. Find dia of particles & its percentage finer?

Solution:

$$d = \sqrt{\frac{18\mu}{(G-1)\gamma_w}} \cdot \sqrt{\frac{He}{t}} = \sqrt{\frac{18 \times 0.001}{(2.65-1)\times 9810}} \times \sqrt{\frac{.05}{4\times 60\times 60}}$$

$$= .00105 \times .00186$$

$$= 1.965\,\mu > 0.2\,\mu.$$

$$\% \text{ finer} = \frac{2}{25} \times 100 = 8\%$$ Ans

- CORRECTIONS IN HYDROMETER READING:

1. Meniscus correction [Cm]:

R_{Ho}: observed hydrometer reading

R_{Ha}: actual hydrometer reading

We have measured less value of hydrometer reading, thus meniscus correction shall be added and its value is positive.

$$R_{Ha} = R_{Ho} + Cm$$

Note: Meniscus correction is always +ve.

2. Dispersion Reagent correction [C_D]: The presence of dispersion reagent increases the density of the suspension. As a result the hydrometer reading is overestimated. That means for actual the dispersion correction shall be subtracted.

$$R_{H1a} = R_{H1O} - C_D$$

Note! Dispersion reagent correction is taken as negative

3. Temperature correction: Reduced Hydrometer readings (R_{H1}) and sampling depth (H_e) are calibrated at 27°C temperature of soil suspension. But in actual the temperature of soil suspension can be < 27°C or > 27°C therefore temperature correction is required in hydrometer readings.

temp ↑↑ → density of liquids ↓ → R_{H1} is underestimated

temp ↓↓ → density of liquid ↑ → R_{H1} is overestimated.

if temp > 27°C

$$R_{H1a} = R_{H1O} + C_T$$

if temp < 27°C

$$R_{H1a} = R_{H1O} - C_T$$

temp > 27°C. Correction is +ve

temp < 27°C, correction is −ve

The combined correction can be written as

$$C_c = C_m - C_O \pm C_T$$

Final accurate hydrometer reading can be given as

$$R_{H1a} = R_{H1b} + C_c$$

SOIL CLASSIFICATION.

The methods of soil classification:

1) Particle size classification
2) Textural soil classification
3) Highway Research Board classification

4) Unified Soil classification system

or

Indian standard soil classification system.

1. Particle size classification: Based on the particle sizes soil can be classified as clay, silt, sand, Gravel, Cobbles and Boulders.

Clay : $d < 2\mu$

Silt : $2\mu < d < 75\mu$

Fine Silt : $2\mu - 10\mu$

Medium Silt : $10\mu - 20\mu$

Coarse Silt : $20\mu - 75\mu$

Sand: $75\mu < d < 4.75\,mm$

Fine silt : $75\mu - 425\mu$

Medium Silt : $425\mu - 2mm$

Coarse Silt : $2mm - 4.75mm$

Gravel: $4.75mm < d < 80\,mm$

Fine Gravel : $4.75mm - 20mm$

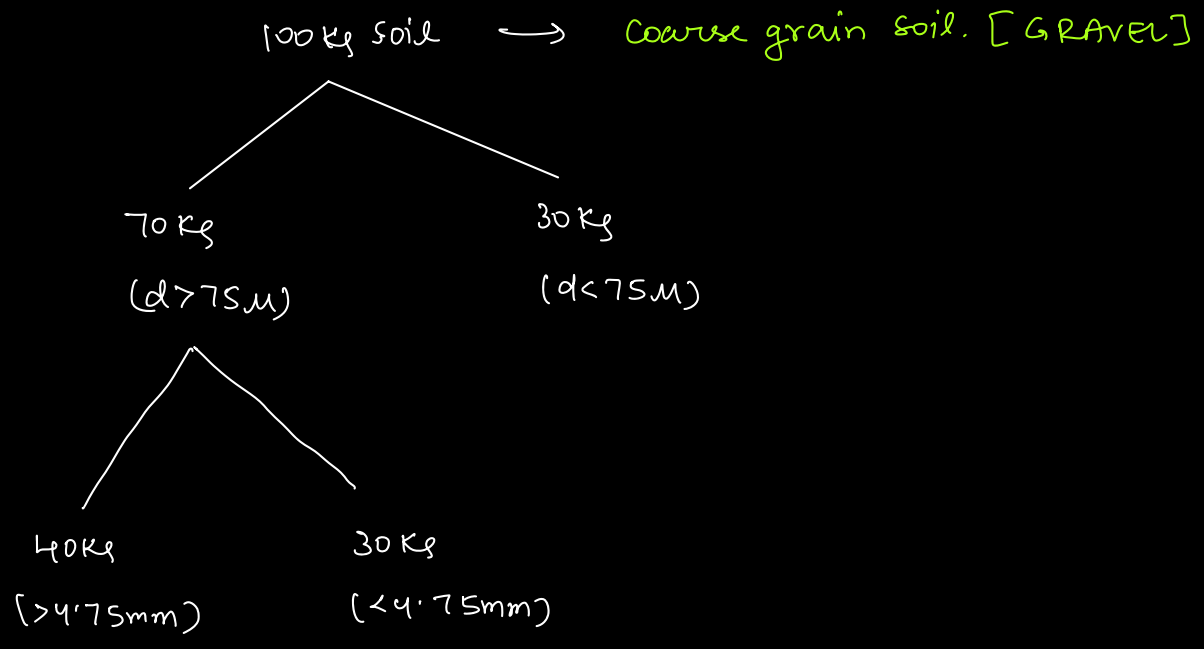
Coarse Gravel : $20mm - 80mm$

Cobbles: $80mm < d < 300mm$

Boulders: $d > 300mm$.

CLAY, SILT } Fine Grained soil ($<75\mu$)

SAND, GRAVEL } Coarse grained soil ($>75\mu$)



Note 1: The given soil sample can be called as fine grain soil if more than 50% material is finer than 50%.

Note 2: The given soil sample can be called as coarse grain soil if more than 50% material is coarser than 50%.

2. Textural soil classification:

→ Generally soil is found in the form of mixture of different sized particles

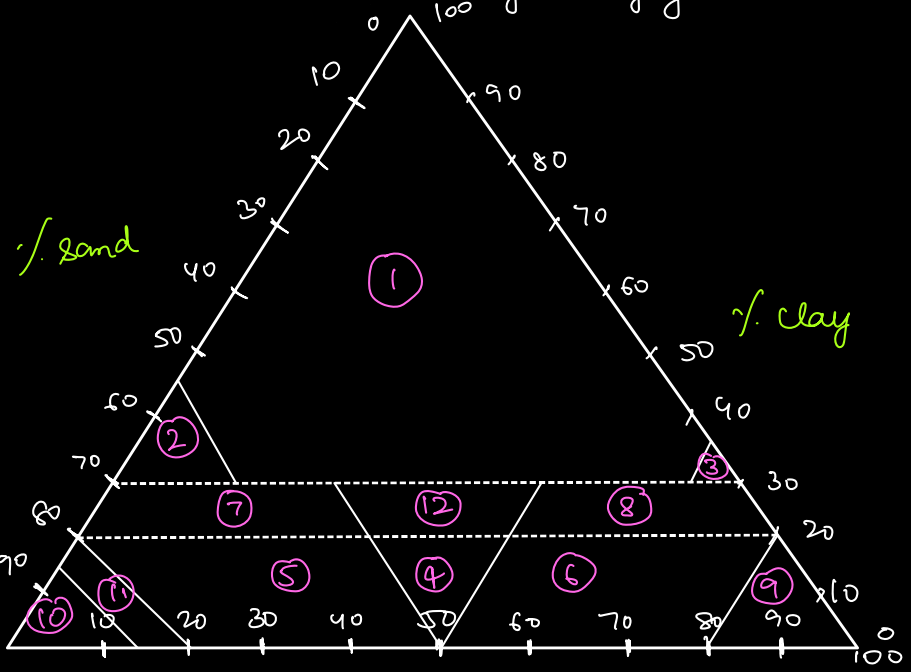
→ For the classification of such soils, public road organisation (PRA) system recommend the use of textural soil classification method.

→ To use this method the percentage of sand, silt and clay present in given soil sample should be known

→ This method consist of equilateral triangle, the sides of which show the percentage sand, percentage silt and % clay from 0 to 100 in anticlockwise direction.

→ The triangular chart is divided into the number of zones

① Clay
② Sandy clay
③ Silty clay
④ Loam
⑤ Sandy loam
⑥ Silt loam
⑦ Sandy clayey loam
⑧ Silty clayey loam
⑨ Silt
⑩ Sand
⑪ Loamy sand.
⑫ Clayey loam



% silt

Question:

Classify the mixture of sand, silt & clay.

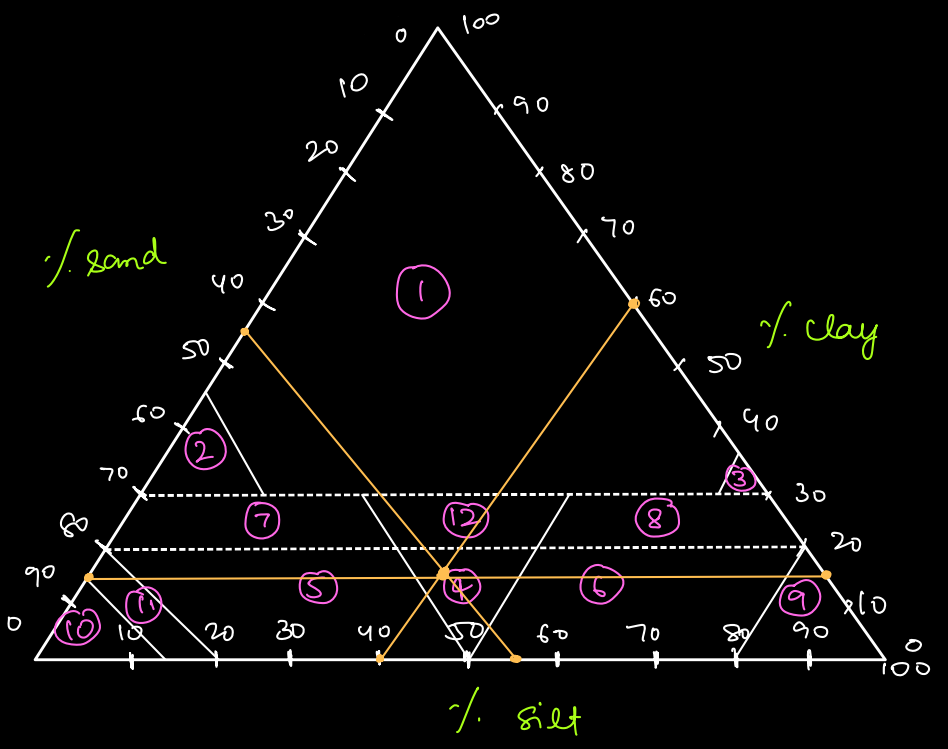
Sand → 45%.

Silt → 40%.

Clay → 15%.

Solution

when we plot the lines in the textural chart they intersect or meet at one common point which lies in the zone of loam as shown in chart.



Thus the mixture is classified as LOAM Soil

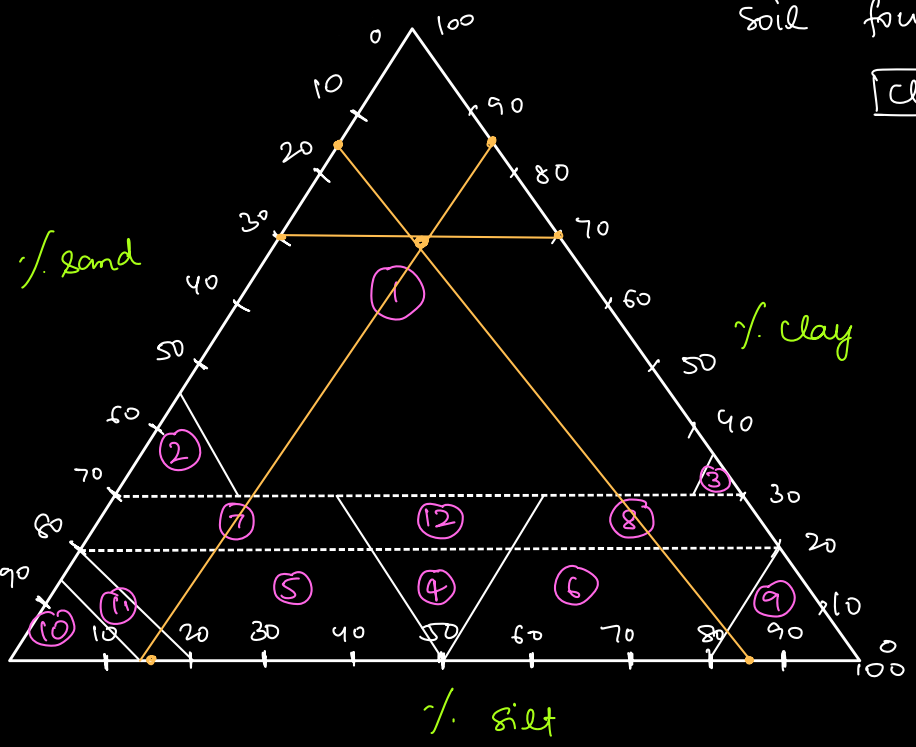
Question

Sand : 15%.

Silt : 15%.

Clay : 70%.

Solution



Soil found out to be clay.

Note: If in the exam textural chart is not given then the given mixture is termed as that soil which has proportion more than 50%.

Note: In the textural soil classification chart, the size of clay, silt and sand particles is according to bureau of united States soil classification system.

According to which:

clay : $d < 5\mu$

Silt : $5\mu < d < 50\mu$

Sand : $50\mu < d < 2mm$

3. Highway Research Board (HRB) Soil classification system

or

Public Road Administration (PRA) soil classification system

or

American association of state highway & transport officials (AASHTO) Soil classification system:

→ In this method, the performance of given soil is checked which is used to be subgrade in pavement construction

→ In this method the soil is classified into the total 8 groups where 7 main groups A_1, A_2, A_3, A_4, A_5, A_6, A_7 and one additional group A_8 for peat or muck

→ In this method, a characteristic parameter termed as Group Index is estimated to check the performance of soil used as a subgrade

→ Higher the value of group index means poorer the performance of soil used as a subgrade in pavement construction

→ The soil with excellent performance is kept in A-1 group, and the soil with poorest performance is kept in A-7 groups

→ These groups can be further classified in different subgroups

→ The value of group index depends upon the liquid limit, plastic limit, Plastic index and % of soil passing through 75μ sieve.

$$GI = 0.2\,a + 0.01\,bd + 0.005\,ac \quad \text{(No unit)}$$

a: The portion of percentage passing through 75μ sieve greater than 35% but not exceeding 75% expressed as a whole number between 0 to 40.

b: The portion of percentage passing through 75μ sieve >15% but not exceeding 55% expressed as whole number between 0 to 40.

Example 1

60% passing 75μ, 40% passing 75μ, $a = 40 - 35 = 5$

$b = 40 - 15 = 25$

Example 2

$\frac{15\%}{85\%}$ 75μ , $a = \frac{\cancel{85}}{75} - 35 = 40$

$b = \frac{\cancel{85}}{55} - 15 = 40.$

Example 3 $\frac{90\%}{10\%}$ 75μ, $a = 0$

$b = 0$

c: The portion of numerical W_L greater than 40% but not exceeding 60% expressed as a whole number b/w 0 to 20.

Ex①: $W_L = 48\%$

$c = 48 - 40 = 8.$

d: The portion of numerical plastic index greater than 10% but not exceeding 30% expressed as a whole number between 0 to 20.

Ex①: $I_P = 27\%$

$d = 27 - 10$

$\{d = 17\}$

Note: a, b, c & d are non negative numbers therefore group index will be always Non-negative.

Estimation of min & max. value of GI.

$GI = 0.2a + 0.01bd + 0.005ca$

$(GI)_{min} = 0$

$(GI)_{max} = 0.2 \times 40 + 0.01 \times 40 \times 20 + 0.005 \times 40 \times 20$

$(GI)_{max} = 20.$

$(GI)_{min} = 0$ $(GI)_{max} = 20$

if GI ↑ : Poorer the performance of soil used as a subgrade in pavement construction.

if GI ↓ : Better the performance of soil used as a subgrade in pavement construction.

Note: The value of group index should be as low as possible.

GI	General Evaluation of Subgrade
0	Excellent
0-1	Good
2-4	Fair
5-9	Poor
10-20	Very Poor

Note: The soil of excellent performance is kept in the A-1 group and the soil with very poor performance is kept in the A-7 Group.

4. Unified Soil classification System
or
Indian Standard Soil classification System

→ Unified soil classification system was originally developed by Casagrand which was adopted by American Society of Testing materials (ASTM) and later by Bureau of Indian Standards and other countries.

→ According to Indian soil classification system soil can be broadly classified into the

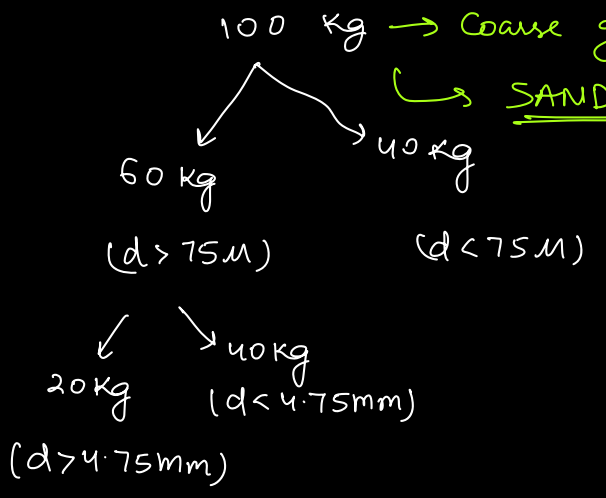
i) Fine Grain Soil

ii) Coarse Grain Soil

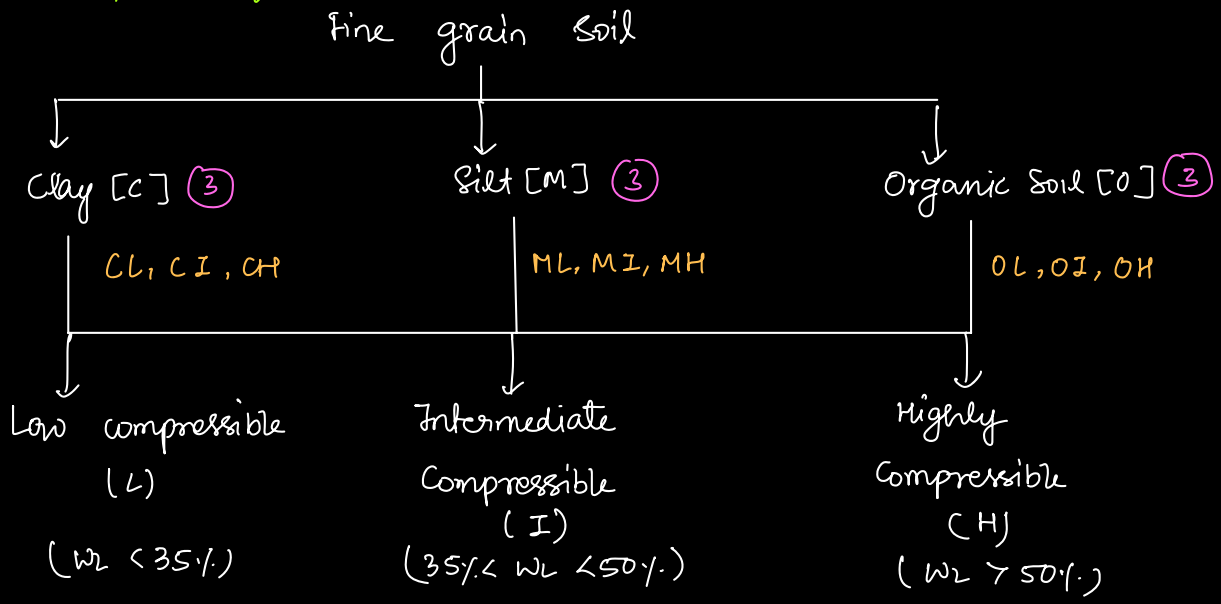
Note: If in the soil more than 50% material is finer than 75 μ sieve the soil is classified as fine grain soil.

Note: If in the given soil more than 50% material is coarser than 75 μ sieve, the soil is termed as coarse grained soil.

for ex:



a) classification of fine grain soil:



CL : low compressible clay

CH : High Compressible clay

CI : Intermediate compressible clay

ML : low compressible silt

MH : High Compressible silt

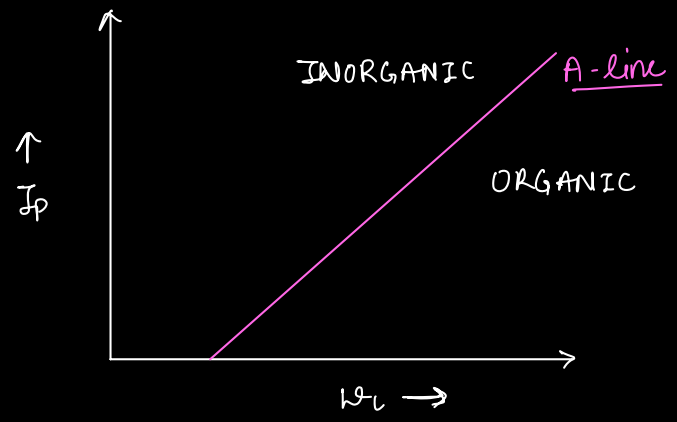
MI : Intermediate compressible silt

OL : low compressible organic soil

OH : High Compressible organic soil

OI : Intermediate compressible organic soil

- In order to seperate Inorganic clay from silt and organic soil A- Casagrand has proposed one curve known as A-line.



→ The soil above the A- line is clay and the soil below the A-line can be either silt or organic soil.

→ The equation of A-line can be given as:

$$(Ip)_{A\text{-}line} = 0.73 (w_L - 20)$$

→ In order to show the upper limit, the U-line is given above which soils are not classified.

→ The equation of U-line can be given as

$$(Ip)_{U\text{-}line} = 0.9 (w_L - 8)$$

RULES:

- if $w_L < 20\%$: identify the soil as
  - if $Ip < 4$ → Silt (M)
  - if $Ip > 7$ → clay (C)
  - if $4 < Ip < 7$ → Silt and clay both [M-C]
- if $w_L > 20\%$: then use A- line to identify the given soil

*** If $(Ip)_{soil} < (Ip)_{A\text{-}line}$

↳ It means below A-line. Then it is "Silt (M)".

Example: $w_L = 40\%$, $w_p = 20\%$

$(Ip)_{soil} = 20$.

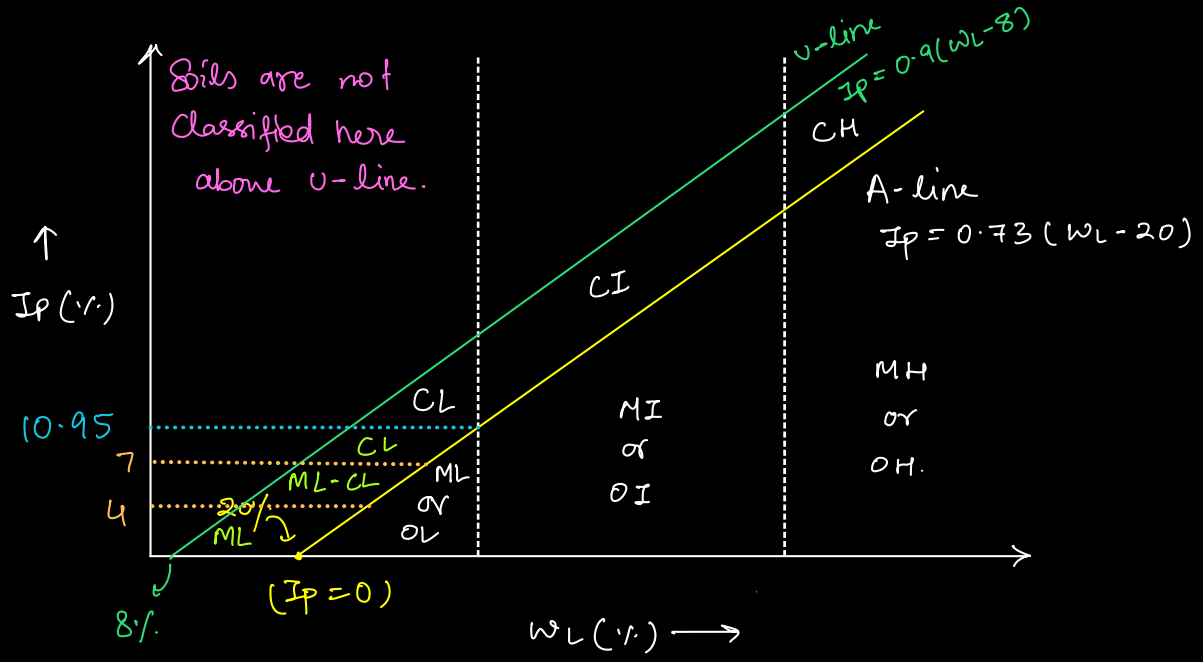
$(I_P)_{A\text{-line}} = 0.73 \times 20 = 14.6$

Soil above A-line, CLAY, $w_L = 40\%$ ($< 50\%$ & $> 35\%$)

CI

*** if $(I_P)_{soil} > (I_P)_{A\text{-line}}$

↳ It means above A-line. Then it is "clay" (C).



→ The soils above A-line is inorganic and below A-line it is organic

→ Below A-line, the soil is either "silt" or "organic soil".

Identification of organic soil:

(i) If soil is found in dark black or brown in colour with bad organic odour (smell) then it can be organic soil.

(ii) Reduction in the liquid limit after oven drying to a value less than 3/4 of the original value of liquid limit (before oven drying or Natural State) is the strong evidence of the presence of the organic matter.

Question: Classify the given fine grain soil for which $w_L = 40\%$ and $w_P = 20\%$.

Solution:

$$(I_P)_{soil} = w_L - w_P = 20\%$$

$$(I_P)_{A\text{-line}} = 0.73(w_L - 20)$$

$$= 14.6\%$$

$\because$ (Plastic Index)$_{soil}$ > (Plastic index)$_{A\text{-line}}$

above a line [C]

$35\% < w_L < 50\%$, CI

Question2: Classify the given soil if $w_L = 60\%$, $w_P = 40\%$.

Solution: $(I_P)_{soil} = 60 - 40 = 20\%$

$(I_P)_{A\text{-line}} = 0.73(60-20) = 29.2\%$

$\because$ $(I_P)_{soil} < (I_P)_{A\text{-line}}$

$\therefore$ Below the A-line, MH $\{w_L > 50\%\}$

Question3: classify the fine grain soil if $w_L = 15\%$ and $w_P = 12\%$.

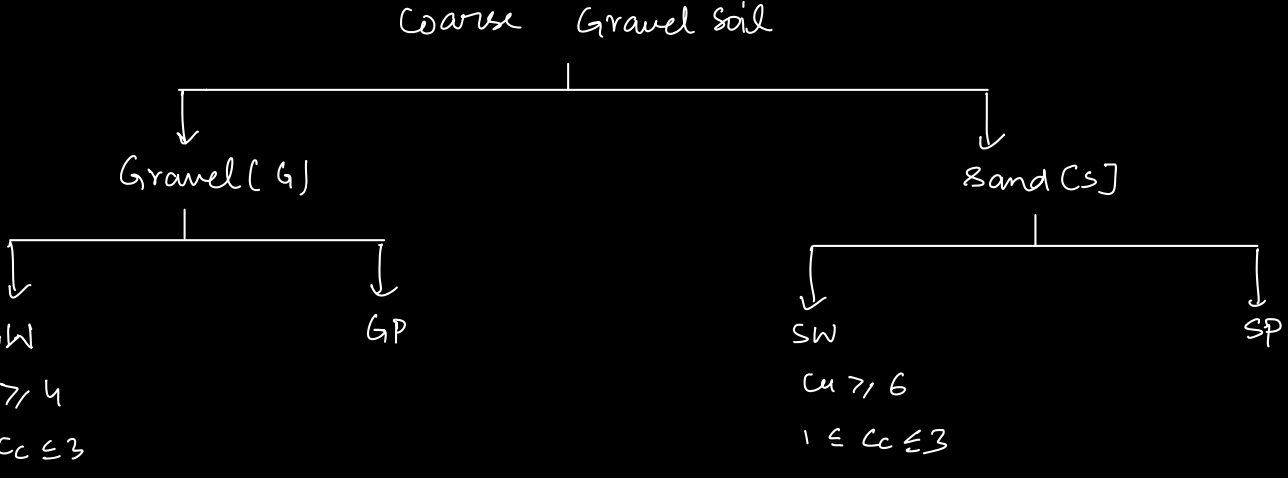
Solution: $\{I_P = 3\%\}$ $\because$ $w_L < 20\%$.

ML $\therefore$ A-line shall not be used to classify the given soil sample

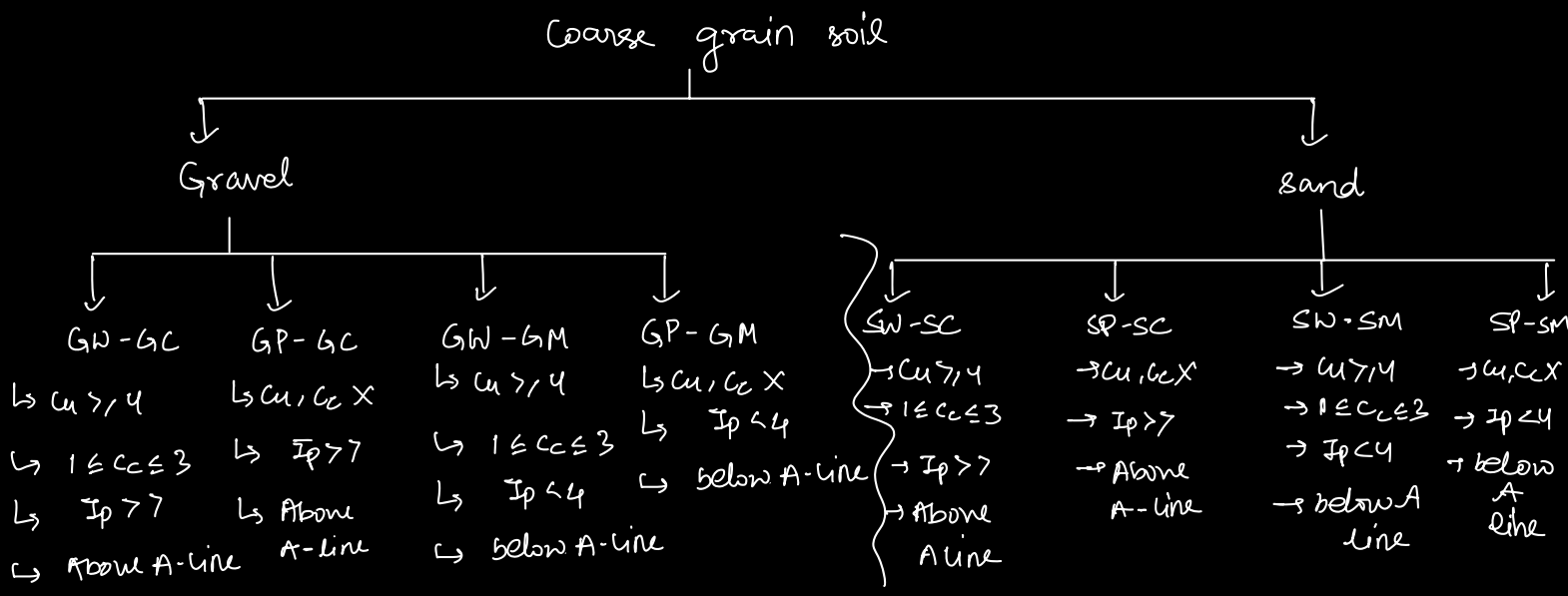
CLASSIFICATION OF COARSE GRAIN SOIL:

The given soil can be called as coarse grained soil if more than 50% material is coarser than 75μ.

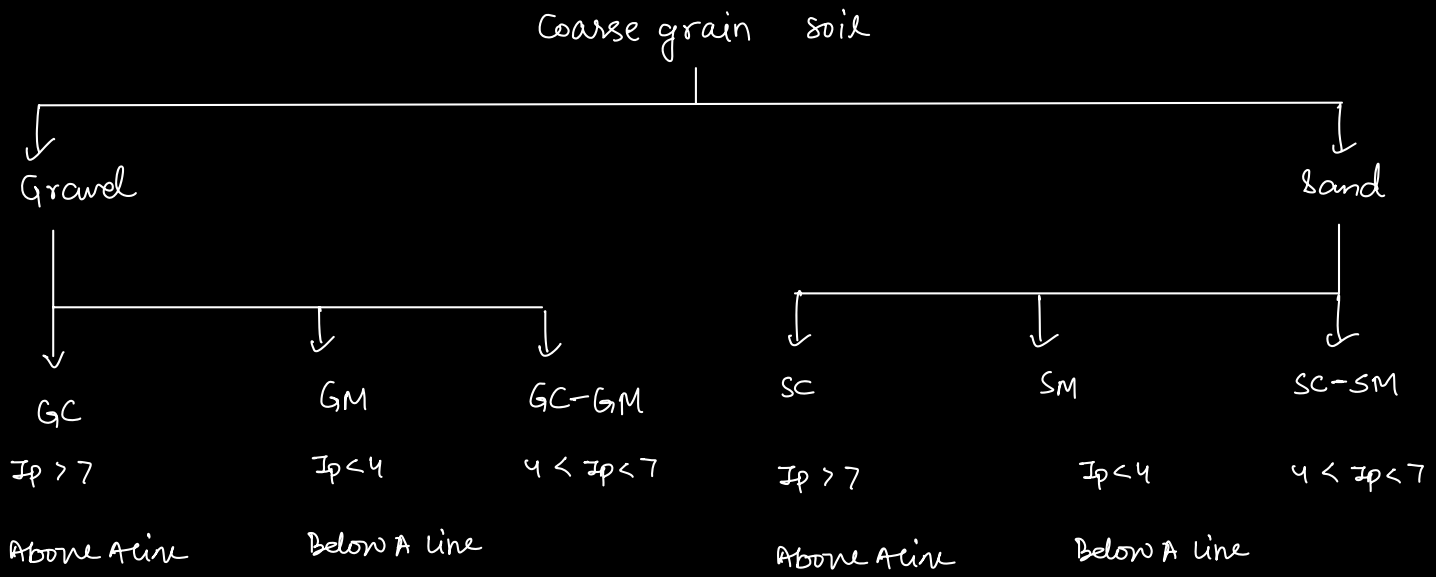
Case 1: When % finer < 5%.



Case 2: when 5% < % fine < 12%.



Case 2: when % finer > 12% (But < 50%):



Question 2: 980g passes through 4.75mm : 98% Sand

270g passes through 75μ : 27% fines

w_L = 40%

w_p = 18%

Solution. w_L > 40% A-line can be used.

20 gm — 4.75 mm

710 gm — 75 μ

270 gm

$(I_p)_{soil} = 40 - 18 = 22$

$(I_p)_{Aline} = 0.73 \times 20 = 14.6$

∴ Above A line : CLAY

SC : [Clayey sand.]

Question3: Percentage finer than 4.75mm : 60%.

Percentage finer than 0.075mm : 30% > 12%. (Case III)

COARSE : 70%.

Sand : 30%.

Gravel : 40% : GRAVEL (G)

W_L : 35% > 20% (A line can be used)

W_L : 27%.

$(I_P)_{soil} = 35 - 27 = 8\%$.

$(I_P)_{A\,line} = 0.73 \times 15 = 10.95$

∴ below line

∴ Silt (M) ⇒ Thus (GM)

(Silty Gravel)

Field observations for coarse grain soil:

1. Visual Inspection: By visual inspection coarse grained particles of sand or gravel can be identified.

Field observations for fine grain soil:

To differentiate between silt & clay following tests or observations can be done.

1. Dispersion test: Take small amount of dry soil sample and put in the one glass of water if soil particles settle in 5-10 minutes, then it is silt or else clay.
2. Dilatency test: Take small amount of sample and add sufficient amount of water to make a ball, take this ball on the palm, demould it, if moisture appears over the surface of the ball, then it is silt, otherwise clay.

* Plasticity is a measure of dry strength.

3. Dry strength test: From the given soil, a ball of 3mm diameter is prepared and then it is dried. After drying press the ball b/w two fingers if it is crushed by small load, then it is silt.

4. Toughness test: The given soil sample is rolled into the thread of 3mm diameter if it is crumbled/craked then it is silt or else clay.

MINERALOGY

The solids in the soil excluding organic matter are termed as minerals. Minerals are essentially inorganic, naturally occurring compound of various elements.

Clay Minerals: The "clay" or "clay minerals" are formed by chemical weathering of primary minerals of rock. Clay minerals have different properties such as:

i) Plasticity
ii) Compressibility
iii) Activity
iv) Consistency
v) Sensitivity
vi) Thixotropy
vii) Collapsibility

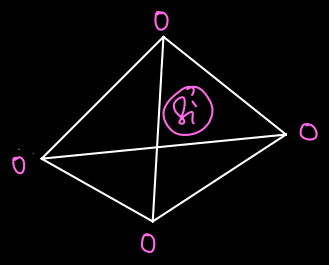
→ The clay minerals have particle sizes < 2μ. They are primarily Hydrous Aluminium Silicates.

→ These clay minerals belong to phyllosilicates group of minerals where phyllo [Greek Word] means leaf. Thus silicates having leaf or sheet shape thin structure termed as phyllosilicates

→ Every clay mineral is made of two basic structural units or building blocks:

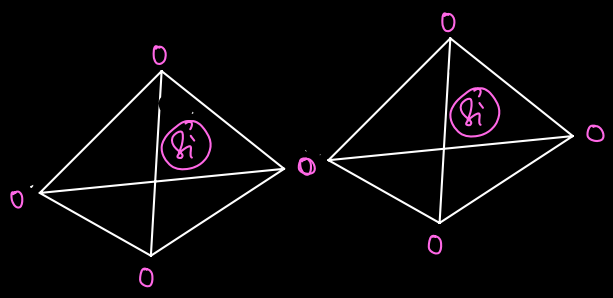
(i) Silica tetrahedral sheet
(ii) Aluminium or Magnesium or Iron octahedral sheet.

- Silica tetrahedral sheet:



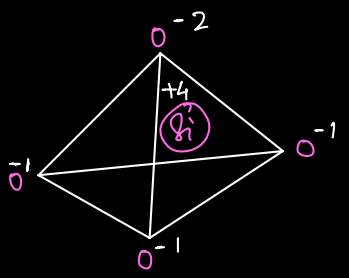
Silica tetrahedral unit

In the silica tetrahedral sheet 4 O atoms enclose the 1 silica atom in the centre in 1 tetrahedral geometry



→ Out of 4 oxygen atom, 3 oxygen atoms which are present at the base are shared between two units of Silica tetrahedral sheets

→ Therefore, the contribution of each bottom oxygen in each unit of silica tetrahedral sheet will be -1.



SiO_4^{-1}

→ Therefore the net charge over the 1 silica tetrahedral unit is -1.

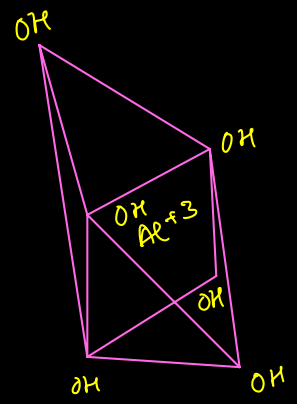
1 Oxygen at the top

$$[3\times(-1) + (-2) + (+4)] = -1$$

3 oxygen at the base — Silica at centre.

→ Silica tetrahedral sheet can be symbolically represented as trapezium .

- **Aluminium Octahedral Sheet:**



→ In octahedral sheet, six hydroxyl atoms encase 1 Aluminium Atom at the centre in the octahedral Geometry.

→ In this sheet, each hydroxyl atom is shared between 3 units of octahedral units therefore the contribution of charge by hydroxyl atom in each unit will be $-1/3$.

→ The net charge over the aluminium octahedral sheet is +1

$$\left[6\times\left(-\frac{1}{3}\right) + (+3)\right] = +1 \qquad [Al(OH)_6]^{+1}$$

(Hydroxyls: $6\times(-\frac{1}{3})$; Aluminium: $+(+3)$)

Very important Note: · If Al^{+3} is present at the centre in the octahedral geometry then it is termed as gibbsite sheet.

· If Fe^{+2} / Fe^{+3} is present at the centre in the octahedral geometry then it is termed as ferrite sheet

· If Magnesium Mg^{+2} is present at the centre, then it is termed as brucite sheet.

→ The Aluminium octahedral sheet is symbolically represented as ▭

Note: These fundamental building blocks arranged themselves in different orientation & pattern results the formation of different clay minerals as follows.

There are following groups of clay minerals:

1. Kaolinite Group (1:1 Mineral)
   - a) Kaolinite Mineral
   - b) Dickite Mineral
   - c) Halloysite Mineral
2. Montmorillonite Group (2:1 Mineral)
   - a) Montmorillonite Minerals
   - b) Beidellite Mineral
   - c) Hectorite Mineral
   - d) Saponite Mineral
3. Mica Group (2:1 Mineral)
   - a) Illite Mineral
   - b) Glauconite Mineral
4. Chlorite group (2:2 Mineral)
   - a) Chlorite Mineral
5. Vermiculite Group (2:1 Mineral)
   - a) Vermiculite Mineral
6. Serpentine Group (1:1 Mineral)
   - a) Chrysotile Mineral.

KAOLINITE MINERAL: A single structural unit of Kaolinite (1:1) is composed of one silica tetrahedral sheet and one aluminium octahedral sheet (Gibbsite sheet)

→ The thickness of a single unit is 7Å [1Å = 10^{-10}m]

→ Several number of these structural units are joined together by very strong H-bonding.

→ Kaolinite is generally electrically neutral

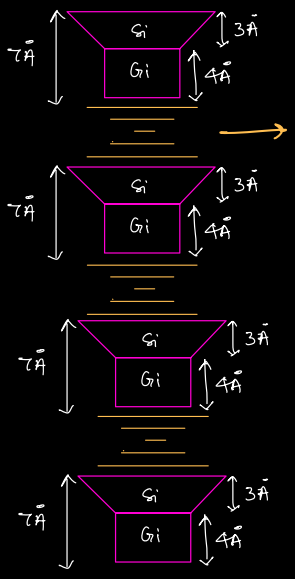
→ Several Number of structural units are joined by very strong H-bond therefore it is relatively difficult to displace them.

As a result less or No volume change [swelling and shrinkage] is observed.

→ It is least active clay mineral with Activity of 0.4 to 0.5

→ It's plasticity Index is approximately 15% which is relatively less compared to Illite & Montmorillonite. The total thickness of these layers extend upto 15-2000 nanometers.

→ The china soil is purest form of Kaolinite



" Kaolinite Minerals "

Note: Little Isomorphous substitution can be observed.

HALLOYSITE MINERAL: When a layer of water molecule exist between structural units of Kaolinite Mineral, then it is termed as Halloysite Mineral.

MONTMORIUONITE MINERAL: Montmorillonite is also known as smectite. (2:1) It is the most common clay mineral which is found in black cotton and bentonite soils

→ A single structural units is composed of 2 silica sheet and 1 Alumina sheet

→ The Gibbsite sheet is sandwiched between silica sheet at top & bottom.

→ Thickness of 1 unit is 10Å.

→ As compare to Kaolinite Isomorphous substitution is more in montmorillonite

→ Isomorphous substitution means replacement of one atom from the sheet by other atom.

→ Generally in silica sheet, silica is replaced by Isomorphous substitution by either Al^{+3} or Mg^{+2} as a result negative charges increases.

→ These negative charges can be balanced by K^+ or Na^+ or Ca^{++} etc. As a result we can have Na- montmorillonite, and Ca- Montmorillonite minerals

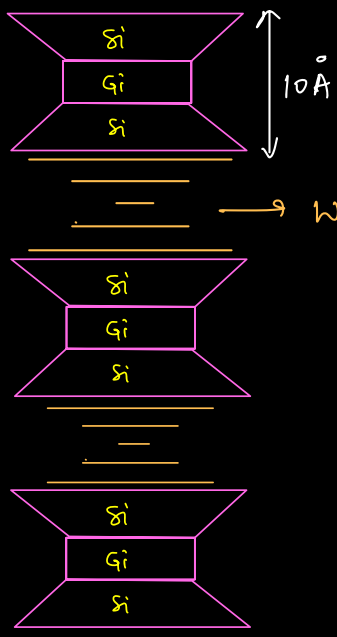
→ Na- Montmorillonite [Activity - 4 to 5]

Ca- Montmorillonite [Activity - 1.5]

Note: If in the soil sodium Montmorillonite is present it shows large activity. By increasing the calcium compounds [gypsum content] it can be made Ca- Montmorillonite which is relatively less active.

→ Several number of structural units are joined together by very weak water bond.

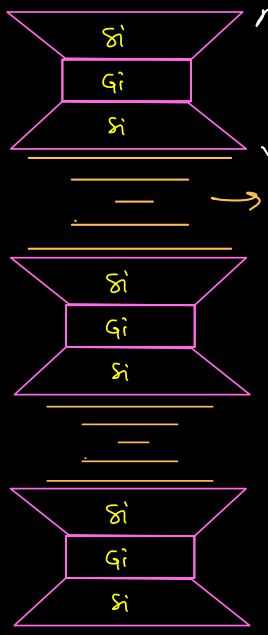
→ Since the bond is very weak therefore structural units can be easily displaced as a result it shows large volume change [large swelling and shrinkage]



"Montmorillonite Mineral"

Note: Plasticity Index for Montmorillonite is approximately 300.

ILLITE MINERAL : The composition of a single structural unit (2:1) is same as that of montmorillonite clay mineral.



→ The several number of structural units are joined together by ionic bond i.e. K+ ionic or Na^+ ionic bond.

→ In Illite also isomorphous substitution can be observed.

→ Since K^+ ionic bond is stronger than water bond but weaker than hydrogen bond as a result medium volume change [intermediate or Normal swelling and Shrinkage] is observed.

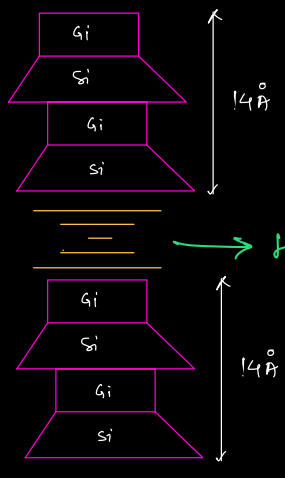
→ It is normal active clay mineral with activity 0.5 - 1.

→ Illite can also be termed as hydrous mica.

→ It shows intermediate plasticity between Kaolinite and Montmorillonite.

CHLORITE MINERAL (2:2 Mineral): Chlorite Mineral is formed when two silica sheets combine alternately with two aluminium or boucite sheets as shown in figure

A single structural unit of chlorite is thus equal to two units of Kaolinite Minerals.



Let

Kaolinite - A

illite - B

Montmorillonite - C

Plasticity in increasing order.

A < B < C

Bond strength in increasing order.

C < B < A

Grain size in increasing order

C < B < A

Activity in increasing order.

A < B < C.

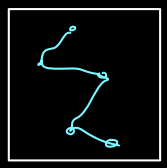
PERMEABILITY

Permeability can be defined as the property of soil which allows the flow of water through its interconnected voids.

(Total No of voids) fine grain soil > (Total volume of voids) CGS

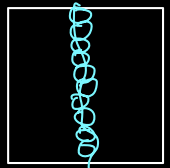
then also (permeability) CGS > (permeability) FGS.

Even if total volume of voids and total number of voids are less in Coarse grain soil then also their permeability is more because their voids are interconnected



clay

(voids not interconnected)



Gravel

Voids are interconnected.

Note: During the flow of water resistance / friction is offered by soil solids over their surface.

- If particle size ↓ → SSA ↑ → friction ↑ → Resistance against flow ↑↑ → Permeability ↓

- If particle size ↑ → SSA ↓ → friction ↓ → Resistance against flow ↓↓ → Permeability ↑

In fluid mechanics (during pipe flow):

$$TH = PH + VH + EH$$

In geotechnical engineering (during seepage):

$$TH = PH + \cancel{VH} + EH$$

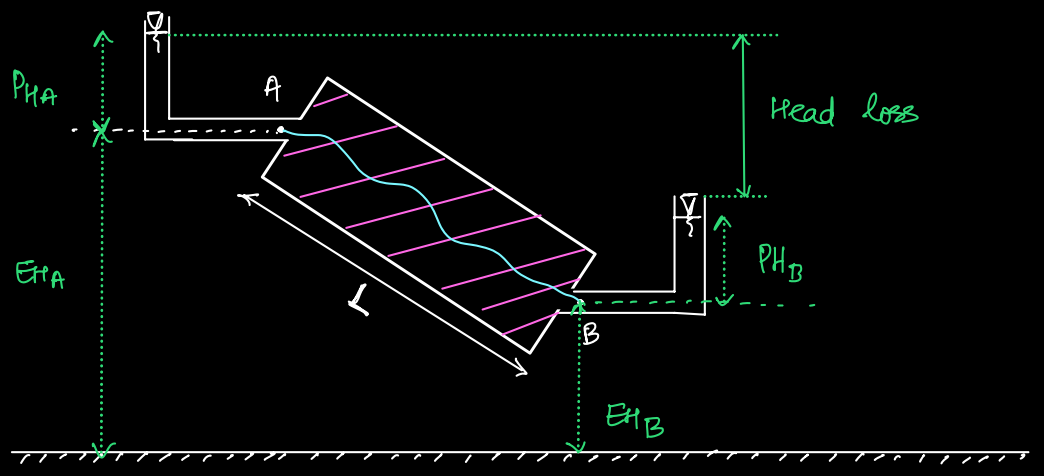
VH → Ignored.

$$TH = PH + EH$$

Note: "VH" is ignored because discharge through the soil is very less therefore velocity head is also very less.

Note: This "total head" (TH) is also termed as "hydraulic head" and "piezometric head".

DARCY'S LAW:



$$TH_A = PH_A + EH_A$$

$$TH_B = PH_B + EH_B$$

$\because$ $TH_A > TH_B$

$\therefore$ Water will flow from high head to low head i.e. point A to point B.

$$\Delta H = TH_A - TH_B$$

where $\Delta H \rightarrow$ Total head loss or total head difference

$$i = \frac{\text{Total head loss}}{\text{Total length of flow path}}$$

$$i = \frac{\Delta H}{L'} = \frac{\Delta H}{L}$$

Where i: hydraulic gradient

L': Actual length of flow path

L: length of soil sample.

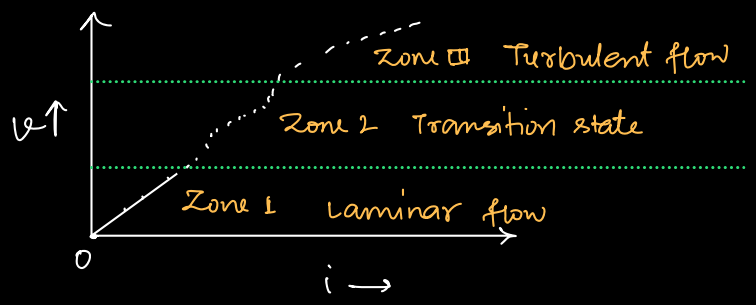
Note: For simplification L' can be taken as L.

→ It is observed that if hydraulic gradient is increased then velocity of flow also increases.

at $i = 0$, $v = 0$

if $i \uparrow \rightarrow v \uparrow$

$i \uparrow\uparrow \rightarrow v \uparrow\uparrow$



→ In the zone 1, the flow is streamlined / laminar in nature therefore a linear relation is observed between v and i.

→ According to darcy's law, for fully saturated homogeneous soil in laminar flow condition, discharge velocity is directly proportional to hydraulic gradient.

$v \propto i$

$v = Ki$ m/s

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where,

v: discharge velocity / Average velocity / virtual velocity / superficial velocity / Macroscopic velocity

i: Hydraulic gradient

K: Coefficient of permeability / hydraulic conductivity [m/s]

$$Q = K \cdot i \cdot A$$

$\therefore$

Q: Discharge

A: Cross sectional Area.

if $i = 1$, $K = \frac{Q}{A}$: The coefficient of permeability can be defined as the rate of flow per unit cross sectional area of the soil sample

units of 'K' $\rightarrow$ $(m^3/s)/m^2 \rightarrow$ m/s.

→ Darcy's law is valid for laminar flow condition

→ For laminar flow (Streamed line flow) through the soil, Reynold's number (Re) shall be less than 1.

→ The flow through Gravels, Brocken bricks is turbulent therefore Darcy's law can not be applied for them.

→ Generally darcy's law is best suitable for silt and fine sand in which laminar flow (Re < 1) is observed

→ v is termed as virtual velocity because it is not the actual velocity of flow of water. it is estimated corresponding to the total c/s area of the soil but actually water flows through the area of voids not through the total cross sectional area of the soil sample.

$v = \frac{Q}{A}$ → Q → discharge (m^3/s)

A → Total c/s area of soil sample

SEEPAGE VELOCITY: Seepage velocity is the actual velocity of flow of water through the voids, It can be denoted by v_s. Seepage velocity can be estimated as,

from the equation of continuity ; $A_1 v_1 = A_2 v_2$

$$A_v \cdot v_s = A \cdot v$$

$$v_s = \frac{A}{A_v} \cdot v$$

$$\frac{v_s}{v} = \frac{A \cdot L}{A_v \cdot L} = \frac{V}{V_v}$$

$$\frac{v_s}{v} = \frac{V}{V_w} \longrightarrow \text{fully saturated } (V_v = V_w)$$

$$\frac{v_s}{v} = \frac{V}{V_w}$$

$$\frac{v_s}{v} = \frac{V}{V_w} \times \frac{V_v}{V_v} = \frac{1}{n} \cdot \frac{1}{S}$$

$$v_s = \frac{v}{S \cdot n}$$

$n \longrightarrow$ porosity.

$S \longrightarrow$ degree of saturation

→ According to Darcy's law, soil is considered to be fully saturated then

∴ when $S = 1$, then $v_s = \frac{v}{n}$ m/s.

→ Actually seepage velocity is estimated through the area of voids therefore it comes out to be greater than discharge velocity

→ ∵ Porosity (n) Ranges from 0 to 1 ($0 < n < 1$) so from eqn $v_s = \frac{v}{n}$ v_s cannot be less than v, v_s is always greater than v.

→ Seepage velocity is also proportional to the hydraulic

gradient - that means $v_s \propto i$

$$v_s = K_p i$$

where, K_p : coefficient of percolation (m/s)

$$\because \quad v_s = \frac{v}{n}$$

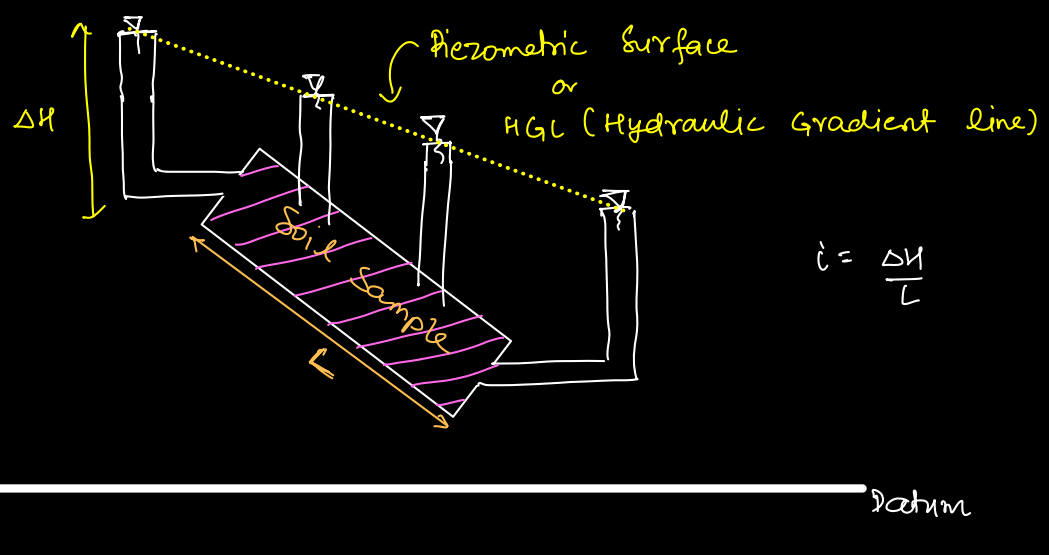
$$\therefore \quad K_p i = \frac{K \cdot i}{n}$$

$$\Rightarrow \quad K_p = \frac{K}{n}$$

Note: The coefficient of percolation corresponding to seepage velocity is greater than the coefficient of permeability corresponding to discharge velocity.

TYPES OF SOIL	K (cm/s)
Gravel	>1 cm/s
sand	10^{-3} to 1 cm/s
Silt	10^{-7} to 10^{-3} cm/s
clay	$<10^{-7}$ cm/s

Note: The clay soil is very less coefficient of permeability therefore it is sometimes treated as impermeable soil.



$$i = \frac{\Delta H}{L}$$

Note: The line joining the water levels in all the piezometers installed throughout the length of soil is termed as piezometric surface.

Note: the slope of the piezometric surface is called as hydraulic Gradient (i) therefore piezometric surface also termed as Hydraulic gradient line (HGL).

Factors affecting coefficient of permeability:

Note: The coefficient of permeability is the function of properties of medium (soil) and properties of fluid (water) both.

$$K = C D_{10} \left(\frac{\gamma_w}{\mu}\right)\left(\frac{e^3}{1+e}\right)\left(\frac{1}{K_k \cdot SSA^2}\right)\left(\frac{1}{T^2}\right) \times A \times \frac{1}{B} \times \frac{1}{C} \times \frac{1}{D}$$

i) Allen Hazen's Equation :

$$K = C D_{10}^2$$

K : Coeff. of permeability [cm/s]

D10 : Effective size of particle [cm]

C : 100

if D10 : in mm

then C : 1

K : cm/s.

from the above equation it is clear that

if particle size ↑ → K ↑ → Because voids are interconnected and due to less surface area, friction or resistance against flow is less.

ii) Louden's Equation:

$$\log_{10}(K \cdot SSA^2) = a + b\,\eta$$

a, b → constant

η → porosity

K → Coefficient of permeability

SSA → specific SA

Note: 'a' and 'b' are the constants and their values are 1·36 & 5·15 respectively.

$$K \propto \frac{1}{SSA^2}$$

if SSA ↑ → K ↓ , if SSA ↓ → K ↑

→ for coarse grain soil → SSA ↓ → K ↑

→ for fine grain soil → SSA ↑ → K ↓

(iii) Kozeny carmanis Equation:

$$K = \left(\frac{\gamma_w}{\mu}\right)_{fluid} \left(\frac{e^3}{1+e}\right) \left(\frac{1}{K_x \cdot SSA^2}\right) \left(\frac{1}{T^2}\right)$$

γ_w → unit wt of fluid

μ → dynamic viscosity of fluid

$\left(\frac{\gamma_w}{\mu}\right)_{fluid}$ → properties of fluid

$\left(\frac{e^3}{1+e}\right)\left(\frac{1}{K_x \cdot SSA^2}\right)\left(\frac{1}{T^2}\right)$ → properties of soil

$$K \propto \frac{1}{\mu} \quad \therefore \mu \uparrow : K \downarrow$$

$\mu \rightarrow 25$ $\quad$ $\mu \rightarrow 50$

$K \rightarrow 100$ $\quad$ $K \rightarrow ?$ $\quad$ 50 !!

if T ↑ → μ ↓ → K ↑

↳ 'e' is the void ratio.

$$K \propto e^2$$

if e ↑ → available effective space for the flow is more

→ K ↑

$\therefore K \propto e^2$

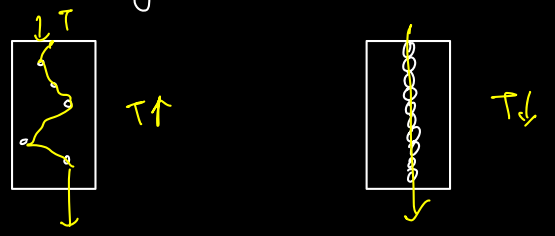
we can't compare Gravel with clay.

↳ SSA → specific surface area $[cm^2/cm^3]$

$$K \propto \frac{1}{SSA^2}$$ (Kk → Kozeny coefficient)

if particle size ↑ → SSA ↓ → K ↑

↳ T → Tortuosity



(fine grain soil) (Coarse grain soil)

Particle size is less → Tortuosity ↑ → K ↓

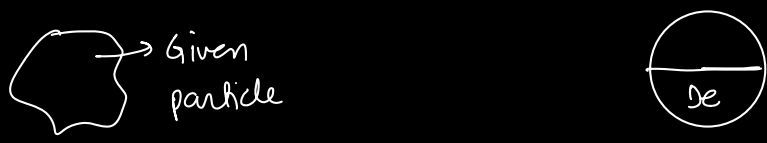
Particle size is more → Tortuosity ↓ → K ↑

iv) Terzaghi's Equation:

$$K = 200\, e^2\, De^2$$

e → void ratio

De → effective grain size considering particles to be spherical in such a way the ratio of volume of particle to the SA of the particle is equivalent to ratio of volume to SA of the given particle.



$$\frac{V_p}{SA_p} = \frac{V_s}{SA_s}$$

→ In the main equation

A: Degree of Saturation C: Impurities

A ↑, K ↑ C ↑ → K ↓

B: Entrapped Gases D: Adsorbed water.

B↑, K↑ D↑ K↓

* Degree of Saturation: If degree of saturation is less then air content present in the voids will be more, this air offers the resistant against the flow acting as a air blockage. As a result permeability decreases

* Entrapped Gases: If % of entrapped gases is more then resistance against the flow will be high, as a result permeability decreases

* Impurities: If content or % of impurities is more then effective space available for the flow reduces and visiosity increases that leads to reduce the permeability

* Adsorbed Water: Due to the presence of adsorbed water the void ratio is reduced by 0.1. That means due to the presence of large amount of adsorbed water the effective space available for the flow reduces as a result permeability decreases.

$$K \propto (e - 0.1)^2$$

Soil Structure: In case of floculated structure, permeability is high whereas in case of dispersed structure permeability is less.

Note: Floculated & dispersed structures are observed in fine grain soil.

Stratified soil deposites: In case of stratified soil deposites the permeability in parallel flow is always greater than permeability in Normal (Perpendicular) flow.

Estimation of permeability through stratified soil Deposites:

* Parallel flow (|| to stratified soils):

Note: Since head causing flow (ΔH) and length of flow path (L) is same for all the three soils therefore hydraulic

$$Q = Q_1 + Q_2 + Q_3$$

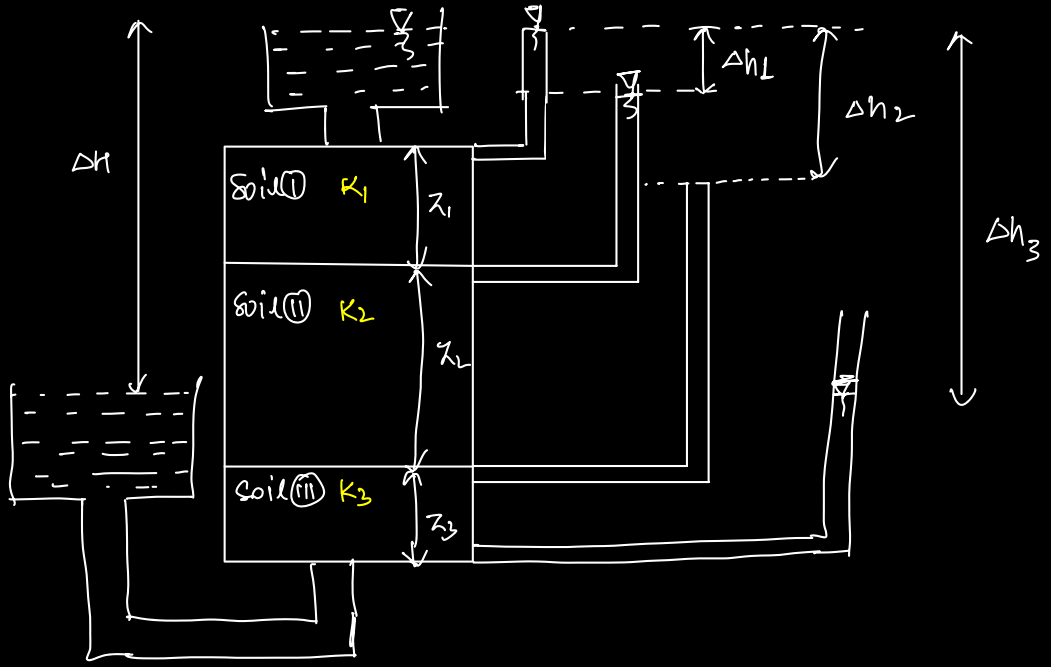
$$K_{eq}\, iA = K_1 \cdot iA_1 + K_2\, iA_2 + K_3\, iA_3$$

$$1 \times [K_{eq}(z_1 + z_2 + z_3)] = [K_1 z_1 + K_2 z_2 + K_3 z_3] \times 1$$

$$K_{eq} = \frac{K_1 z_1 + K_2 z_2 + K_3 z_3}{z_1 + z_2 + z_3}$$

K_{eq}: Equivalent coefficient of permeability

Water flow is perpendicular to stratified soil deposits:



$$\Delta H = \Delta H_1 + \Delta H_2 + \Delta H_3$$

$$i(z_1 + z_2 + z_3) = i \cdot z_1 + i z_2 + i z_3$$

$$\therefore \quad i = \frac{v}{k} \quad \text{(from darcy's law)}$$

$$\frac{v}{K_{eq}}(z_1 + z_2 + z_3) = \frac{v}{K_1} z_1 + \frac{v}{K_2} z_2 + \frac{v}{K_3} z_3$$

$$K_{eq} = \frac{z_1 + z_2 + z_3}{\left(\frac{z_1}{K_1}\right) + \left(\frac{z_2}{K_2}\right) + \left(\frac{z_3}{K_3}\right)}$$

Note:

$$K_{eq} = \frac{\Sigma K z}{\Sigma z} \quad \therefore \text{ parallel flow}$$

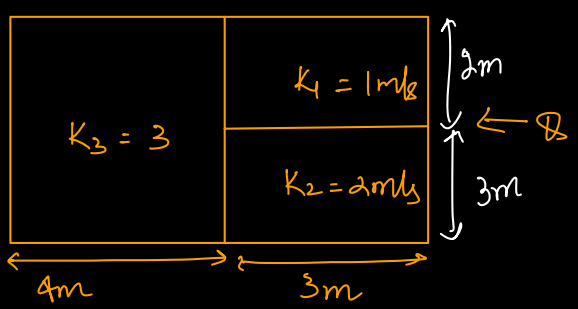
$$K_{eq} = \frac{\Sigma z}{\Sigma \frac{K}{z}} \quad \therefore \text{ perpendicular flow condition}$$

Note: $(K_{eq})_{\text{parallel flow}} > (K_{eq})_{\text{perpendicular flow}}$

for example:

$$\left\{ K_{eq} = \frac{\Sigma z}{\Sigma \frac{z}{K}} \right\}$$

Question Calculate K_{eq}



1st stage : $K_H = \frac{2 \times 1 + 3 \times 2}{5} = \frac{8}{5} = 1.6$ m/s

2nd stage : $\frac{7}{\frac{3}{1.6} + \frac{4}{3}} = 2.18$ m/s.

DETERMINATION OF COEFFICIENT OF PERMEABILITY

Indirect Method

- Allen Hazen eqn
- louden's equation
- Coreny carman's equation
- Terzaghi Equation
- Consolidation equation ($K = C_v m_v \gamma_w$)

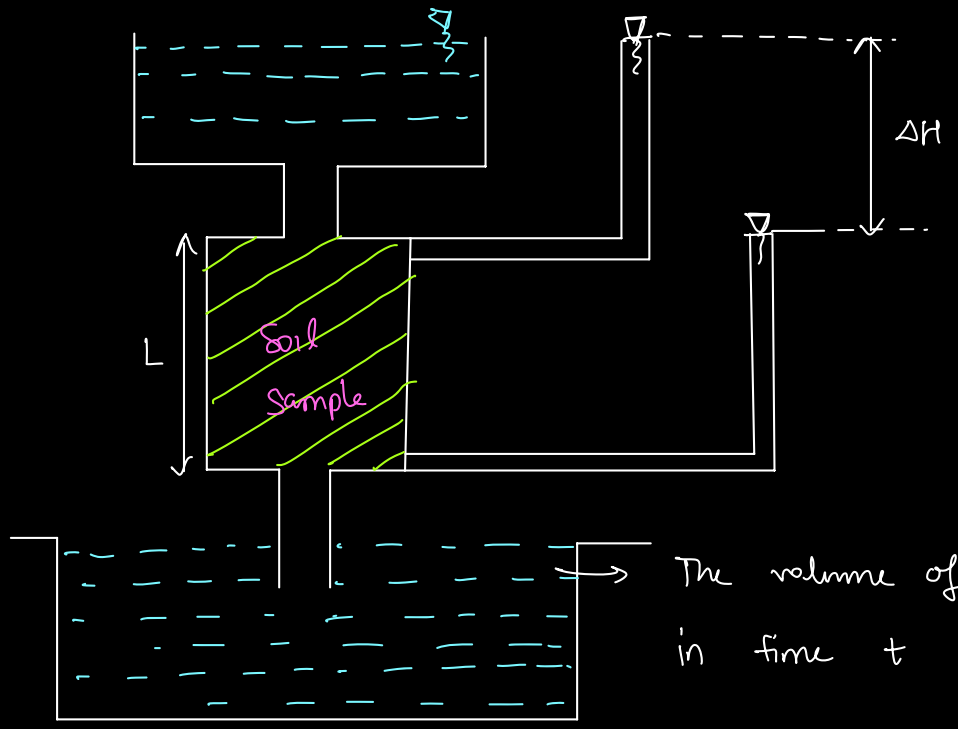
Field Method

- Pumping in Method
- Pumping out method

Laboratory Method

1. Constant head permeability test
2. Falling head permeability test
3. Capillary permeability test.

1. CONSTANT HEAD PERMEABILITY TEST: This test is suitable to determine the permeability of coarse grain soil.



The volume of water (v) collected in time t can be measured.

According to the darcy's law

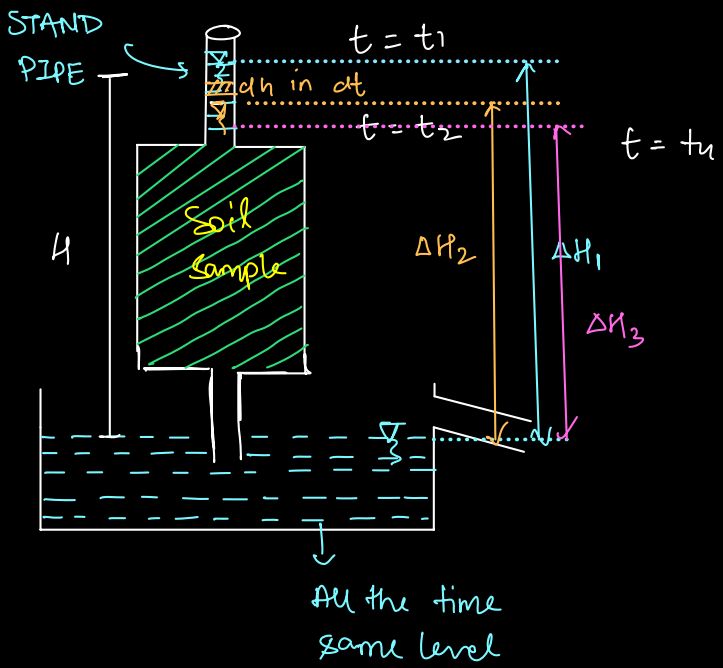
$Q = K \cdot i \cdot A$

$$\frac{V}{t} = K \cdot \left(\frac{\Delta H}{L}\right) \cdot A$$

$$K = \frac{VL}{t \cdot \Delta H \cdot A}$$

A: cross sectional Area

2. FALLING HEAD PERMEABILITY TEST: It is also termed as variable head permeability test. It is used to find permeability of fine grain soil.



→ The discharge through stand pipe = discharge through soil sample.

$$Q = A \cdot V$$

Let us assume Area of stand pipe = a,

$$Q_{st} = -a \cdot \frac{dh}{dt} = K\,i\,A$$

$$= -a \cdot \frac{dh}{dt} = K \cdot A \cdot \frac{h}{L}$$

$$= -a \int_{H_1}^{H_2} \frac{dH}{H} = \int_{t_1}^{t_2} \frac{KA\,dt}{L}$$

$$= -a\left[\ln H_2 - \ln H_1\right] = \frac{KA}{L}(t_2 - t_1)$$

$$\Rightarrow \quad a \ln\left(\frac{H_1}{H_2}\right) = \frac{KA}{L}(t_2 - t_1)$$

$$\Rightarrow \quad \ln\left(\frac{H_1}{H_2}\right) = \frac{KA}{aL}(t_2 - t_1)$$

$$\Rightarrow \quad K = \frac{2.303\, aL}{AT} \log_{10}\left(\frac{H_1}{H_2}\right) \qquad \text{Ans}$$

Let, @ any time t_1, head is h_1
at any time t_2, head is h_2
at any time t_3, head is h_3

Note: let time interval (T) is same ($t_2 - t_1 = t_3 - t_2$)

$$K = \frac{2.303\, aL}{AT} \log_{10}\left(\frac{H_1}{H_2}\right) \qquad \text{— (I)}$$

$$K = \frac{2.303\, aL}{AT} \log_{10}\left(\frac{H_2}{H_3}\right) \qquad \text{— (II)}$$

Equating (I) & (II):

$$\frac{H_1}{H_2} = \frac{H_2}{H_3}$$

$$H_2 = \sqrt{H_1 \cdot H_3} \qquad \text{— eq A}$$

→ Using above concept any missing readings can be determined at any given time.

Let.

at $t_1 \rightarrow h_1$

$t_2 \rightarrow h_2$?

$t_3 \rightarrow h_3$

'h_2' can be determined as

$$\{h_2 = \sqrt{h_1 \cdot h_3}\}$$

Note: In this test number of readings are measured at different times. let time interval is kept same and readings are

$h_1, h_2, h_3, h_4, h_5, h_6$ - - - - -

Then from the above equation A.

$$\frac{h_1}{h_2} = \frac{h_2}{h_3} = \frac{h_3}{h_4} = \frac{h_4}{h_5} = \frac{h_5}{h_6} \quad - - - - - - -$$

$$h_2 = \sqrt{h_1 \cdot h_3}, \quad h_3 = \sqrt{h_2 \cdot h_4}, \quad h_4 \sqrt{h_5 \cdot h_3}$$

These results shows that readings are consistent in nature

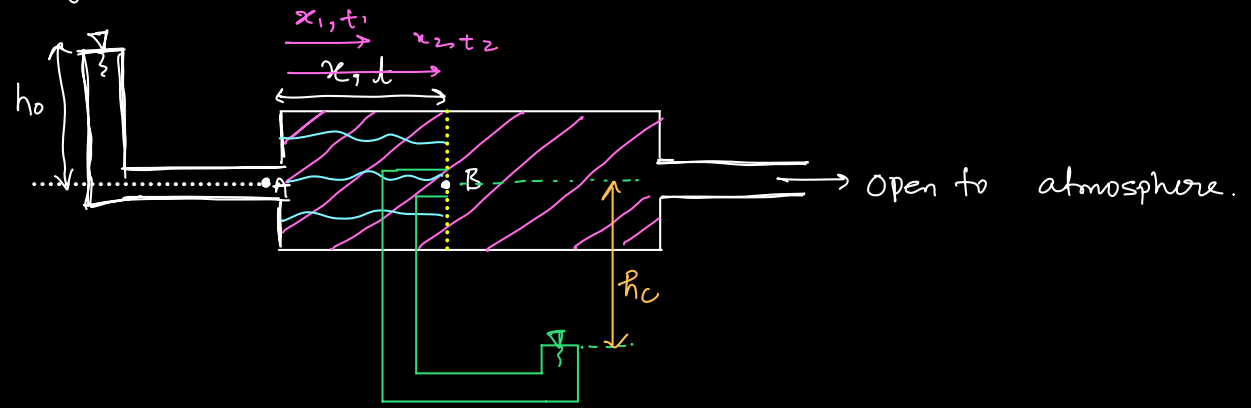
3. Horizontal capillary permeability test:

→ This test is suitable to determine the coeff. of permeability of partially saturated soil

→ The test is performed in a glass tube of 35cm in length & 4 cm in diameter.

→ At one end water is supplied and at the other end air is allowed to escape from the tube

→ In this test dry soil sample can be filled within the glass tube.



→ The head at the point A = h_0

The head at the point B = $-h_c$

The head causing flow (Head loss or head difference) = $h_A - h_B$

$$\Delta H = h_0 - (-h_c)$$

$$\Delta H = h_0 + h_c$$

Hydraulic gradient, $i = \frac{\Delta H}{L} = \frac{h_0 + h_c}{x}$

$$i = \frac{h_0 + h_c}{x}$$

Seepage velocity, $v_s = \frac{dx}{dt}$

$$v_s = \frac{v}{Sn} = \frac{K_u\, i}{Sn}$$

$$\frac{dx}{dt} = \frac{K_u\, i}{Sn}$$

$$\frac{dx}{dt} = \frac{K_u (h_0 + h_c)}{x \cdot S \cdot n}$$

$$\int_{x_1}^{x_2} x \cdot dx = \int_{t_1}^{t_2} \frac{K_u (h_0 + h_c)\, dt}{S \cdot n}$$

$$\frac{(x_2^2 - x_1^2)}{2} = \frac{K_u (h_0 + h_c)}{S \cdot n} (t_2 - t_1)$$

$$\frac{x_2^2 - x_1^2}{(t_2 - t_1)} = \frac{2 K_u (h_0 + h_c)}{S \cdot n}$$

h_0: Initial head at point A

h_c: capillary head which is negative

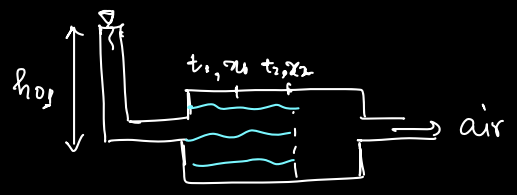
S: degree of saturation

n: porosity

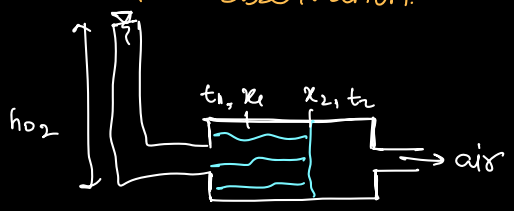
K_u: coeff. of permeability of partially saturated soil.

→ In the above equation there are two unknowns K_u & h_c. Therefore two sets of observations are required with two different heads to determine the permeability.

1st observation



2nd observation.



$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2 K_u}{S\eta} (h_{o_1} + h_c) \quad - \quad \text{eq(i)}$$

$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2 K_u}{S\eta} (h_{o_2} + h_c) \quad - \quad \text{eq(ii)}$$

→ Solve the above two equations and determine 'K_u' and 'h_c'

→ After completion of test the wet soil sample shall be tested for η, S, w, γ etc.

when $S < 1$

$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2 K_u}{S\eta} (h_o + h_c)$$

when $S = 1$, the above equation is reduced to

$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2K}{\eta} (h_o + h_c)$$

Question.

Q.1 **A capillary permeability test was performed in two stages. In the first, stage, the wetted surface advanced from it's initial position of 3cm to 10 cm. In 8 minutes, under a head of 60 cm at the entry of water.**

$S = 92\%$
$\eta = 32\%$

60 cm → 23 cm
24 minutes
230 cm

Solution

$h_o = 60\text{cm}$

$t_2 - t_1 = 480$ second

$x_1 = 3\text{cm}$

$x_2 = 10\text{cm}$

$$\frac{10^2 - 3^2}{480} = \frac{2 \times K_u}{S\eta} (60 + h_c)$$

$$0.19 = \frac{2 \times K_u}{.92 \times .32} (60 + h_c)$$

$$K_u\,(60+h_c) = 0.028 \quad \text{——} \quad (1)$$

$$\frac{23^2 - 10^2}{24\times 60} = \frac{2\times K_u}{0.92\times .32}\,(230+h_c)$$

$$\frac{429}{1440} = \frac{2K_u}{.92\times .32}\,(230+h_c)$$

$$(230+h_c)\,K_u = 0.044 \quad \text{—} (11)$$

(1) / (11) :

$$\frac{60+h_c}{230+h_c} = \frac{0.028}{0.044} = .636$$

$$60+h_c = 146.28 + .636\,h_c$$

$$(h_c = 237.033 \text{ cm})$$

$$K_u = \frac{0.044}{230+237.033} = 0.000094 \text{ cm/s}$$

Absolute Permeability / Intrinsic permeability [K_0] :

The coefficient of permeability depends upon the properties of medium (soil) & properties of fluid both. The study becomes complex when two different fluids are flowing through two different soil medium. In such cases a parameter termed as absolute or Intrinsic permeability is introduced which is independent of properties of fluid.

$K = f$[properties of medium, properties of fluid]

$K_0 = f$[properties of medium]

$$K_0 = \frac{K}{\text{properties of fluid}} = \frac{K}{\left(\frac{\gamma_w}{\mu}\right)_{\text{fluid}}}$$

$$K_0 = K\cdot\left(\frac{\mu}{\gamma_w}\right)_{\text{fluid}}$$

K: Coefficient of permeability

Ko: Coefficient of permeability

$$K_o = K\left(\frac{\mu}{\gamma_w}\right) = \frac{m}{s}\left(\frac{\frac{N\text{-}s}{m^2}}{N/m^3}\right) = m^2$$

Unit of Absolute Permeability: cm^2, m^2, Darcy.

$$1 \text{ Darcy} = 9.87 \times 10^{-13}\ m^2$$

IMPORTANT NOTES:

i) K_{CGS} : depends upon the d, SSA, e, T etc

ii) K_{FGS} : other than, d, SSA, e, T, it depends upon type of minerals and types of cations adsorbed over the surface of the particles.

iii) In case of fine grain soil permeability in increasing order for the adsorbed cations can be given as

In case of montmorillonite:

$$K^+ < Na^+ < H^+ < Ca^{+2}$$

In case of Kaolinite:

$$Na^+ < K^+ < Ca^{2+} < H^+$$

(iv) It is observed that if adsorption of sodium cations is more permeability will be less in both the minerals, thus in many engineering projects salted water is used to reduce permeability

v) To study the permeability topic, the knowledge of water content is required.

STRESS IN SOIL

TOTAL STRESS (σ): Total stress can be defined as the total load per unit area at any given plane in the soil due to its self weight or overburden pressure.

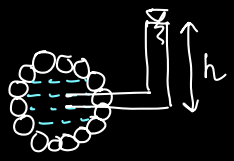
$$\sigma = \frac{\text{load}}{\text{Area}} = \frac{\text{self wt}}{\text{Area}} = \frac{\gamma z \cdot A}{A} = \gamma \cdot z$$

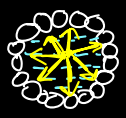
$\sigma = \gamma . z$ (Partially saturated)

$\sigma = \gamma_{sat} \cdot z$ (when soil is fully saturated)

- There are two components included in total stress
  - i) Pore water pressure / Neutral stress
  - ii) Effective stress / Intergranular stress

1. PORE WATER PRESSURE (u): Pore water pressure is the pressure transmitted by the pore water or can be defined as the pressure exerted by water of equal amount in all the directions





h : pressure head

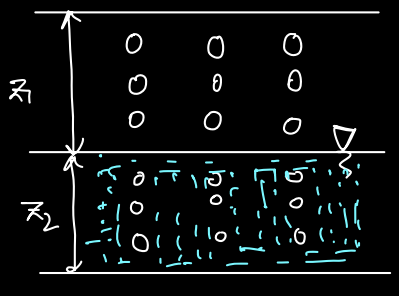
u : Pressure head $\times \gamma_w$

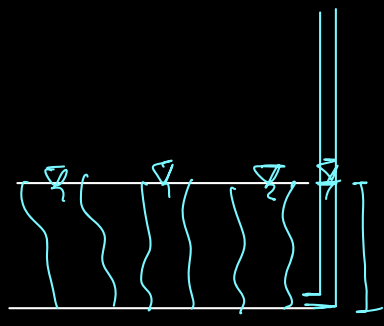
$u = h\gamma_w$

Since pore water pressure acts all around the soil particles therefore it does not bring a particle closer therefore it is called neutral pressure or stress.

Pore water pressure = Pressure head $\times$ unit weight of water

$u = h\gamma_w \quad kN/m^2$





($u = 0$) at free water surface

$u = z_2 \gamma_w$

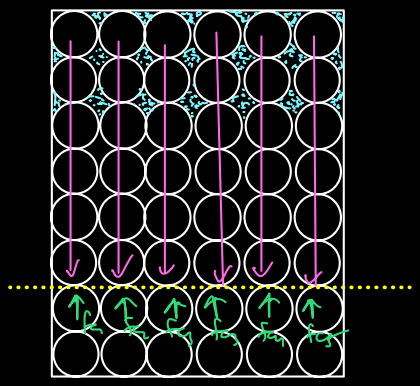
2. EFFECTIVE STRESS ($\bar{\sigma}$) : It is the pressure transmitted by particle to particle contact area.

According to terzaghi (1936),

$$\bar{\sigma} = \sigma - u \quad \text{— eqn (A)}$$

$$\bar{\sigma} = \frac{\text{Total Contact Force}}{\text{Total contact Area.}}$$

↳ later it was modified by lambe and whitman (1969.)



* we have taken one section at any depth in the soil. let the total weight (load) over this section is P.

This total load shall be shared between Contact forces and pore water

F_c : Total contact force

A_c : Total contact Area.

$$P = F_c + u(A - A_c)$$

$$\frac{P}{A} = \frac{F_c}{A} + u\left(\frac{A}{A} - \frac{A_c}{A}\right)$$

$$\frac{P}{A} = \frac{F_c}{A} + u\left(1 - \frac{A_c}{A}\right)$$ → Ignored.

$$\frac{P}{A} = \frac{F_c}{A} + u$$

$$\frac{F_c}{A} = \bar{\sigma} - u \quad \text{— eqn (B)}$$

Comparing eqn (A) and eqn (B)

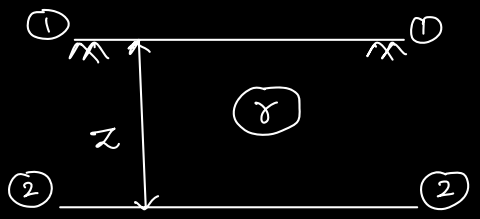
$$\therefore \bar{\sigma} = \frac{F_c}{A} = \frac{\text{Total contact force}}{\text{Total crosssectional Area.}}$$

Note: As we have seen, effective stress is total contact force to the total crosssectional area, thus it is physically meaningless, ∴ it can't be measured directly. To calculate the effective stress $\bar{\sigma}$, first estimated total stress (σ) and pore pressure (u).

$$\boxed{\bar{\sigma} = \sigma - u} \quad KN/m^2$$

Note: The compressibility, shear strength, bearing capacity, all depends upon the effective stress.

Case I: NO WATER TABLE:



at (2),

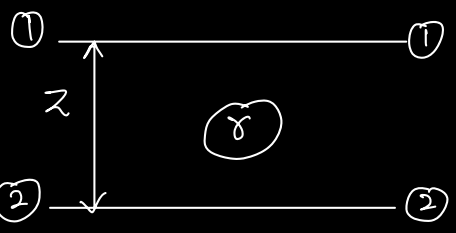
$\sigma_{22} = \gamma \cdot z$

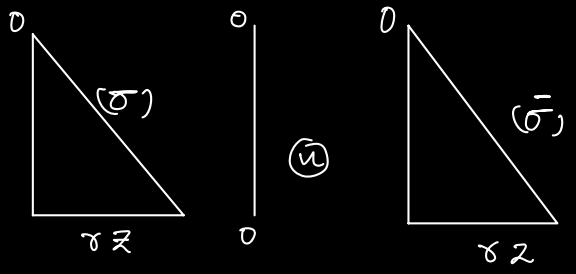
$u_{22} = 0$

$\bar{\sigma}_{22} = \gamma z - 0 = \gamma z$

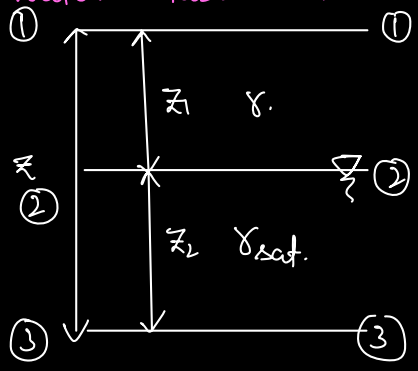
at (1),

$\sigma_{11} = 0, \quad u_{11} = 0, \quad \bar{\sigma}_{22} = 0$





Case II: Water table at any depth



$\sigma_{11} = 0, \quad u_{11} = 0, \quad \bar{\sigma}_{11} = 0$

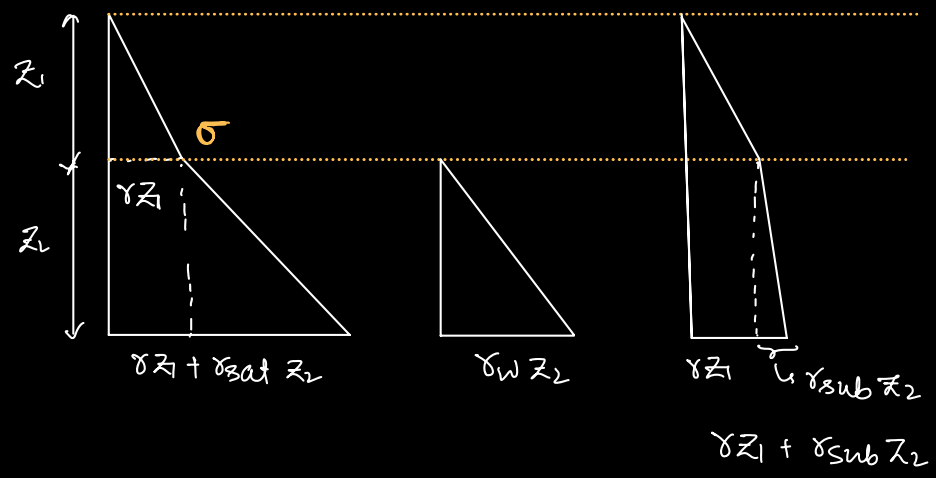
$\sigma_{22} = \gamma z_1, \quad u_{22} = 0, \quad \bar{\sigma}_{22} = \gamma z_1$

$\sigma_{33} = \gamma z_1 + \gamma_{sat} z_2$

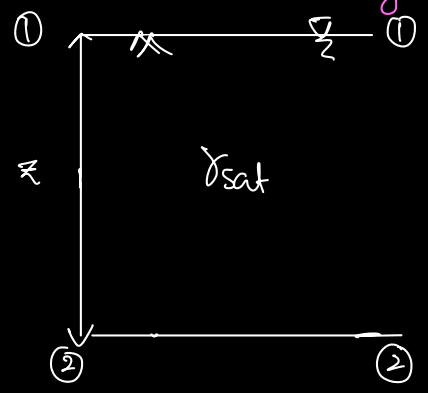
$u_{33} = \gamma_w z_2$

$$\boxed{\bar{\sigma}_{33} = \gamma z_1 + \gamma' z_2}$$

γ' → Submersed unit weight



Case III: Water table at ground table:

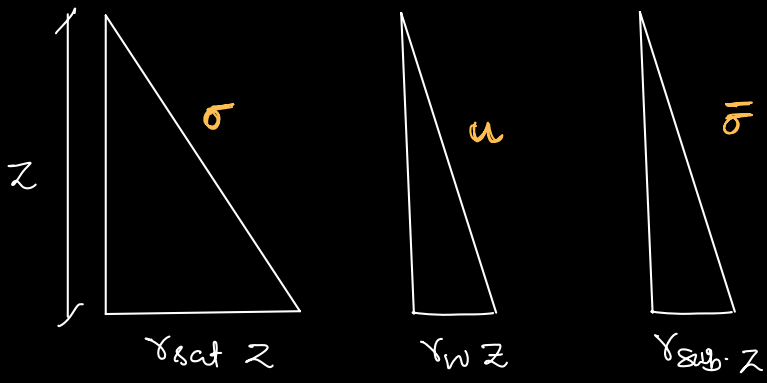


$\sigma_{11} = 0, \quad u_{11} = 0, \quad \bar{\sigma}_{11} = 0$

$\sigma_{22} = \gamma_{sat} Z$

$u_{22} = \gamma_w \cdot Z$

$\bar{\sigma}_{22} = \gamma_{sub} Z$



Question

$Z = 5m$ $\quad \gamma = 16\ KN/m^3$

$Z_1 = 2m$ $\quad \gamma_{sat} = 20 KN/m^3$

$Z_2 = 3m$ $\quad \gamma_w = 10 KN/m^3$

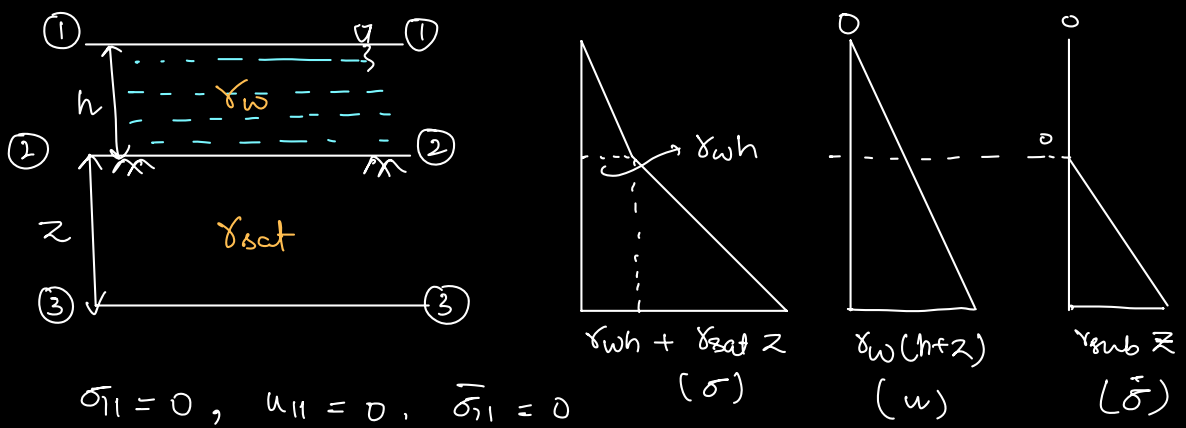
$\bar{\sigma}_① = \gamma \cdot Z = 16 \times 5 = 80\ KN/m^2$

$\bar{\sigma}_② = \gamma Z_1 + \gamma_{sub} Z_2 = 16 \times 2 + (20-10)3$

$= 32 + 30 = 62\ KN/m^2$

$\bar{\sigma}_③ = \gamma_{sub} Z = 10 \times 5 = 50\ KN/m^2$

Case IV: Water table above the ground table:



$\sigma_{11} = 0$, $u_{11} = 0$, $\bar{\sigma}_{11} = 0$

$\sigma_{22} = \gamma_w h$, $u_{22} = \gamma_w h$, $\bar{\sigma}_{22} = 0$

$\sigma_{33} = \gamma_w h + \gamma_{sat} z$, $u_{33} = \gamma_w (h+z)$, $\bar{\sigma}_{33} = (\gamma_{sat} - \gamma_w) z$

$\bar{\sigma}_{33} = \gamma_{sub} z$

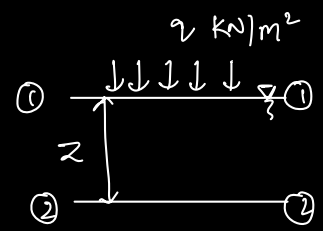
Note ① Below ground surface, if water surface rises, then effective stress decreases

Note ② Below ground surface, if water surface drops, then effective stress increases.

Note ③ Above ground surface, if water table fluctuates (rise or fall) it doesn't lead to any change in the effective stress of soil.

Case V: When soil is subjected to uniform surcharge

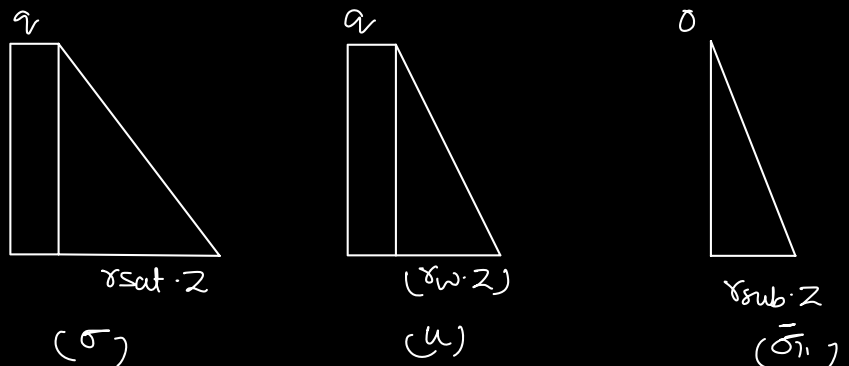
Condition: (SHORT TERM ANALYSIS)



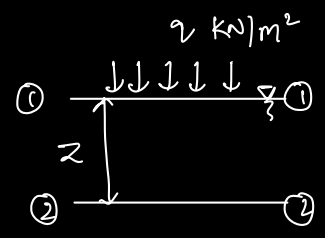
$\sigma_{11} = q$, $u_{11} = q$, $\bar{\sigma}_{11} = 0$.

$\sigma_{22} = q + \gamma_{sat} \cdot z$, $u_{22} = q + \gamma_w z$

$\bar{\sigma}_{22} = \gamma_{sub} \cdot z$



Condition: LONG TERM ANALYSIS

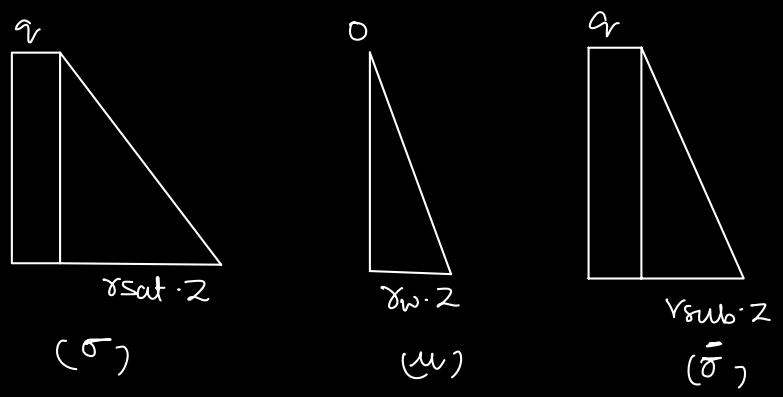


$\sigma_{11} = q$, $u_{11} = 0$, $\bar{\sigma}_{11} = ?$

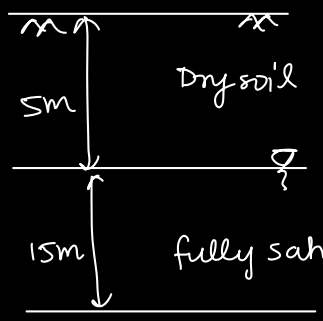
$\sigma_{22} = q + \gamma_{sat} \cdot z$

$u_{22} = \gamma_w z$

$\bar{\sigma}_{22} = q + \gamma_{sub} \cdot z$



Question



$\gamma_w = 10\ KN/m^3$

$G = 2.65$

$e = 0.65$

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 10}{1.65} = 16.06\ KN/m^3$$

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.65+0.65) \times 10}{1.65} = 20\ KN/m^3$$

$\bar{\sigma} = \sigma - u$

$\sigma = 16.06 \times 5 + 20 \times 15 = 380.3\ KN/m^2$

$u = 15 \times 10 = 150\ KN/m^2$

$\bar{\sigma} = 230.3\ KN/m^2$

If when fresh water is present in the soil & soil is fully saturated.

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w = \frac{G\gamma_w + e\gamma_w}{1+e}$$

when any other fluid is present & soil is fully saturated

$$\gamma_{sat} = \frac{G\gamma_w + e\gamma_{fluid}}{1+e}$$

For example seawater is present with unit weight γ_{wsea}.

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_v} = \frac{GV_s\gamma_w + \gamma_{wsea} \cdot V_w}{V_s + V_v}$$

$$= \frac{GV_s \cdot \gamma_w + \gamma_{wsea} \cdot V_w}{V_s\left[1 + \frac{V_v}{V_s}\right]} = \frac{G\gamma_w + \gamma_{wsea} \cdot \frac{V_w}{V_s}}{1 + \frac{V_v}{V_s}}$$

$$= \frac{G\gamma_w + \gamma_{wsea}\frac{V_w}{V_v} \cdot \frac{V_v}{V_s}}{1+e} = \frac{G\gamma_w + \gamma_{wsea} x e S}{1+e}$$

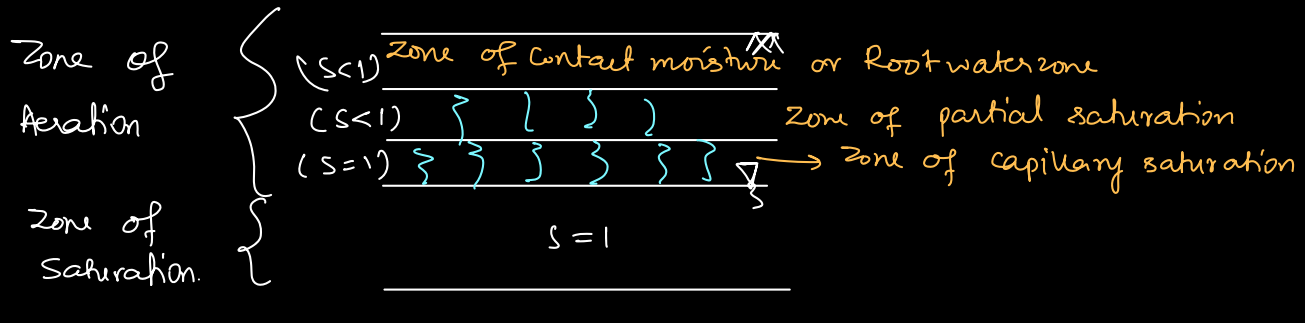
$$(\gamma_{sat})_{\text{(SEA WATER)}} = \frac{G\gamma_w + eS(\gamma_w)_{sea}}{1+e}$$

CAPILLARITY IN THE SOIL:

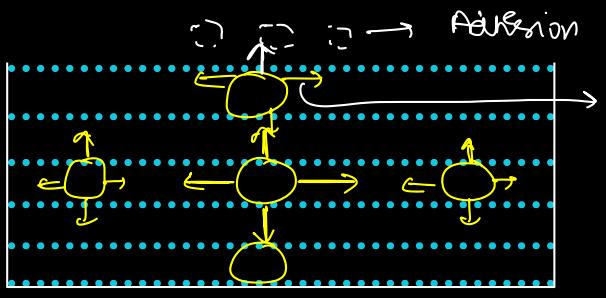
→ The capillary water is the water which is lifted above the free water surface [Groundwater table] by surface tension force [capillary force]

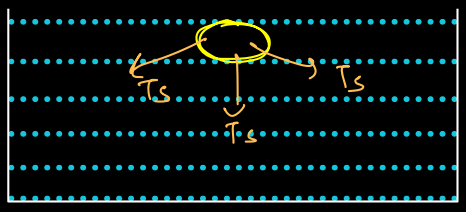
→ The height upto which water rises above the free water surface is termed as height of capillary rise. this phenomena is termed as capillarity

→ This capillary water can be classified into three zones,

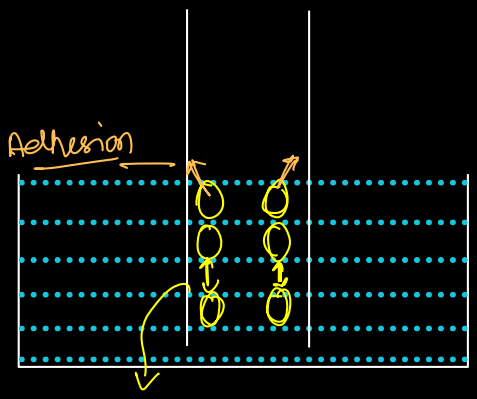


SURFACE TENSION: Net force of Cohesion & adhesion





→ The above net force is termed as surface tension.



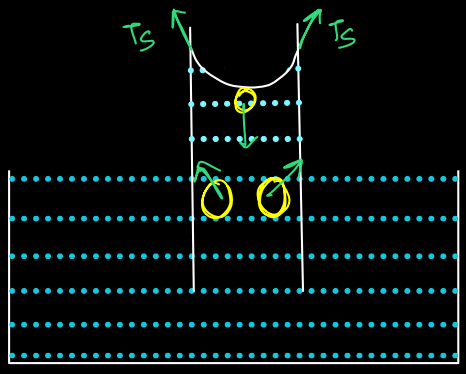
water – water – Cohesion

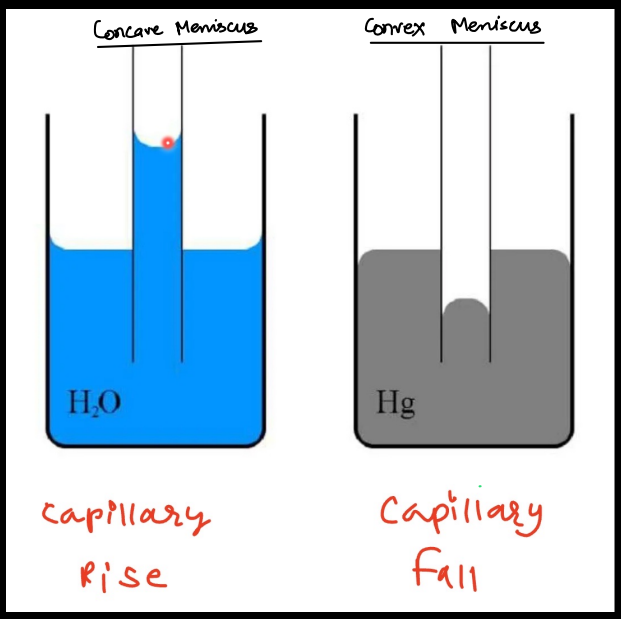
water – Glass – adhesion

water – air – adhesion

Here adhesion > Cohesion

thus Net force = Adhesion - Cohesion.





↳ There can be two types of forces of attraction

Force of attraction → Cohesion → same material

→ Adhesion → Different material.

→ When a tube is inserted in a container filled with water then triple interface can be seen

water – water : Cohesion

water – Glass : adhesion

water – air : adhesion

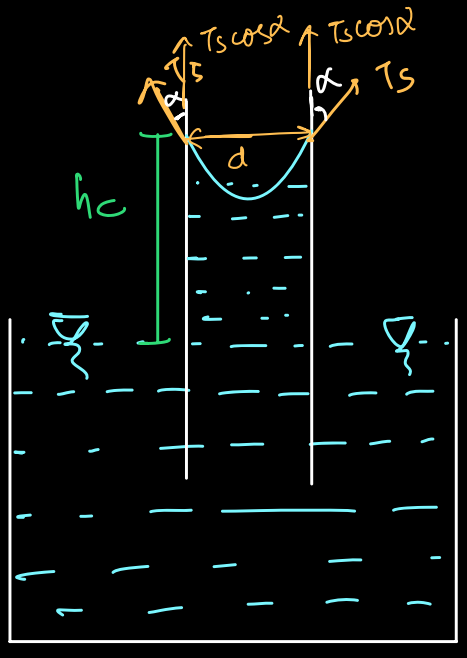
→ The adhesion between water & glass tube is greater than internal cohesion of water, as a result net force of attraction (surface tension force) pulls the water in the tube above the free water surface.

→ But in case of mercury (Hg) the internal cohesion is much greater that the adhesion (b/w Hg & glass tube). As a result depressed meniscus with convex nature is observed

→ But in case of water capillary rise is observed with concave meniscus.

→ If internal surface of the tube is wet then only capillary rise will occur. If it is dry then capillary fall may occur in case of water

→ The capillary water rises in the capillary tube to the height when the force of surface tension becomes equal to weight of the capillary water above free water surface



Total upward force : $T_s \cos\alpha \, (\pi d)$

wt of capillary water : $\gamma_w \cdot h_c \cdot \frac{\pi}{4} d^2$

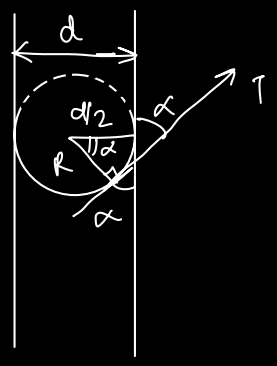
$$T_s \cos\alpha \, (\pi d) = \gamma_w \cdot h_c \cdot \frac{\pi}{4} \cdot d^2$$

$$\frac{4 T_s \cos\alpha}{\gamma_w \cdot d} = h_c$$

$$h_c = \frac{4 T_s \cos\alpha}{\gamma_w \cdot d}$$

$\because$ Since $h_c = \frac{4T_s \cos\alpha}{\gamma_w \cdot d}$

$\therefore$ $h_{cmax} = \frac{4T_s}{\gamma_w \cdot d}$ [when $\cos\alpha \to 1$]



$\cos\alpha = \frac{\frac{d}{2}}{R} = \frac{d}{2R}$

$d = 2R\cos\alpha$

$h_c = \frac{4T_s \cos\alpha}{\gamma_w \cdot d} = \frac{4T_s \cos\alpha}{\gamma_w \cdot 2R \cdot \cos\alpha}$

$$h_c = \frac{2T_s}{\gamma_w \cdot R}$$

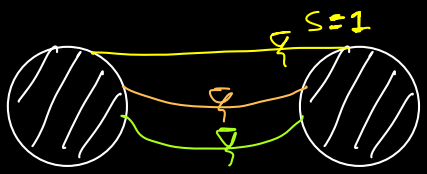
{ Radius of Meniscus }

Free Water Surface: Ground water table

Glass Tube: Soil Solids

Hollow space in Glass tube: Voids

Note: As water content is increased in the soil, degree of saturation increases and as a result, radius of meniscus also increases, when soil becomes fully saturated then radius of meniscus becomes ∞, in that case no capillarity occurs, $h_c = 0$.



Curvature $= 0 = \frac{1}{R} \Rightarrow \{R = \infty\}$

if $S \uparrow \to R \uparrow \to h_c \downarrow$

if $S = 1$, $R = \infty$, $h_c = 0$

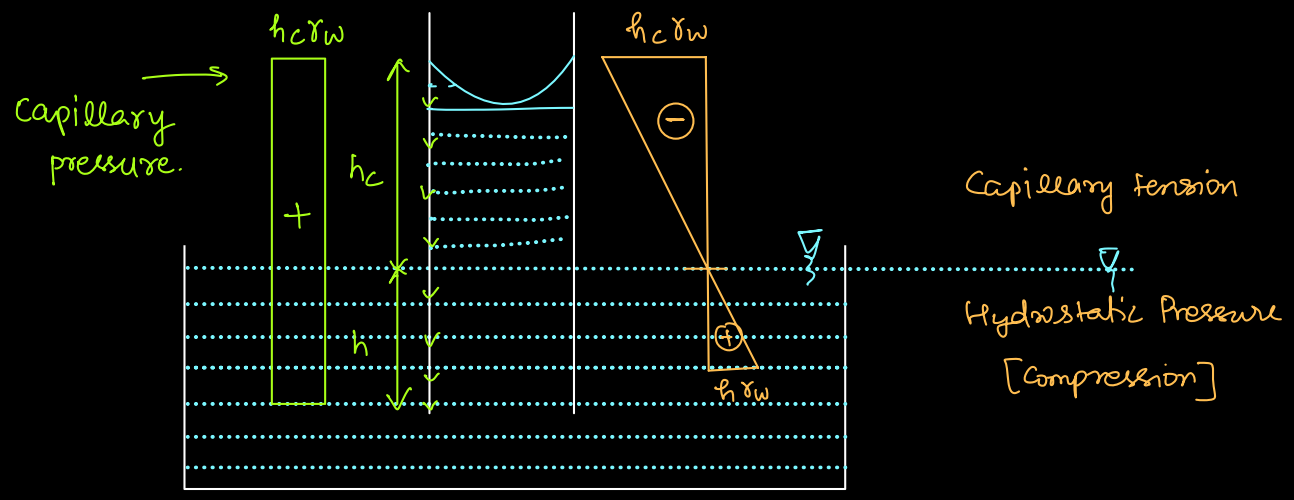
Note: That means capillary rise ocurs only in partially saturated soil. In general soil is always partially saturated, thus capillary fall is not observed in soil.

CAPILLARY TENSION AND PRESSURE: When water rises in the tube by the force of surface tension, the water is under the

tension and the meniscus transfer this tension to the glass tube in the form of compression. This tension is termed as capillary tension, soil suction, suction pressure, pressure deficiency, capillary potential. And the compression force occuring in the glass tube is termed as capillary pressure.

Exactly same picture occurs in case of soil. The water is lifted above the ground water table in the soil by the force of surface tension b/w water and soil solids and the cohesion b/w water. This lifted water is termed as capillary water. This capillary water is under the tension. The concave meniscus transfer this capillary tension to the soil solids in the form of compression termed as capillary pressure. This capillary tension is maximum at the level of meniscus and reduces to zero at the free water surface whereas the magnitude of capillary tension can be taken as $h_c \cdot \gamma_w$.

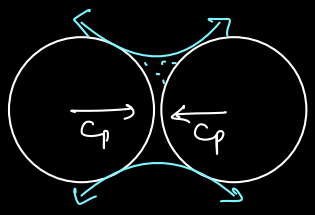
The magnitude of the capillary pressure is equal to the magnitude of the capillary tension at the level of meniscus and this capillary pressure acts throughout the depth of soil.



→ Due to capillary pressure [compressive pressure] the grain to grain contact increases therefore it is also termed as inter-granular pressure or effective stress.

→ Due to capillarity pressure particles come closer and void spaces reduce. As a result soil goes towards densification which leads to an increase in effective stress, degree of denseness, shear strength & bearing capacity.

CONTACT MOISTURE: The water can also be held by the surface tension force at the point of contact b/w the two particles.



This capillary water is termed as "contact Moisture".

This capillary water is under the capillary tension which is transferred to the solid particles in the form of contact pressure which push the particles in closer state. As a result apparent cohesion is developed. This apparent cohesion may be developed in case of sand also.

In this stage if water content is increased, then the DOS & radius of meniscus both increases as a result when S = 1, the meniscus is completely destroyed then no capillary pressure / contact pressure and apparent cohesion is found.

→ This theory of capillary rise in the glass tube cannot be exactly applied to be soil. Because, in the case of soil, voids are not only connected in vertical direction even in horizontal direction also they are connected. Therefore this theory of capillary tube shall be modified for soil.

(i) The diameter of the tube or diameter of meniscus (d) can be taken as 20% of the effective size of the particle.

$$d = 20\% \text{ of } D_{10}$$

$$d = \frac{20}{100} \times D_{10} = \frac{D_{10}}{5}$$

$d = \frac{D_{10}}{5}$

diameter of pipe

$$h_{c_{max}} = \frac{4T_s}{\gamma_w \cdot d} = \frac{20\,T_s}{\gamma_w \cdot D_{10}}$$

$h_{cmax} = \frac{20T_s}{\gamma_w \cdot D_{10}}$

This formula can be used in case of soil

at $t = 20^\circ C$, $T_s = 72.8 \times 10^{-8}$ KN/cm

at $t = 4^\circ C$, $T_s = 75.8 \times 10^{-8}$ KN/cm

(ii) As per **Allen hazen** the maximum height of capillary rise can be given as,

$$h_{cmax} = \frac{C}{e\,D_{10}}$$

D_{10} : in cm C : 0.1 – 0.5 cm^2

e : void ration h_{cmax} : in cm.

(iii) The maximum height of capillary rise considering particles & voids to be perfectly spherical.

$$h_{cmax} = \frac{4T_s}{\gamma_w \cdot d}$$

$$e = \frac{V_v}{V_s} = \frac{\frac{\pi}{6} \cdot d^3}{\frac{\pi}{6} D_{10}^3} = \left(\frac{d}{D_{10}}\right)^3$$

$$d^3 = e \cdot D_{10}^3 \Rightarrow d = e^{1/3} \cdot D_{10}$$

$$h_{cmax} = \frac{4T_s}{\gamma_w \cdot e^3 \cdot D_{10}}$$

Type of soil	h_c (in cm)
Gravel	2 - 10
Sand	10 - 100
Silt	100 - 1000
clay	1000 - 3000

Note: The h_c is more in FGS & less in CGS.

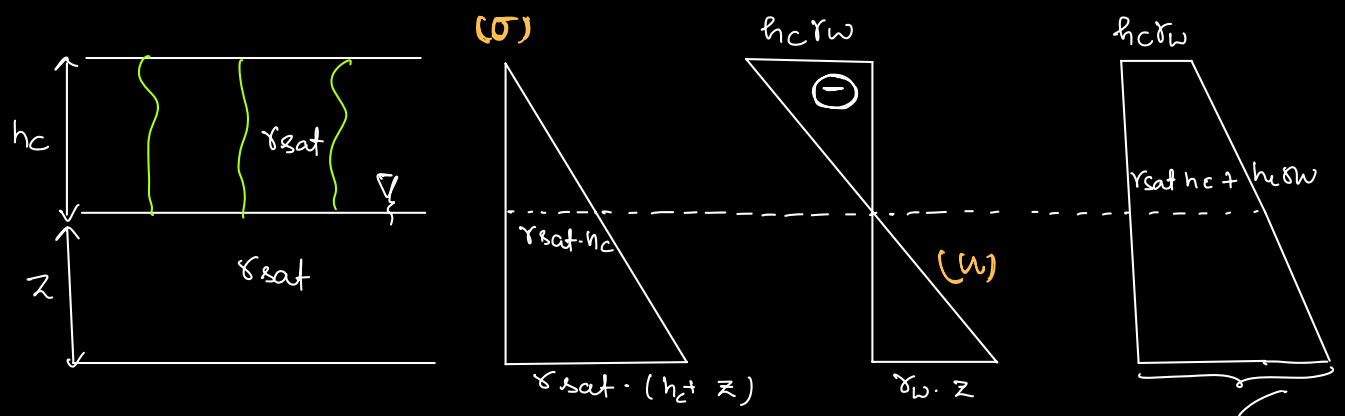
Increasing order of soils according to their height of capillary rise:

Gravel < Sand < Silt < clay < soil colloids

SOIL COLLOIDS: Soil colloids are the non-Soluble suspended particles of size $\leq \frac{1}{1000}$ nm. Soil colloids may be the mineral particles or organic matter. Soil colloids are the finest clay particles. The surface electrical forces are predominant in soil colloids

Effect of capillarity on effective stress:

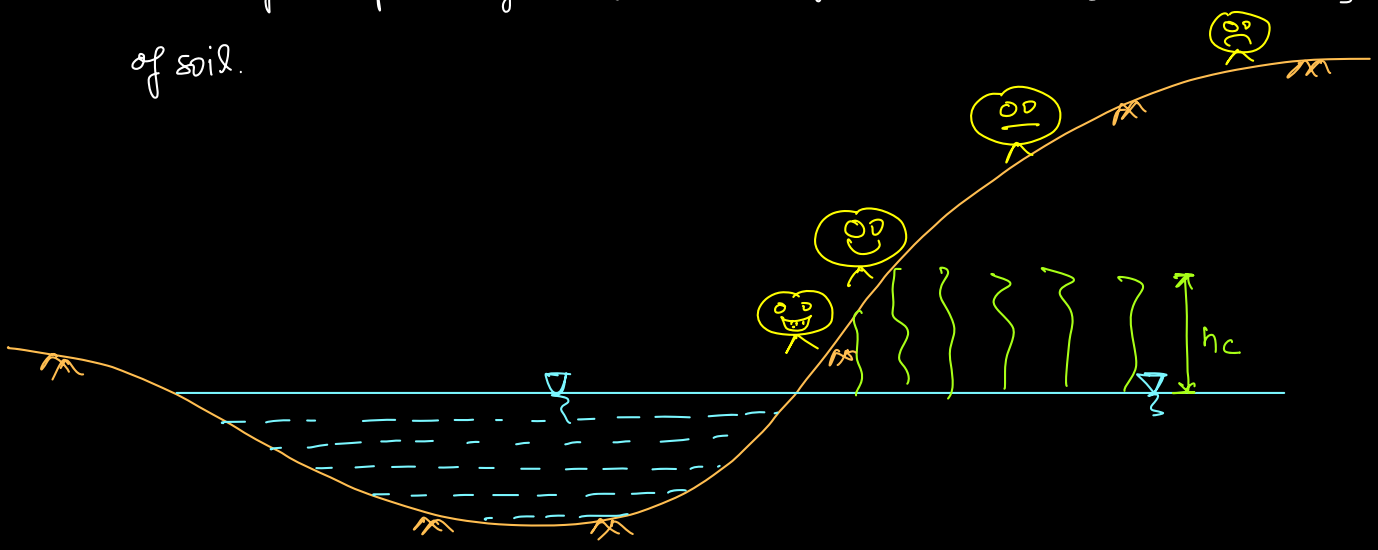
Due to capillary rise the water above the GWT is under the capillary tension which cause negative pore water pressure in the soil. As a result the effective stress increases in the soil.



$\gamma_{sat} h_c + \gamma_{sub} z$

Example: The capilarity can be observed in the wells.

Example: It is observed near the beach. Difficult to walk away from seacost and easy to walk near the sea cost Zone of capillarity rise occur which increases the strength of soil.



The height of capillary rise & capillary tension depends upon the following factors:

Factors	h_c	Capillary Tension ($h_c \gamma_w$)
(i) Particle Size ↑	↓	↓
(ii) Void ratio ↑	↓	↓
(iii) Temperature ↑	↓	↓
iv) Denseness of soil ↑	↑	↑
v) Plasticity index ↑	↑	↑
vi) water content ↑ (or S)	↓	↓

→ The effect of capilarity or application of capillarity can also be seen in different phenomenon as follows:

(i) Baking of clay.

(ii) Frost action.

(iii) Bulking of Sand.

(iv) Capillary Syphonage

(i) Slaking of clay: The slaking of clay is the breakdown of a lump of a soil into small fragments on wetting. When the clay swells the trapped air bursts out.

If the dry soil sample of clay is immersed in the water then water enters in the voids that leads to the formation of menisacs. As a result contact pressure is built which tend to bursts the soil and air comes out. This phenomena disintegrate the lumps of the clay soil into the small fragments. It is not observed in case of sand because contact pressure is less so no bursting or explosion of voids occur.

(ii) Bulking of Sand: When dry sand is slightly moistened and dumped then increase in the volume of sand is observed as compared to dry volume.

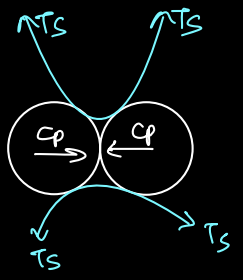
Because as water is added in the sand it forms the meniscus at the point of contact between two particles. As a result capillary pressure [contact pressure] is developed which tend to bring the particle to closer state and develope the apparent cohesion in the sand. This apparent cohesion hold the particles in the large volume of the clusters of honeycomb structures.

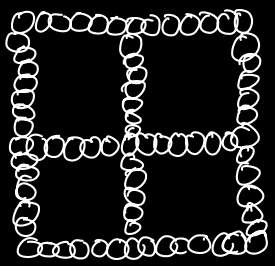
The sand particles do not settle due to gravity because apparent cohesion hold the particles in the clusters of honey comb.

This honeycomb structure encloses large volume of voids filled with air because moisture is held at the point of contact between the particles. This honeycomb structure is not permanent it may be destroyed on increasing the water content.

The maximum bulking increase in volume is observed

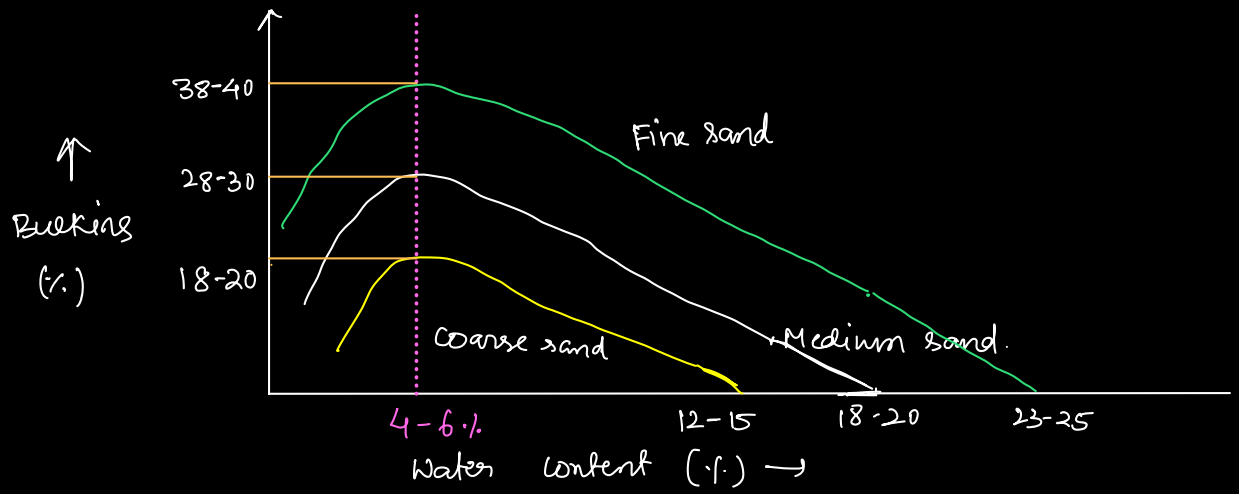
at 4-6% water content If water content is raised beyond the 4-6% then degree of saturation increases and that results the decrease in the apparent cohesion. Therefore volume of soil (Bulkins of sand) starts decreasing. when degree of saturation becomes one in the sand, the meniscus is completely destroyed as a result, apparent cohesion is finished so the particles of sand in the clusters of honey comb seeths down due to their self weight and soil reaches to dense state. on increasing the water content decrease in volume is being observed that can be termed as collapsibility.





Clusters of Honeycomb [4-6% water content]

→ saturated volume of sand S=1



Note: As water content is increased beyond 4-6% → S↑ → capillarity ↓ → Radius of meniscus ↑ → capillary pressure ↓ → Apparent cohesion ↓ → Bulking ↓

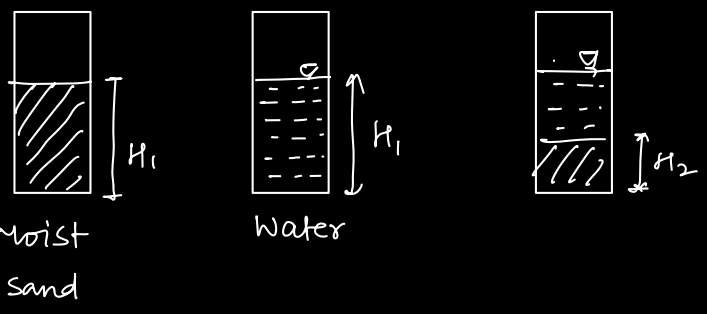
Note: If particle size ↓ → h_c ↑ → capillary tension ↑ → capillary pressure ↑ → Apparent cohesion ↑ → Bulking ↑.

Note: That means large bulking is observed in fine sand as compared to medium & coarse sand.

(Bulking) fine sand > (Bulking) medium sand > (Bulking) coarse sand.

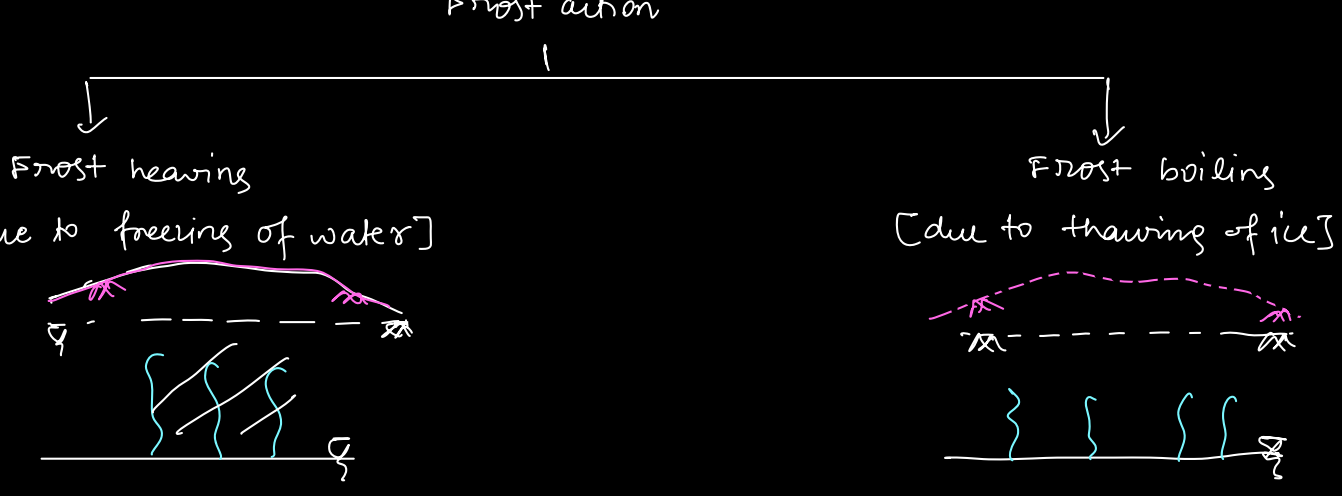
Procedure to measure the bulking of sand:

1. Fill the moist sand (Partially saturated) in the normal container upto the $\frac{2}{3}$rd height
2. Measure the height of the Moist sand by scale as (H_1)
3. Fill the 2nd container with the water upto the same height
4. Now the moist sand is transferred to container 2.
5. Wait for particles to settle down.
6. Measure the height of the settled sand as (H_2).



$$\% \text{ Bulking of sand} = \frac{H_1 - H_2}{H_2} \times 100$$

3) Frost action: Frost action occurs due to freezing & thawing process of water.

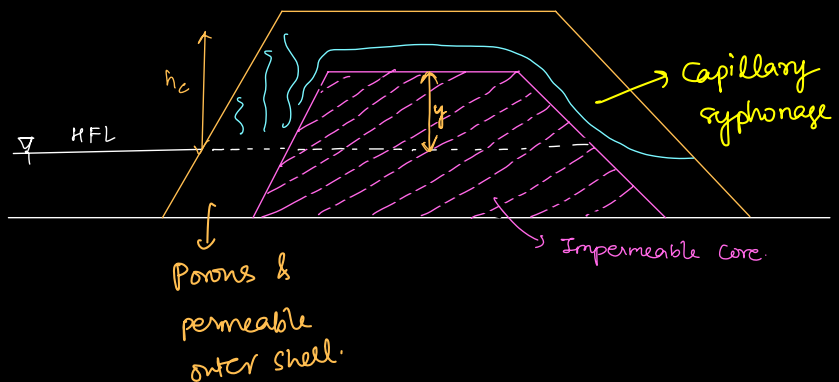


i) Frost heaving: due to decrease in temperature below freezing point, water present in the voids of soil freezes as a result ground level rises up. This phenomenon is termed as frost heaving. As water freezes it gets converted into the ice & the volume of ice is more than the volume of water (approximately 8–9% more). As a result increase in the volume of soil is observed that tends to rise the ground level

(ii) Frost boiling: Due to increase in temperature above the freezing point, the soil in which frost heaving was taken place shows decrease in volume due to melting of ice. This frost boiling makes the soil softer and increase in water content is also observed.

Note: The frost action is generally found in the soil which has porosity, permeability and water.

4) Capillary Syphonage:



→ In order to avoid the seepage through the body of earth dam, a core of impermeable material is constructed at the centre of the earth dam. Now the earth dam is constructed to be termed as composite earth dam.

If height of the core above heighest flood level (HFL) [y] is less than the height of capillary rise (h_c) then water tends to flow towards the downstream slope This phenomenon is termed as capillary syphonage.

→ The capillary syphonage is not only responsible for loss of water from the storage even it cause damages in the downstream slope

→ To avoid the capillary syphonage : $y > h_c$

SEEPAGE ANALYSIS

→ Seepage means Flow of water thorough the inter connected voids of the soil

→ Water always flows from high head to low head.

$$TH = PH + VH + EH$$

VH ignored (flow of water or discharge is very less in the soil thus it is ignored)

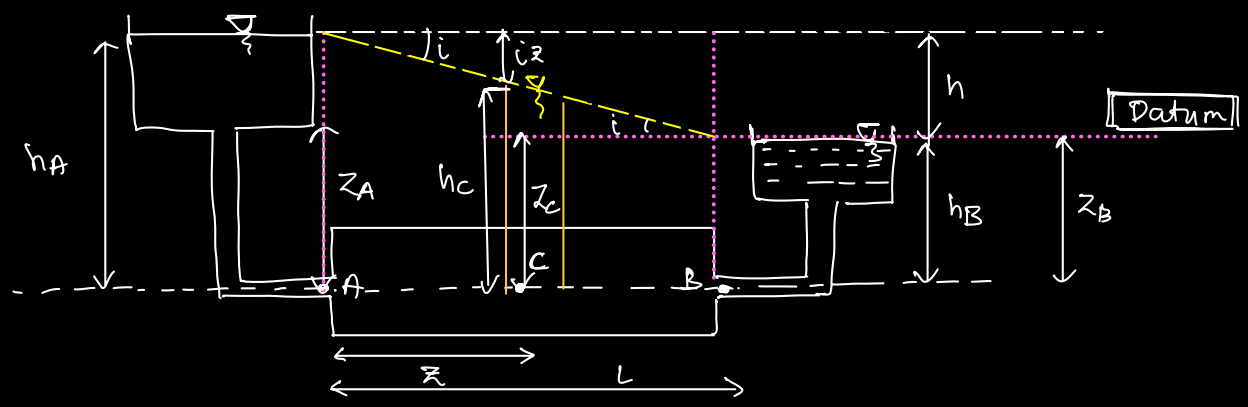
$$TH = PH + EH.$$

↳ Total head / Piezometric head / hydraulic head.

PH: It is the height of the water in the piezometer above the given point

EH: It is the height of given point above the assumed datum line.

Note: If the given point is above the datum line, it is considered as +ve whereas if it is below, it is considered as negative.



$$i = \frac{h}{L}$$

Points	pressure head	Elevation head	Total head
A	h_A	$-Z_A$	$h_A - Z_A$
B	h_B	$-Z_B$	$h_B - Z_B = 0$
C	h_C	$-Z_C$	$h_C - Z_C$

$$h = TH_A - TH_B$$

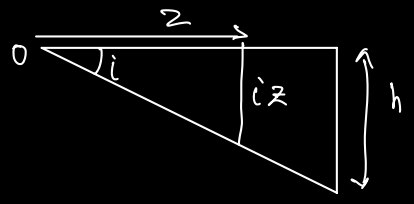
$$h = h_A - Z_A - 0$$

$$\{h = h_A - Z_A\}$$

h: total head loss between point A and B

iZ - total head loss b/w point A and C.

Note: total head loss or hydraulic head loss or piezometric head loss all are same.



$y = iz$

↳ when water covers z distance in direction A → B the head loss is iz.

Question: Find total head at point C?

Total head loss = 80m

$$i = \frac{100}{50} = 2$$

head @ C: $0 + 2 \times 10 = 20m$ Ans

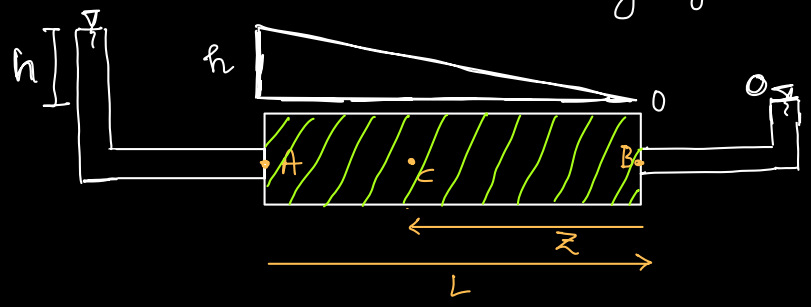
{Head loss = 80 m @ c}

→ when water flows through the soil it exerts the pressure on the soil solids by the virtue of friction drag, it is termed as seepage pressure

→ Seepage pressure always acts in the direction of flow

→ Seepage pressure depends upon the hydraulic head loss or hydraulic head available

let available head at any given point be : h.



Maximum seepage pressure = $h\gamma_w$ KN/m²

Maximum seepage pressure = $(iL)\gamma_w$ KN/m² (where $i = h/L$)

Seepage pressure at any intermediate point C = $i \cdot z \cdot \gamma_w$ KN/m²

Seepage force = Seepage pressure x Projected Area of soil

$f_s = P_s \cdot A$ KN

$f_s = i \cdot z \cdot \gamma_w \cdot A$ KN

$f_s = i \cdot \gamma_w \cdot (A \times z)$ KN

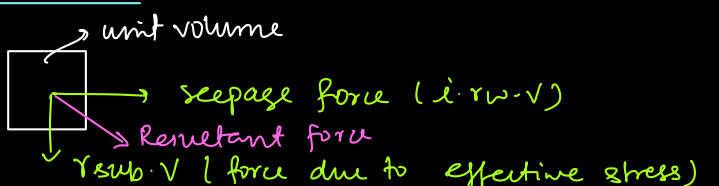
$$\boxed{f_s = i \cdot \gamma_w \cdot V} \text{ KN}$$

→ The specific seepage force can be estimated as seepage force per unit volume.

$$f_{ss} = \frac{f_s}{V} = \frac{i \cdot \gamma_w \cdot V}{V} = i \cdot \gamma_w \text{ KN per unit volume}$$

f_p · Seepage force = $i \cdot \gamma_w$.

In case of horizontal seepage :



Resultant force = $\sqrt{(\text{force due to eff stress})^2 + (\text{seepage force})^2}$

$$FR = \sqrt{(i\gamma_w V)^2 + (\gamma_{sub} V)^2}$$

$$FR = V\sqrt{(i\gamma_w)^2 + (\gamma_{sub})^2}$$

∴ The resultant force for unit volume :

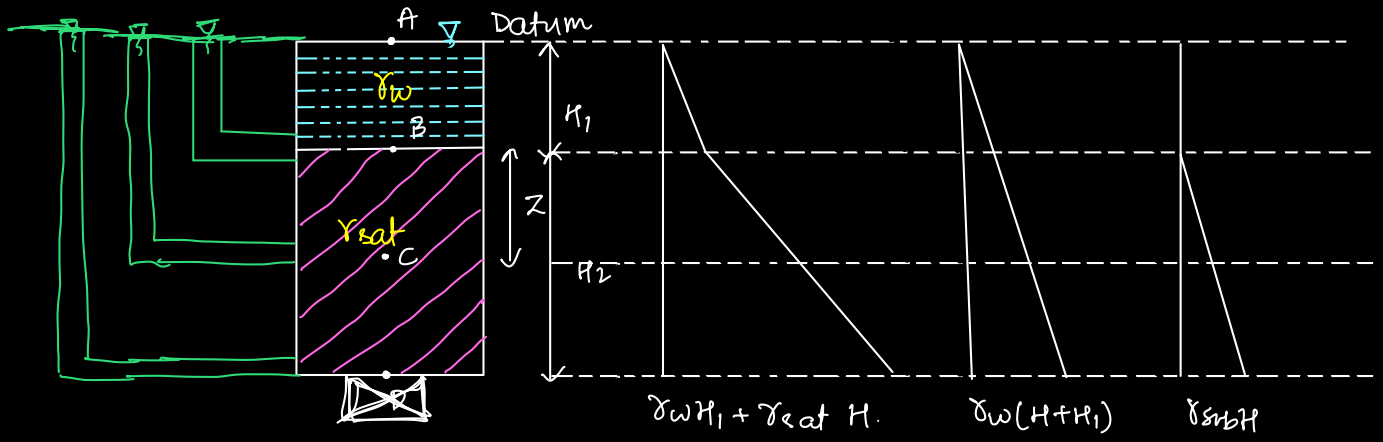
$$(FR)_{V=1} = \sqrt{(i\gamma_w)^2 + (\gamma_{sub})^2} \quad (KN)$$

↙

This is the resultant force in unit volume of soil under the hosizontal seepage condition.

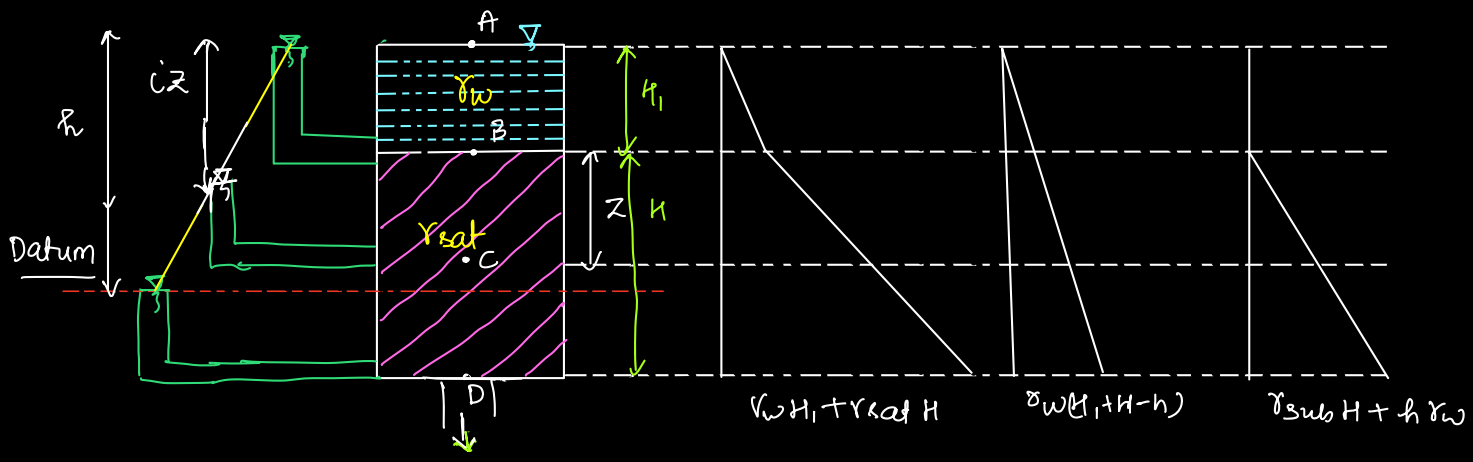
Effect of seepage on effective stress:

(Case I) : No flow condition :



Points	PH	EH	TH	$\bar{\sigma}$	u	$\bar{\sigma}$
A	0	0	0	0	0	0
B	H_1	$-H_1$	0	$\gamma_w H_1$	$\gamma_w H_1$	0
C	H_1+Z	$-H_1-Z$	0	$\gamma_w H_1 + \gamma_{sat} Z$	$\gamma_w(H_1+Z)$	$\gamma_{sub} Z$
D	$H+H_1$	$-H_1-H$	0	$\gamma_w H_1 + \gamma_{sat} H$	$\gamma_w[H_1+H)$	$\gamma_{sub} H$

Case II : Downward flow condition.

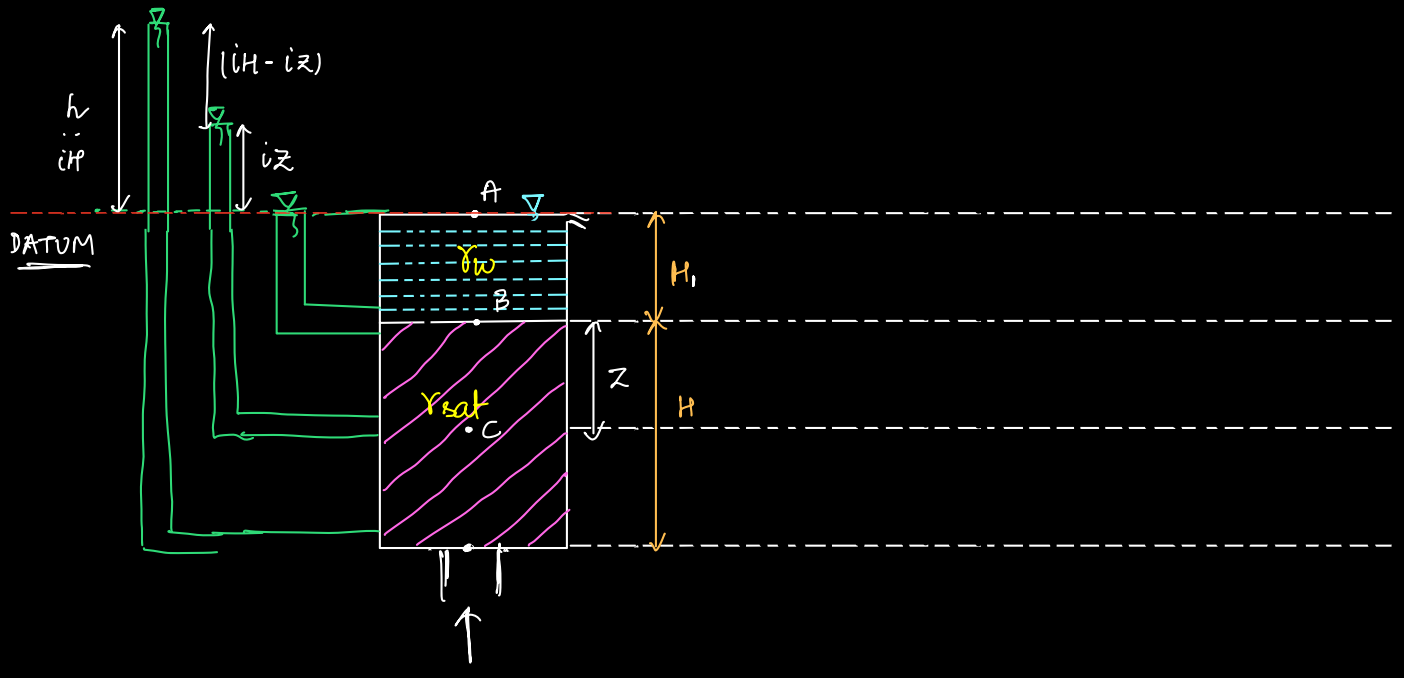


Points	PH	EH	TH	σ	u	$\bar{\sigma}$
A	0	h	h	0	0	0
B	H_1	$h - H_1$	h	$\gamma_w H_1$	$\gamma_w H_1$	0
C	$(H_1 + Z - iZ)$	$h - (H_1 + Z)$	$h - iZ$	$\gamma_w H_1 + \gamma_{sat} Z$	$\gamma_w (H_1 + Z - iZ)$	$\gamma_{sub} Z + \gamma_w (iZ)$
D	$(H_1 + H - h)$	$-[H_1 + H - h]$	0	$\gamma_w H_1 + \gamma_{sat} H$	$\gamma_w (H_1 + H - h)$	$\gamma_{sub} H + \gamma_w h$

Note: The effective stress at any point in the soil under the downward flow condition can be given as

$$\bar{\sigma}_{xx} = \gamma_{sub} \cdot Z + \gamma_w (iZ)$$

Case III : upward flow condition:



Point	PH	EH	TH	σ	u	$\bar{\sigma}$
A	0	0	0	0	0	0
B	H_1	$-H_1$	0	$\gamma_w H_1$	$\gamma_w H_1$	0
C	$Z+H_1+iZ$	$-H_1-Z$	iZ	$\gamma_w H_1 + \gamma_{sat} Z$	$\gamma_w (Z+H_1+iZ)$	$\gamma_{sub} \cdot Z - \gamma_w i Z$
D	$H+H_1+h$	$-H_1-H$	h	$\gamma_w H_1 + \gamma_{sat} H$	$\gamma_w (H+H_1+Z)$	$\gamma_{sub} H - \gamma_w h$

Note: The net effective stress at any section under the upward flow condition is given as

$$\bar{\sigma}_{xx} = \gamma_{sub} Z - iZ\gamma_w$$

Note: On comparing Case I and Case II it is observed that under the downward seepage effective stress increases

Note: On comparing Case I and Case III it is observed that under the upward seepage effective stress decreases.

Quick sand condition / Sand boiling / piping failure:

It is the special case of upward flow condition when seepage pressure becomes equal to submerged weight of the soil. The effective stress becomes zero, as a result soil starts flowing with the water in the direction of flow. This phenomenon is termed as quick sand condition or sand boiling or piping failure or floating condition or bursting condition.

From the upward flow condition,

$$\bar{\sigma}_{xx} = \gamma_{sub} Z - iZ\gamma_w$$

In quick sand condition,

submerged weight ($\gamma_{sub} \cdot Z$) = seepage pressure ($iZ\gamma_w$)

$\therefore \bar{\sigma} = 0, \quad i \to i_c$

$$0 = \gamma_{sub} Z - i_c Z \gamma_w$$

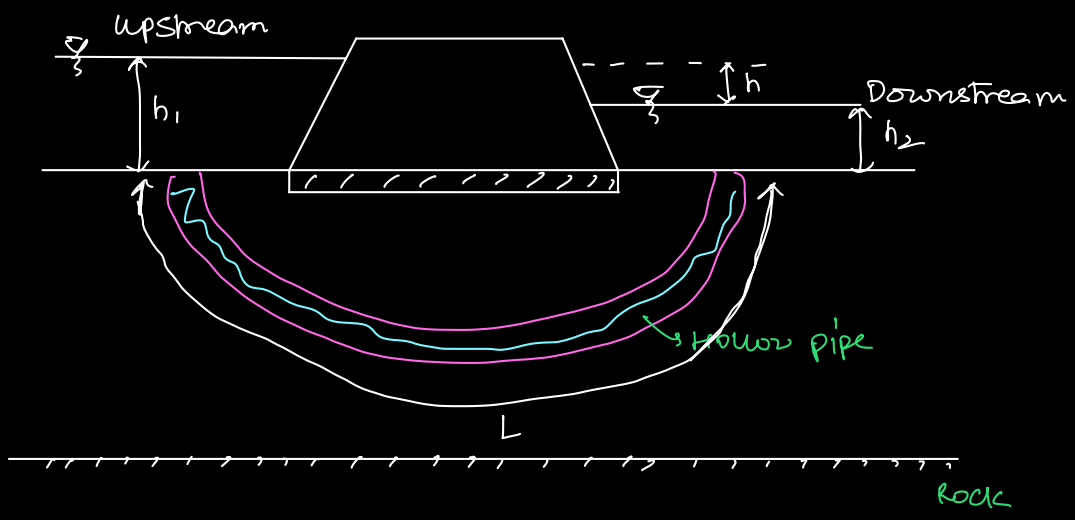
$$i_c = \frac{\gamma_{sub}}{\gamma_w} = \frac{\frac{(G-1)\gamma_w}{1+e}}{\gamma_w} = \frac{G-1}{1+e}$$

$$i_c = \frac{G-1}{1+e}$$

i_c: critical hydraulic gradient / floating gradient / floating gradient.

→ This critical hydraulic gradient can be defined as the maximum hydraulic gradient at which seepage pressure becomes equal to the submerged weight of the soil. As a result effective stress becomes zero.

→ This quick sand condition can be observed at the downstream side of the gravity dam.



$$i = \frac{\Delta H}{L}$$

→ At the downstream side, upward seepage is observed.

→ If the seepage pressure becomes equal or greater than the submerged mass of the soil. Then soil particles starts flowing with seepage water. As a result, hollow pipe is formed below the base of gravity dam. This type of failure is termed as piping failure.

→ The FOS against piping failure can be given as:

$$FOS = \frac{\text{Critical hydraulic gradient}}{\text{exit hydraulic gradient}} = \frac{i_c}{i_{exit}}$$

$$FOS = \frac{i_c}{i}$$

let P_s: Seepage pressure (↑)

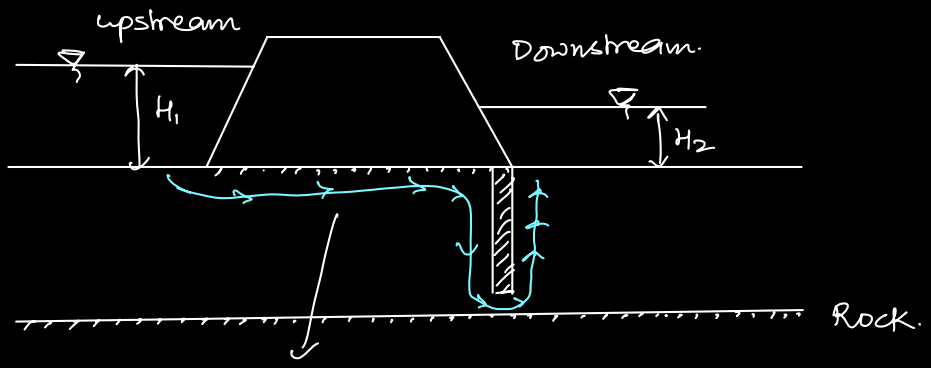
w': submersed weight (↓)

if $i < i_c$, $P_s < w'$, $\bar{\sigma} > 0$, FOS > 1, safe & stable condition.

if $i = i_c$, $P_s = w'$, $\bar{\sigma} = 0$, FOS = 1, critical gradient

if $i > i_c$, $P_s > w'$, $\bar{\sigma} = 0$, FOS < 1, unsafe condition

→ In order to avoid the piping failure the length of flow path can be increased by providing the sheet pile at the downstream side.



Length of flow path is increased.

→ This quick sand condition is observed in case of sands only not in clay & Gravel.

In cohesionless soil (Sands):

$\tau_f = \cancel{c'}^{\,0} + \bar{\sigma} \tan\phi'$

$\tau_f = \bar{\sigma} \cdot \tan\phi'$

$\because P_s = w', \ \bar{\sigma} = 0$

$\{\tau_f = 0 \times \tan\phi'\}$

In case of cohesive soils (Clay):

$\tau_f = c' + \bar{\sigma} \tan\phi'$

$\because P_s = w', \ \bar{\sigma} = 0$

$\tau_f = c' + 0 \times \tan\phi' = c'$

$\boxed{\tau_f = c'}$

→ In case of sand shear strength (τ_f) becomes zero if $\bar{\sigma} = 0$. Therefore, particles of soil start flowing with the water and piping is observed. whereas in case of cohesive soils like

[clay and silt] even if $\delta = 0$ then also $\tau_f \neq 0$, shear strength will be equal to cohesion. Therefore particles don't flow with the water even if the effective stress becomes zero, but in case of gravels cohesion is zero then also piping failure doesn't occur because gravels are highly permeable therefore very large amount of discharge is required to make the piping failure.

$\because Q = KiA$

for unit cross sectional area.

$\therefore i = \frac{Q}{K}$

$\because$ Since hydraulic gradient is very small then it doesn't attain the it's critical value (i_c) so,

No piping failure is observed in case of gravels.

Question

Q. The relationship between the specific gravity of sand (G) and the hydraulic gradient (i) to initiate quick condition in the sand layer having porosity of 30% is

(A) $G = 0.7i + 1$ (B) $G = 1.43i - 1$

(C) $G = 1.43i + 1$ (D) $G = 0.7i - 1$

[GATE 2016 : IISc Bangalore]

Solution

For quick sand condition

$$i_c = \frac{G-1}{1+e}$$

$$1 - n = \frac{1}{1+e}$$

$n = 30\%$

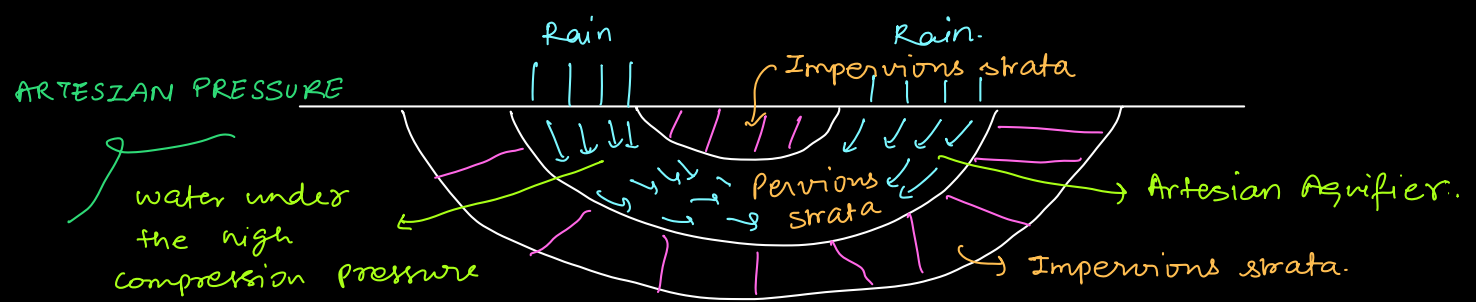
$\frac{1}{1+e} = 0.7$

$e = .43$

$$i_c = \frac{G-1}{1.43}$$

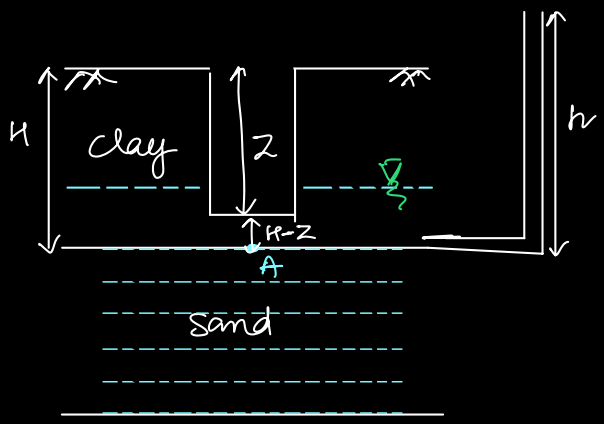
$1.43i + 1 = G$

ARTESIAN PRESSURE:



→ In the artesian aquifer [confined aquifer], water is under the high compression pressure termed as artesian pressure. This artesian pressure can be greater than hydrostatic pressure

→ The quick sand condition can be practically observed during the excavation made in the clay soil which is underlain the sand layer of artesian pressure.



For stability : $\bar{\sigma}_A \geq 0$

$$\gamma_{sat}(H-Z) - h\gamma_w \geq 0$$

$$\gamma_{sat}(H-Z) \geq h\gamma_w$$

$$\gamma_{sat} H - h\gamma_w \geq \gamma_{sat} Z$$

$$\boxed{Z \leq \frac{\gamma_{sat} H - h\gamma_w}{\gamma_{sat}}}$$

→ Z is the maximum depth of excavation upto which quick sand condition is not developed. If excavation is made with the depth greater than Z then quick sand condition will occur in the sand layer and that will not support the clay soil.

Note: Point 'A' is being taken just below the interface of the clay layer and sand layer.

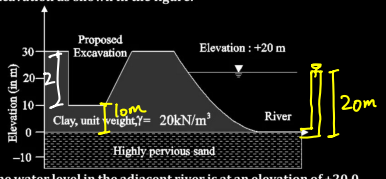
The FOS can be estimated as:

$$FOS = \frac{\text{weight of the soil at the concerned point}}{\text{Total artesian pressure at the concerned point}}$$

$$\left\{ FOS = \frac{\gamma_{sat}(H-Z)}{\gamma_w h} \right\}$$

Question :

Q. At a construction site, a contractor plans to make an excavation as shown in the figure.



The water level in the adjacent river is at an elevation of +20.0 m. Unit weight of water is 10 kN/m³. The factor of safety (up to two decimal places) against sand boiling for the proposed excavation is _____.

[GATE 2018 : IIT Guwahati]

{10x20 / 10x20}

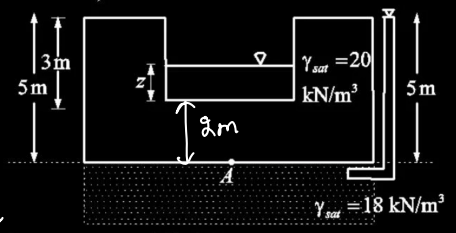
$h = 20\,m$

$H = 30\,m$

$$FOS = \frac{\gamma_{sat} \times 10}{20 \times 10} = \frac{20 \times 10}{20 \times 10} = 1.00$$

Question

Q. A cut is made in a stiff, saturated clay that is underlain by a layer of sand. What should be the height of the water "z" in the cut so that the stability of the saturated clay is not lost. ($\gamma_w = 10 kN/m^3$)



(A) 1 m (B) 1.5 m
(C) 2 m (D) 2.5 m

$$\frac{\gamma_{sat} \times 2 + \gamma_w \times z}{5 \times 10} = 1$$

$$20 \times 2 + 10 \times z = 5 \times 10$$

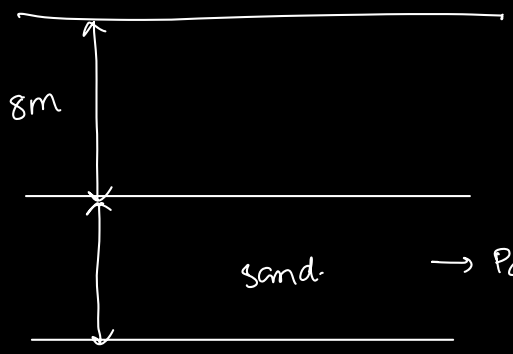
$$10 \times z = 50 - 40$$

$$z = 1m$$

minimum depth for no quick sand condn.

Question

Q. At a given location, 8 m thick saturated clay with water content of 30% and specific gravity of 2.7 is underlain by sand. This sand layer is under artesian pressure equivalent to 3 m head of water. It is proposed to make an open excavation in the clay. How deep can this excavation be made before bottom heaves?



$\rightarrow P_a = \gamma_w \cdot 3$

$w = 30\%$, $G = 2.7$

$eS = Gw$

$e = Gw$

$= 2.7 \times .3 = .81$

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w$$

$$\gamma_{sat} = \frac{2.7 + .81}{1.81} \times 9.81$$

$$= 19.02$$

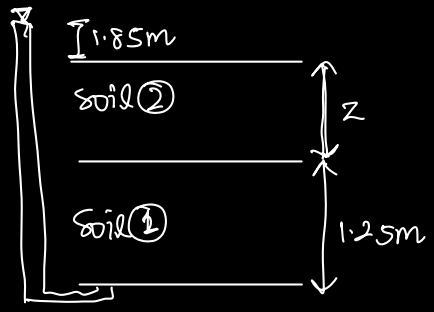
$$\gamma_{sat}(8-2) = 3\gamma_w$$

$$19.02 \times 8 - 19.02 \cdot z = 30$$

$$z = 6.42m$$

Question :

Q. A 1.25 m layer of soil having porosity of 0.35 and specific gravity of 2.65 is subjected to an upward seepage head of 1.85 m. What depth of coarse sand would be required above the existing soil to provide a factor of safety of 2 against piping? Assume that coarse sand has the same porosity and specific gravity as that of soil.



$n = .35$

$G = 2.65$

$h = 1.85m$

$$n = \frac{e}{1+e}$$

$$1 - .35 = \frac{1}{1+e}$$

$$(e = 0.53)$$

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w$$

$$= \left(\frac{2.65 + .53}{1.53}\right)\gamma_w$$

$$= 20.78 \, kN/m^3$$

1st approach: $FOS = \frac{i_c}{i}$

$$i_c = \frac{G-1}{1+e} = (G-1)(1-n) = 1.65 \times (1-.35) = 1.0725$$

$$i = \frac{1.0725}{2} = 0.53625$$

$$0.53625 = \frac{h}{L} = \frac{1.85}{(1.25+Z)}$$

$(Z = 2.2m)$

2nd Approach:

$$W' - P_s \geq 0$$

$$\gamma_{sub}(Z+1.25) - 2 \times h\gamma_w \geq 0$$

$$10.78\,(Z+1.25) - 2 \times 1.85 \times 10 \geq 0$$

$(Z = 2.18m)$

LAPLACE EQUATION AND FLOW NET:

1) Laplace Equation: When the flow is two dimensional / threedimensional through the soil then to analyse the seepage, laplace equation is used which represents the hydraulic head loss due to the flow in the given medium (soil).

Assumption in laplace equation of flow through soil:

- → Soil should be homogeneous & anisotropic
- → Darcy's law is valid. (Soil is fully saturated, laminar flow)
- → Flow is steady state
- → Porewater and soil solids are considered to be incompressible. that means no change in the volume of void during flow.
- → The amount of water entering the soil element must be equal to amount of water leaving the soil element.

Laplace equation can be given as:

$$K_x \frac{\partial^2 H}{\partial x^2} + K_y \frac{\partial^2 H}{\partial y^2} + K_z \frac{\partial^2 H}{\partial z^2} = 0 \quad (3D)$$

$$K_x \cdot \frac{\partial^2 H}{\partial x^2} + K_y \frac{\partial^2 H}{\partial y^2} = 0 \quad (\text{in } 2D)$$

→ where H is the hydraulic head difference or loss

→ K_x, K_y, K_z are the coefficient of permeability in x, y & z directions respectively.

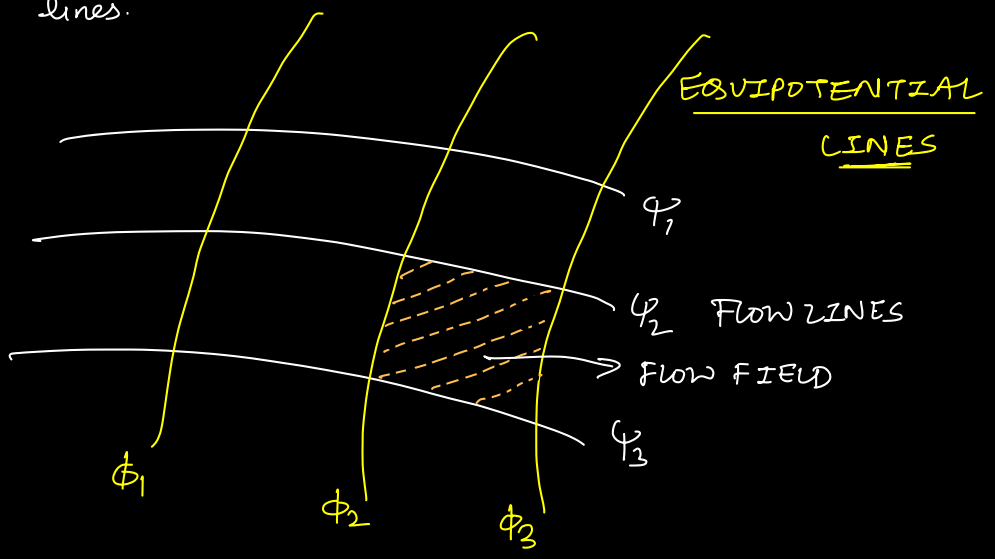
→ for anisotropic soil $[K_x \neq K_y]$

$$K_x \frac{\partial^2 H}{\partial x^2} + K_y \frac{\partial^2 H}{\partial y^2} = 0$$

→ For isotropic soil, $[K_x = K_y]$

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = 0$$

FLOW NET: It is the graphical representation of the solution of the laplace equation. The flow net consists of flow lines & equipotential lines.



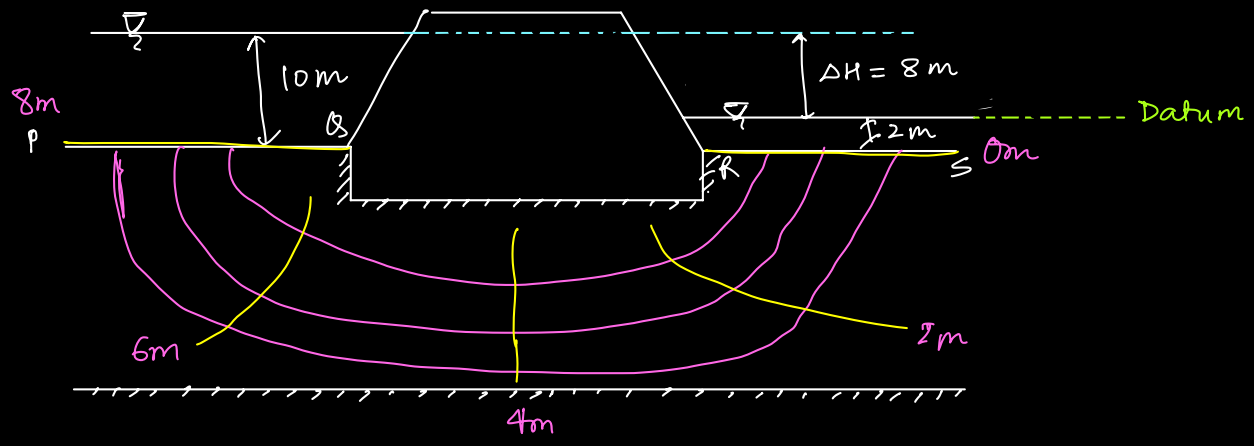
i) FLOW LINES: Flow lines are also termed as stream lines or ψlines. It shows the path of flow of water. The tangent drawn at any point of flow line gives velocity & direction of flow.

ii) EQUIPOTENTIAL LINES: Also termed as ϕ-lines. It is the contour line which connects the points of equal /same total head in the soil.

* Properties of flow net:

* Two flow lines or equipotential lines never intersect each other
* Flow lines and equipotential lines are orthogonal to each other.
* The area bounded between two flow lines is termed as Flow channel. The discharge through each flow channel would be same. The total seepage discharge will be taken as discharge through a single flow channel multiplied by number of flow channels
* Number of flow channels (N_f) will be equal to number of flow lines minus 1.
* The area bound by two flow lines and two equipotential lines is called as flow field.
* The flow field is treated as square in case of Isotropic soil & as rectangle in case of anisotropic soil.
* The flow field [square or rectangular] can be of linear or Non-linear shape
* The total head loss [hydraulic head loss] between two consecutive equipotential lines is always equal or same. The headloss is termed as equipotential drops.
* The number of equipotential drops (N_d) can be taken as Number of equipotential lines minus 1.
* The head loss per drop between two equipotential lines can be taken as total head loss divided by No of equipotential drops
* The flow net remains unchanged even if upstream & downstream heads are reversed, direction of flow is reversed hydraulic head difference is changed, coefficient of permeability of soil is changed. It depends only upon the boundary condition

Note: Smaller the size of flow field means higher the velocity of seepage discharge.



Number of flow lines = 5

$N_f = 5-1 = 4$, $N_f = 4$

Number of equipotential line = 5

$N_d = 5-1 = 4$, $N_d = 4$

$(TH)_{PQ} = (PH)_{PQ} + (EH)_{PQ}$

$(TH)_{RS} = (PH)_{RS} + (EH)_{RS}$.

$\Delta h = \frac{\Delta H}{N_d}$

$(TH)_{PQ} = 10m - 2m = 8m$

$(TH)_{RS} = 2m - 2m = 0m$

$\Delta H = (TH)_{PQ} - (TH)_{RS}$

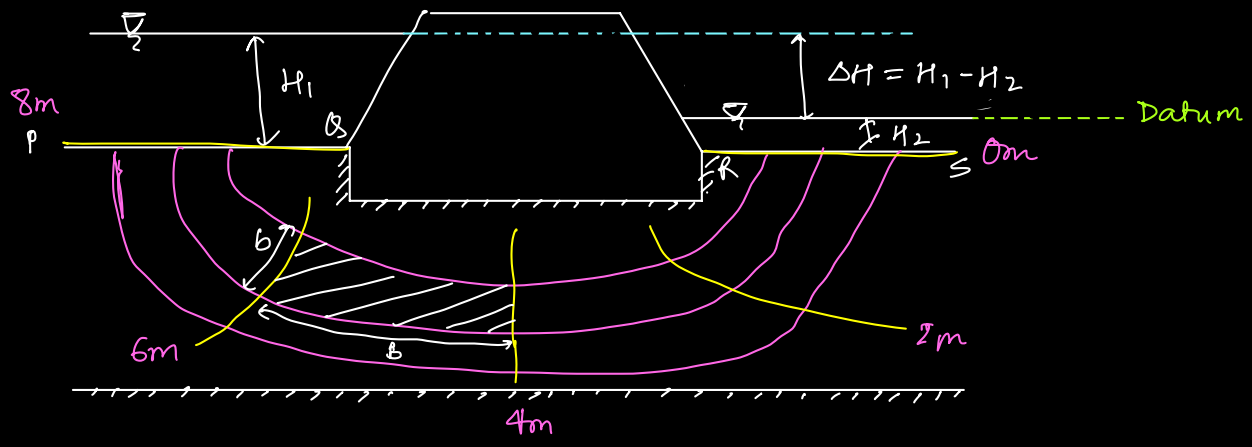
$= 8 - 0 = 8m$

Number of drops : 4

Head loss per drop $= \frac{8}{4} = 2m$

Application of Flow net:

1. Estimation of seepage discharge using flow net:



Discharge through one flow field can be given as

$$q = K \cdot i \cdot A$$

$$q = K \cdot \frac{\Delta h}{b} \cdot b \cdot 1$$

$$q = K \cdot \Delta h$$

$$q = K \cdot \frac{\Delta H}{N_d}$$

The discharge through one flow field = discharge through single flow channel.

∴ Total discharge = No of flow channels × Discharge thru a single channel

$$Q = N_f \times q$$

$$Q = K \cdot \Delta H \cdot \frac{N_f}{N_d} \quad m^3/s \text{ per meter length of the dam}$$

↓

For isotropic soils.

(because we have considered a square flow field)

$\frac{N_f}{N_d}$: shape factor which depends upon the boundary conditions

K: Coefficient of permeability

ΔH: Total head loss

2. Estimation of seepage discharge for anisotropic soil:

The laplace eqn for anisotropic soil given by

$$K_x \frac{\partial^2 H}{\partial x^2} + K_y \frac{\partial^2 H}{\partial y^2} = 0 \rightarrow \text{flow field rectangular}$$

$$\left(\frac{K_x}{K_y}\right)\left(\frac{\partial^2 H}{\partial x^2}\right) + \frac{\partial^2 H}{\partial y^2} = 0$$

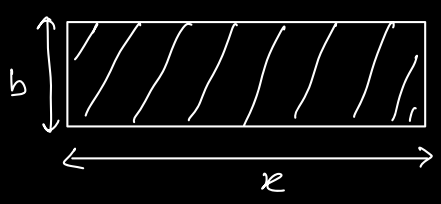
$$\frac{\left(\frac{\partial^2 H}{\partial x^2}\right)}{\left(\frac{K_y}{K_x}\right)} + \frac{\partial^2 H}{\partial y^2} = 0$$

$$\frac{\partial^2 H}{\partial x_T^2} + \frac{\partial^2 H}{\partial y^2} = 0 \rightarrow \text{Flow field square.}$$

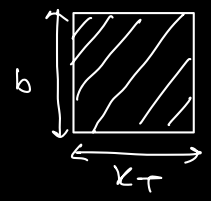
where, $x_T : \sqrt{\frac{K_y}{K_x}} \cdot x$

x_T : transformed dimension of flow field

Actual flow field:



Transformed flow field:



$$x_T < x$$

$\because K_y << K_x$

$\therefore \sqrt{\frac{K_y}{K_x}} < 1$

$\therefore x_T < x$

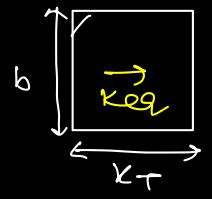
x : Actual size of flow field

x_T : Transformed size of flow field.

→ In the analysis of seepage through anisotropic soil the actual flow field which is rectangular is converted into a transformed flow field which is square.

Actual flow field:

Transformed flow field:



$$\left\{ Q = K_{eq} \cdot \Delta H \cdot \frac{N_f}{N_d} \right\}$$

Discharge through actual flow field

= Discharge through transformed flow field

$$K_x \cdot i_x \ (b \times 1) = K_{eq} \ i_T \ (b \times 1)$$

$$K_x \cdot \frac{\Delta h}{x} = K_{eq} \cdot \frac{\Delta H}{x_T}$$

$$K_x \cdot \frac{1}{x} = K_{eq} \cdot \frac{1}{x_T}$$

$$K_x \cdot \frac{1}{x} = K_{eq} \cdot \frac{1}{\sqrt{\frac{K_y}{K_x}}}$$

$$K_x \cdot \sqrt{\frac{K_y}{K_x}} = K_{eq}$$

$$K_{eq} = \sqrt{K_x \cdot K_y} \quad \text{(in 2D flow condn)}$$

K_{eq} → permeability of transformed flow field

$$K_{eq} = \sqrt[3]{K_x \cdot K_y \cdot K_z}$$

Now the seepage discharge through Anisotropic soil can be given as,

$$Q = K_{eq} \cdot \Delta H \cdot \frac{N_f}{N_d}$$

$$Q = \sqrt{K_x \cdot K_y} \cdot \Delta H \cdot \frac{N_f}{N_d}$$

→ seepage discharge through Anisotropic soil.

(3) Determination of seepage pressure.

The seepage pressure at the concerned point can be given as:

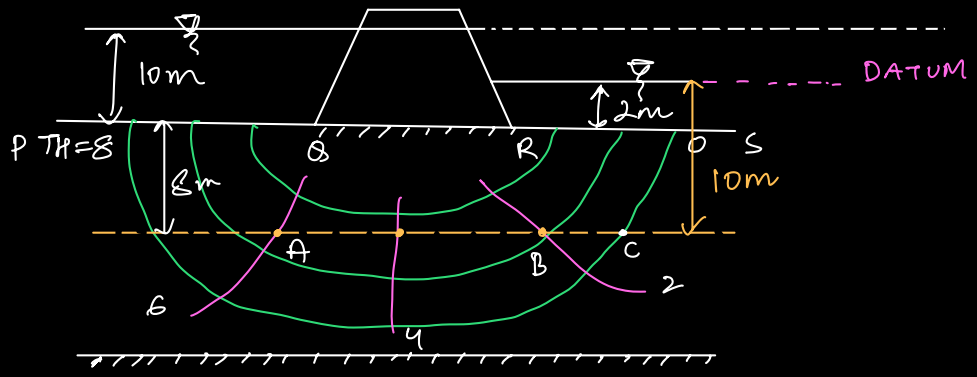
$$P_s = h \cdot \gamma_w$$

h: available hydraulic head at the concerned point

(4) Determination of hydrostatic pressure.

To determine the hydrostatic pressure (pore water press) the pressure head can be multiplied with the γ_w at the concerned point.

For example:



No of flow lines = 5

$N_f = 4$

No of equipotential lines = 5

$N_d = 4$

$$TH_{PQ} = PH_{PQ} + EH_{PQ}$$

$$TH_{PQ} = 10 - 2 = 8m$$

$$TH_{RS} = PH_{RS} + EH_{RS}$$

$$TH_{RS} = 2 + (-2)$$

$$\boxed{TH_{RS} = 0}$$

Total head loss, $\Delta H = TH_{PQ} - TH_{RS}$

$= 8 - 0 = 8m$

Total head loss per drop $= \Delta h = \frac{\Delta H}{N_d} = \frac{8}{4} = 2m.$

SEEPAGE PRESSURE

$SP_A = 6 \cdot \gamma_w$

$SP_B = 4 \cdot \gamma_w$

$SP_C = 2 \cdot \gamma_w.$ $\quad \because \gamma_w = 9.81$

$THA = PHA + EHA$

$Cm = PHA - 10$

$PHA = 16m$

$PHB = 14m$

$PHC = 12m$

hydrostatic pressure$_A$ $= 16 \cdot \gamma_w = 156.96$ (w)

" $_B = 14\gamma_w$

" $_C = 12\gamma_w$.

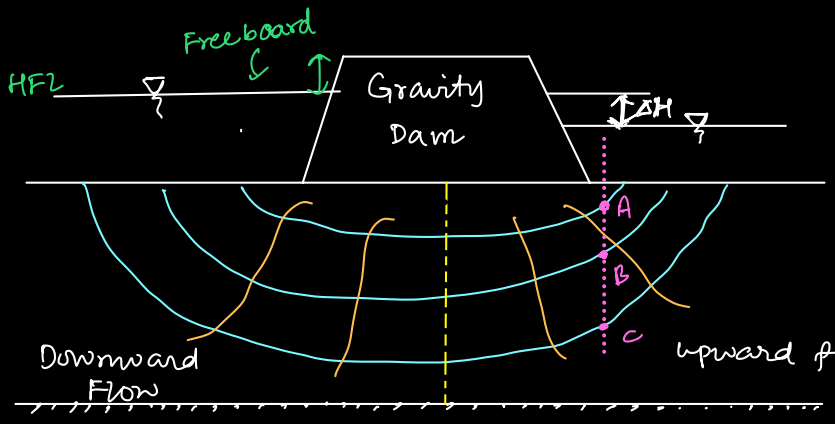
Calculation of effective stress

$\gamma_{sat} = 20\ KN/m^3$

$\bar{\sigma}_A = (10 \times \gamma_w + 8 \times \gamma_{sat}) - (156.96)$

$= 101.14\ KN/m^2$

Determination of exit hydraulic gradient:



→ In the zone of upward seepage, effective stress in the soil decreases. If we compare point A, B and C, then point 'A' has smallest value of effective stress because the submersed weight of the soil present above the point A is very less. Therefore chances of occuring quick sand conditions or piping failure is maximum near downstream side.

→ Therefore we are interested in finding out hydraulic gradient near downstreams. Among all the hydraulic gradients estimated at the different location in the flow net, the exit hydraulic gradient is governing gradient for piping failure.

→ The exit hydraulic gradient can be given as

$$i_{exit} = \frac{\text{Total head loss}}{\text{Total length of flow path}}$$

$$i_{exit} = \frac{\Delta H}{L}$$

Total length of flow path (L) = size of one flow field × No of equi. drops

$$i_{exit} = \frac{\Delta H}{L} = \frac{\Delta H}{bN_d} = \frac{\Delta h}{b}$$

Δh: head loss per drop

b: size of one flow field.

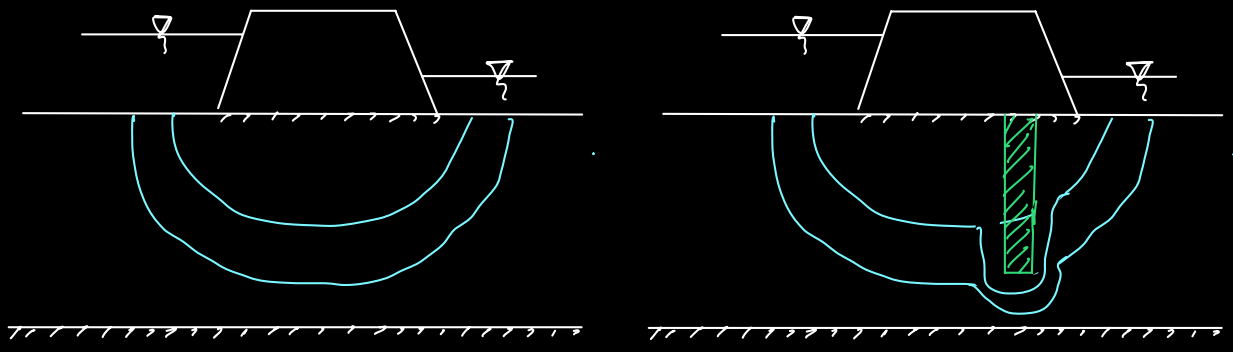
The factor of safety against piping failure can be given as

$$FOS = \frac{i_c}{i_{exit}}$$

Note: The factor of safety against piping failure can be provided as 6 to 7 for fine sand & 4 to 5 for coarse sand.

Solution to avoid the piping failure:

(i) Provide sheet pile wall: This sheet pile wall is the impervious wall provided at the downstream side (in General) throughout the length of dam



→ After providing sheet pile, the length of the flow path is increased as a result

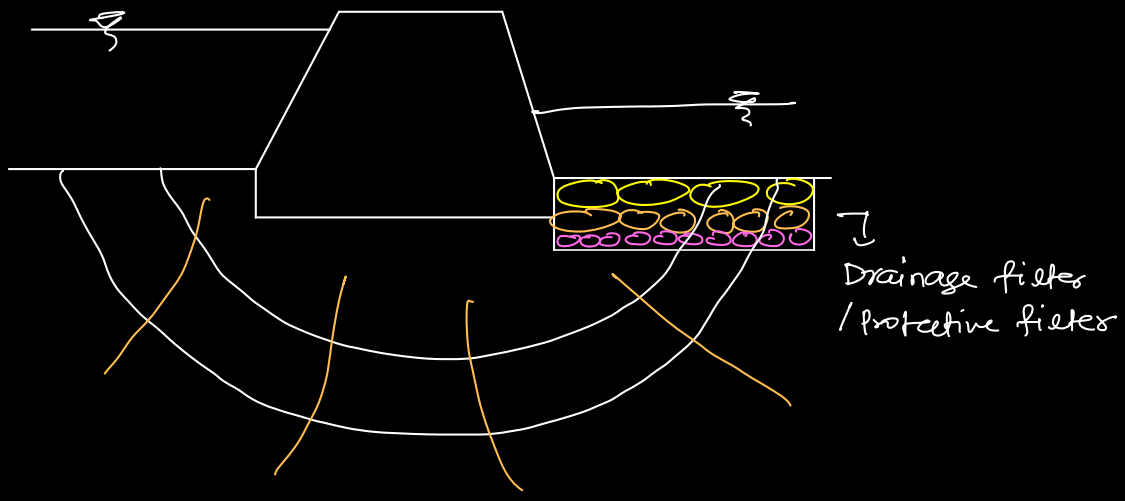
a. hydraulic gradient reduces ($\downarrow i = \frac{\Delta H}{L_T}$)

b. The seepage head gets reduced as a result seepage

pressure gets reduces.

(ii) Provide protective filters / Inverted filters / Graded filters:

In order to avoid the piping failure, protective filters can be provided at downstream of sufficient thickness



* In the protective filters, a layer of coarse particles (highly permeable) can be provided over the layers of fines particles [less permeable] therefore it is also termed as inverted filter.
* In the design of protective filter following points should be kept in the mind.
  - There should be sufficient coarse sized particles so that water can easily drain out
  - There should be sufficient fine grained particles so that soil can't move with water. with FGS erosion of the soil mass from the base can be avoided.
  - The filter material should be coarse enough so as not to be carried away through the opening of the drainage pipe.
  - There should be sufficient thickness such that particles of every size should be accomodated

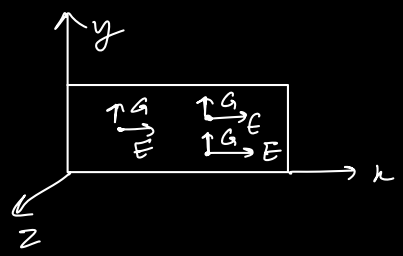
Design of criteria:

$$\frac{D_{15}\ (\text{filter})}{D_{85}\ (\text{protected soil})} < 5$$

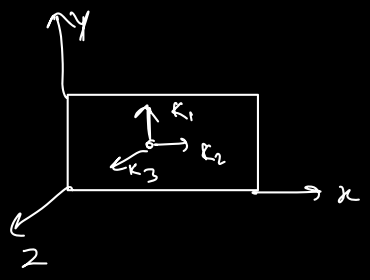
$$4 < \frac{D_{15}\ (\text{filter})}{D_{15}\ (\text{protected soil})} < 20$$

$$\frac{D_{50}\ (\text{filter})}{D_{50}\ (\text{protected soil})} < 25$$

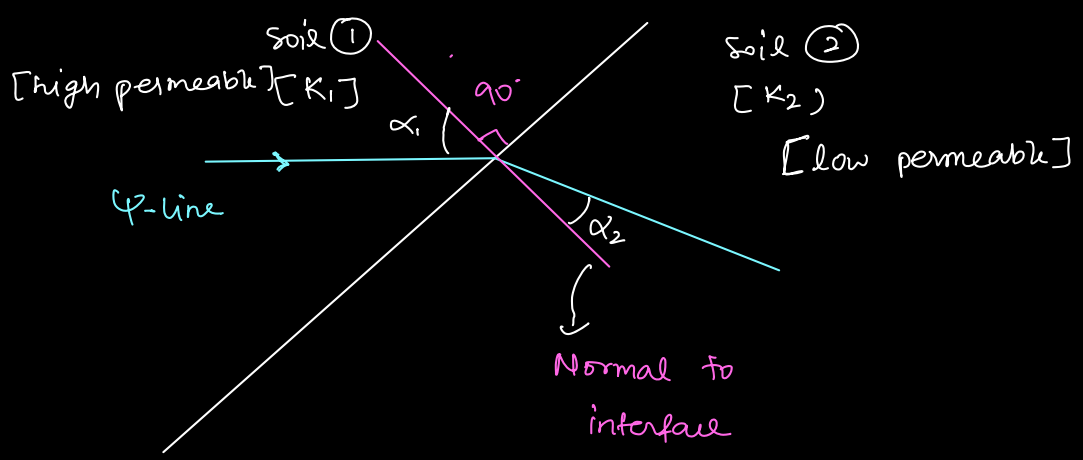
Homogeneous Material: The material in which, in the given direction the properties are same at each & every point.



Isotropic material: The material in which at a given point, properties are same in all the direction



Seepage through Non-homogeneous soil deposition

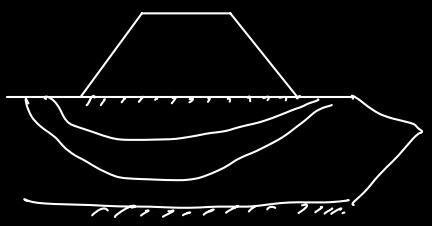


$$K_1 \tan\alpha_1 = K_2 \tan\alpha_2$$

Note: from the above equation it is clear that if water flows from highly permeable soil ① to low permeable soil ② ($K_1 > K_2$) then flow lines gets deflected toward the normal and viceversa. [$\alpha_2 < \alpha_1$ if $K_1 > K_2$ and viceversa]

CONFINED AND UNCONFINED FLOW:

i) Confined flow: When seepage or flow occurs between two impervious strata then the flow is termed as confined flow. For example - flow under the gravity dam, sheet pile wall, masonry dam etc.

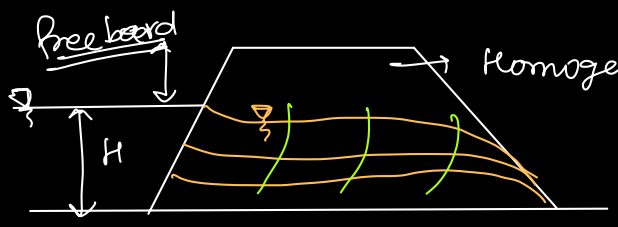


ii) Unconfined flow: When the seepage or flow of water occurs through the body of the homogeneous earth dam then it is termed as unconfined flow.

→ In case of unconfined flow the top boundary of the flow is not known, whereas in case of confined flow top & bottom boundaries are already known hence the flow net can be easily drawn.

→ Therefore in case of unconfined flow first we plot the top most flow line and below that flow net is drawn to determine the seepage discharge.

Note: This top most flow line is termed as phreatic line.

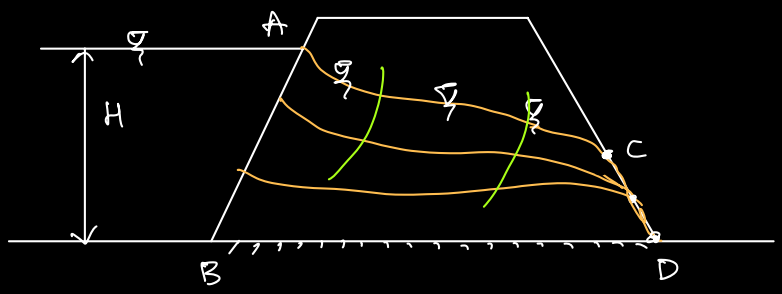


Seepage discharge through earth dam can be given as

$$Q = K \cdot H \cdot \frac{N_f}{N_d} \quad \text{(for isotropic soil)}$$

$$Q = \sqrt{K_x \cdot K_y} \cdot H \cdot \frac{N_f}{N_d} \quad \text{(for Anisotropic soil)}$$

PHREATIC LINE AND ITS CHARACTERISTICS:



ACD: Phreatic line

AB: 100% equipotential line.

BD: Can be considered as one flow line.

Characteristics:

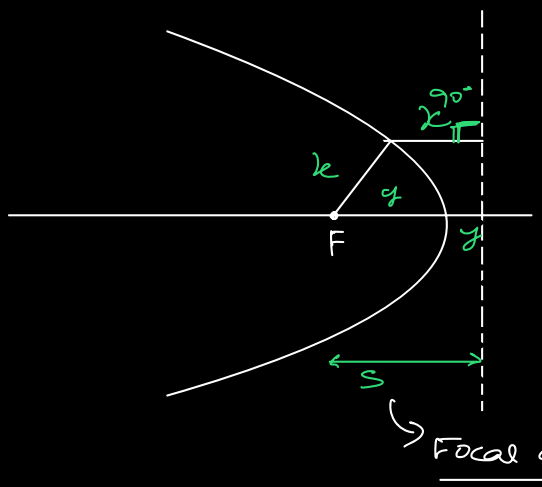
- It is the topmost flow line which seperates the saturated & unsaturated soil mass.
- It follows the path of base parabola. Although small deviations are observed at starting and ending of the flow for which corrections can be applied
- The pressure above the phreatic line is atmospheric & below the phreatic line is hydrostatic.
- The pressure head at every point on the phreatic line is zero, therefore the total head will be equal to the datum head at every point on the phreatic line
- Using phreatic line seepage discharge can be estimated.

Types of feilures in unconfined flow:

i) Piping failure

ii) Capillary syphonage

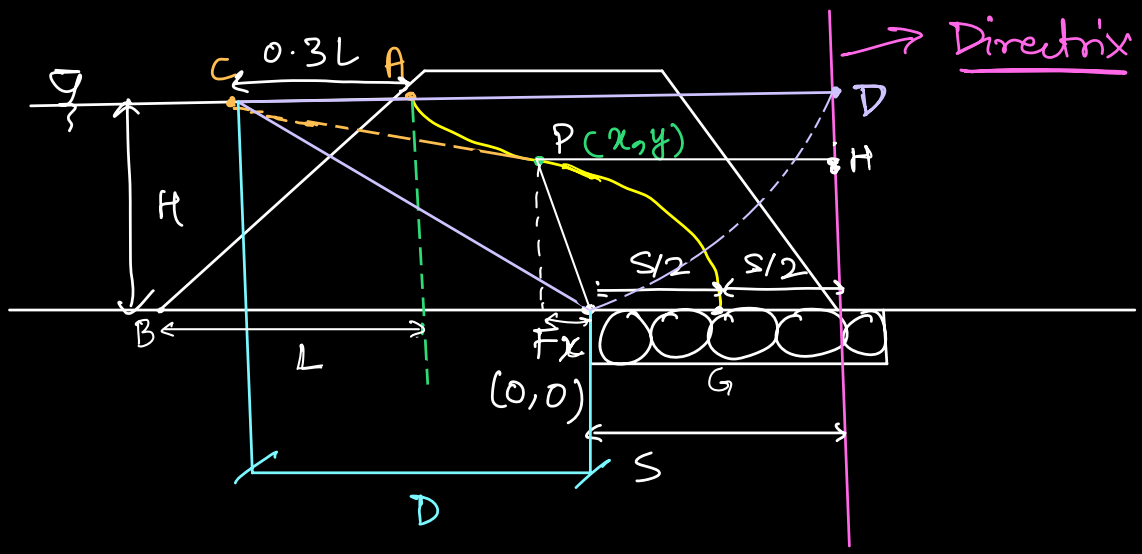
Property of base parabola Base parabola is parabola in which every point is equidistant from its focus & directrix.



$$y = \frac{s}{2}$$

Estimation of seepage discharge using Phreatic line

Case I: when drainage filter is provided:



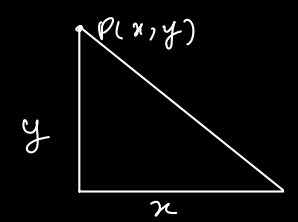
APG: Top-most flow line

AB: 100% Equipotential line

from the property of base parabola:

at Point P:

PF = PH



$$\sqrt{x^2+y^2} = x+s$$

$$x^2+y^2 = (x+s)^2$$

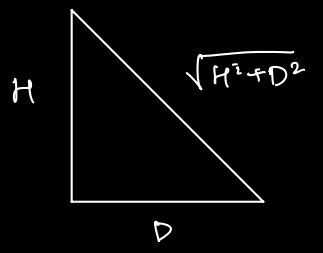
$$x^2+y^2 = x^2+s^2+2sx$$

$$(y^2 = s^2 + 2sx)$$

using this equation also phreatic line can be drawn which is termed as Analytical approach.

$$y = (s^2 + 2sx)^{1/2} \quad - \text{eqn (A)}$$

At point C,



$$CF = CD$$

$$\sqrt{H^2+D^2} = D+S$$

$$S = \sqrt{H^2+D^2} - D \quad - \text{eqn (B)}$$

$\therefore S$ focal distance.

Seepage discharge can be given as

$$Q = K \cdot i \cdot A$$

$$Q = K \cdot \frac{dy}{dx} \cdot (y \times 1)$$

$$Q = K \cdot \frac{dy}{dx} \cdot y$$

From the equation ①:

$$\frac{dy}{dx} = \frac{1}{2}\left[s^2 + 2sx\right]^{-1/2} (0 + 2s)$$

$$Q = K \cdot \left[s^2 + 2sx\right]^{-\frac{1}{2}} (s) \left[s^2 + 2sx\right]^{\frac{1}{2}}$$

$$\boxed{Q = Ks} \quad m^3/s/m$$

Q: Seepage discharge through unit length of Earth dam

K: Coefficient of permeability

S: $\sqrt{H^2 + D^2} - D$ (focal distance)

→ If anisotropic soil is present then seepage discharge through body of the earth dam can be given as

$$Q = K_{eq} \cdot S.$$

$$Q = \sqrt{K_x \cdot K_y} \cdot S.$$

* Procedure to plot the phreatic line when drainage filter is provided:

*GRAPHICAL METHOD:

i) Let AB is the 100% equipotential line and upstream slope.

ii) Let 'L' is the horizontal distance between A & B points.

iii) Locate the point 'C' at a distance 0.3L from the point 'A' on the free water surface

iv) Let F is the focus of the base parabola located at the interface between the low & high permeable zones.

v) With 'C' as a centre draw a circle of radius equal to CF which intersects the horizontal projection of the free water surface at the point D

vi) Draw a perpendicular line (DF) from the point 'D' which is termed as directrix of base parabola.

vii) Locate the point G in the middle of FE where FE is the focal distance (s).

viii) So, till now we have located starting point C and Ending point G of the base parabola.

ix) To locate the intermediate points of the base parabola, draw a circle of radius $s+x$, considering 'F' as a center which intersect a line projected from the distance of x from the focus 'F'.

x) In this way, intermediate points P(x,y) are located for which PF = PH [from the property of base parabola.

xi) In this way, number of intermediate points can be located and by connecting them base parabola can be drawn as CPG

(xii) Since phreatic line starts from the point A and it is perpendicular to 100% equipotential line. Some corrections.

(xiii) Now after correction we have phreatic line 'APG' starting from the point 'A' at an angle of 90° with the equipotential line AB.

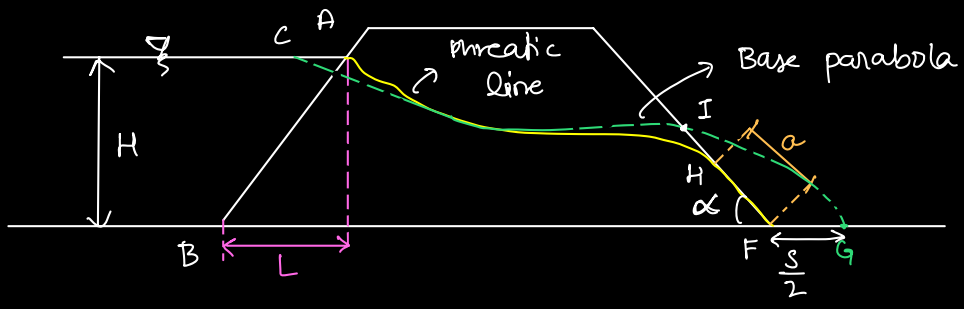
* ANALYTICAL METHOD:

We have already derived, $y = (S^2 + 2Sx)^{\frac{1}{2}}$

using this equation, for different values of x, the values of y can be calculate taking S as focal dist. Locate & connect all the points & get phreatic line

— x —

(Case(ii)) When drainage filter is not provided.



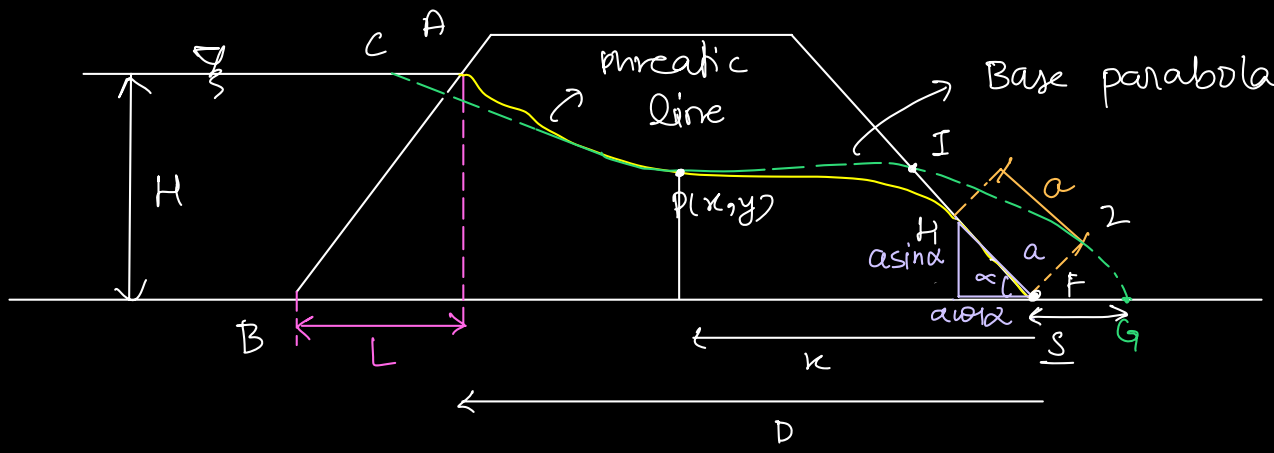
→ In this case where drainage filter is not being provided, base parabola extends beyond the boundary of the earth dam (CIG)

→ But in actual phreatic line [AGF] passes through the downstream slope at the exit

→ The 'HF' is termed as discharge face of length 'a' which always remains wet

→ α is the angle of discharge face at exit condition.

(i) Estimation when seepage discharge when $\alpha < 30°$.



Discharge Q at any point P(x, y)

$$Q = K \cdot i \cdot A$$

$$= K \cdot \frac{dy}{dx} \cdot y \cdot 1$$

$$Q = K \cdot \frac{a\sin\alpha}{a\cos\alpha} (a\sin\alpha \cdot 1)$$

$$Q = K \cdot a\sin\alpha \cdot \tan\alpha \quad m^3/s/m$$

a: length of discharge face and it can be determined as

$$a = \frac{d}{\cos\alpha} - \sqrt{\frac{d^2}{\cos^2\alpha} - \frac{H^2}{\sin^2\alpha}}$$

(ii) Estimation when seepage discharge when $30 < \alpha < 60$:

$$Q = K \cdot \tan\alpha \cdot a\sin\alpha$$

↳ As per Leo. Casagrand when $\alpha > 30°$ then $\tan\alpha$ must be replaced by $\sin\alpha$

then $Q = K \cdot a \cdot \sin^2\alpha \quad m3/s/m$

$\therefore$

$$a = \sqrt{H^2 + D^2} - \sqrt{D^2 - H^2\cot^2\alpha}$$

Question

Q.6 Calculate the seepage through an earth dam resting on an impervious foundation.

The relevant data are given below :

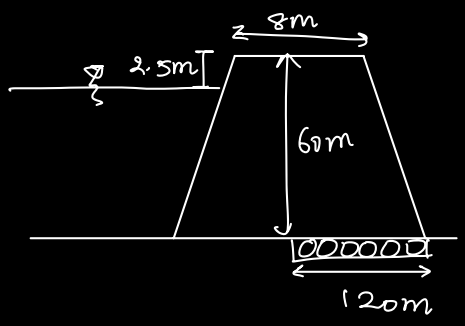
Height of a dam = 60.0 m

Upstream slope = 2.75 : 1 (H : V)

Downstream slope = 2.50 : 1 (H : V)

Free board = 2.5 m; Crest width = 8.0 m; Length of drainage blanket = 120.0 m

Solution

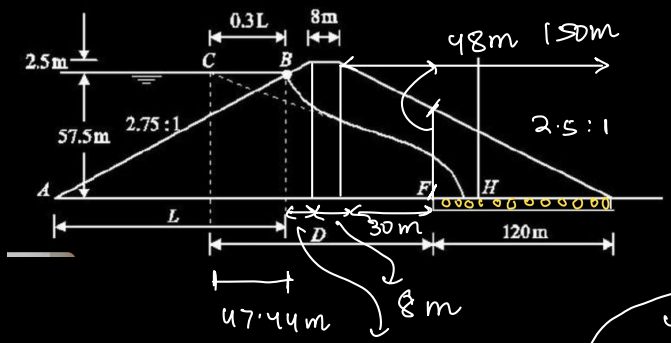


$K_x = 8 \times 10^{-7}$ m/s

$K_y = 2 \times 10^{-7}$ m/s

$K_{eq} = \sqrt{16} \times 10^{-7}$

$= 4 \times 10^{-7}$ m/s



$H = 57.5$m

$L = 2.75 \times 57.5 = 158.125$m

$BC = 0.3 \times 158.125 = 47.44$m

2.75×2.5

$\Rightarrow 6.875$m

$\frac{48}{120} = \frac{60}{x}$

$(x = 150\text{m})$

$D = 47.44\text{m} + 6.875 + 8 + 30$

$= 92.315$ m

$S = \sqrt{H^2 + D^2} - D$

$= 108.75 - 92.315$

$= 16.44$m.

$Q = K \cdot S$

$= 4 \times 10^{-7} \times 16.44$

$Q = 6.576 \times 10^{-6}$ m²/s/m

WELL HYDRAULICS

Types of Geological formation:

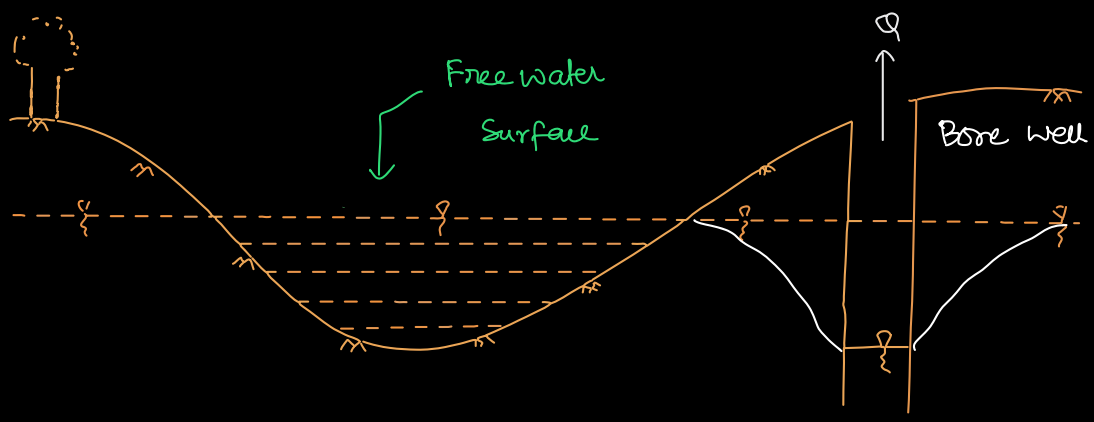
1. AQUIFER : It is porous and permeable geological formation which contains water and capable in transmitting or yielding the water.

There are two types of aquifer

i) Confined aquifer

ii) Unconfined aquifer

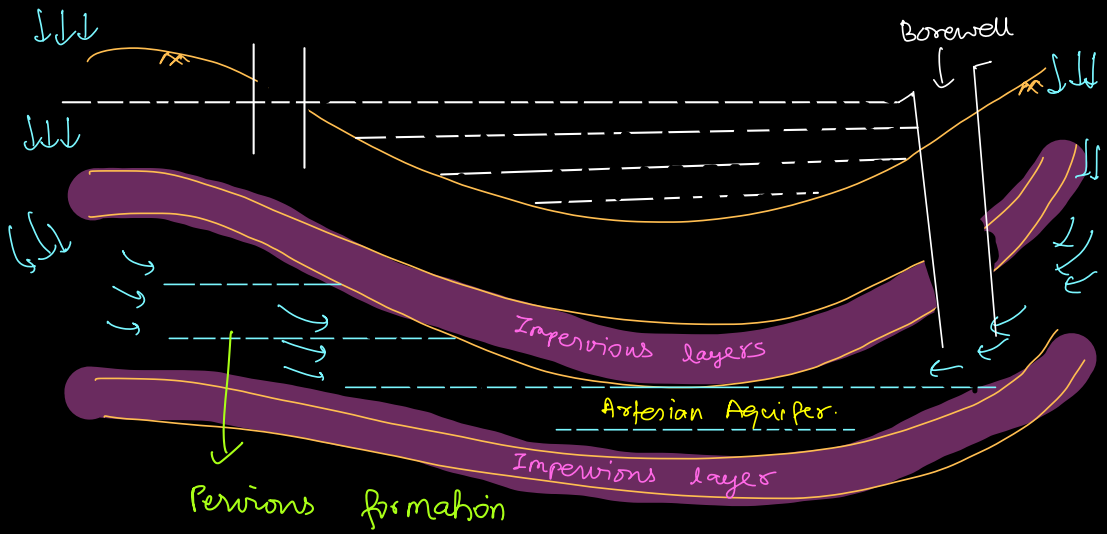
i) UNCONFINED AQUIFER: The unconfined aquifer or water table aquifer fills all the voids of soil by ground water upto



UNCONFINED AQUIFER

the free water surface in the zone of saturation. It is also termed as free, phreatic or Non-artesian aquifer. The pressure at the free water surface of the aquifer is atmospheric and below that the pressure is hydrostatic.

(ii) CONFINED AQUIFER



when the water is trapped between two impervious formations such geological formation is termed as confined aquifer. The Ground water in the confined aquifer is under the higher compression pressure known as artesian pressure. Therfore this geological formation can also be termed as Artesian Aquifer.

The artesian head might be greater than the local hydro head.

2. AQUICUDE : It is the impermeable geological formation which contains the water but not capable in transmitting /yielding of water

3. AQUIFUGE: It is impermeable geological formations which neither contains water nor transmits the water

4. AQUITARD: It is the impermeable geological formation, which contains water but does not yield significant amount of water. Only seepage is possible.

Specific yield & specific Retention: The water storage capacity of any aquifer is measured by its porosity. If porosity of any aquifer is more, that indicates it has more water storage capacity. The water storage capacity of any aquifer doesn't mean its yielding capacity of any aquifer so the well depends upon the size, shape of particles of soil and distribution of voids.

Specific yield (Sy): used to express the water yielding capacity of an unconfined aquifer.

Specific yield can be defined as the ratio of the volume of water drained by gravity to the total volume of aquifier

$$Sy = \frac{\text{Volume of water drained by gravity}}{\text{Total volume of aquifier}}$$

or

$$Sy = \frac{\text{Change in volume of change in storage}}{\text{Total volume of aquifier}}$$

Specific Retention (S_R): The aquifer never yields its complete water. Some water is always retained by the surface tension forces. The amount of water retained is expressed in terms of specific retention. The specific retention can be defined by the ratio of volume of water retained (by the force of surface tension against gravity) to the volume of aquifer.

$$S_R = \frac{\text{Volume of water retained}}{\text{total volume of aquifer}}$$

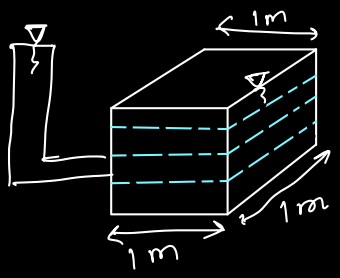
Relation between S_R, S_y and n:

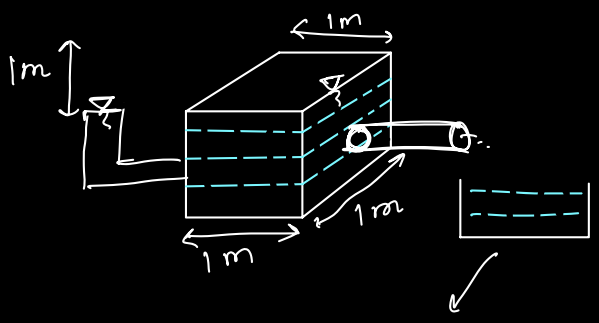
$$S_y + S_R = \frac{(V)_{drained} + (V)_{undrained}}{V}$$

$$S_y + S_R = \frac{V_w}{V} = \frac{V_v}{V} = n$$

$$S_y + S_r = n$$

Specific Storage (S_s):





This volume of water is termed as specific storage

→ Specific storage is a volume of water drawn from a unit volume of soil per unit drop in head freely under gravity

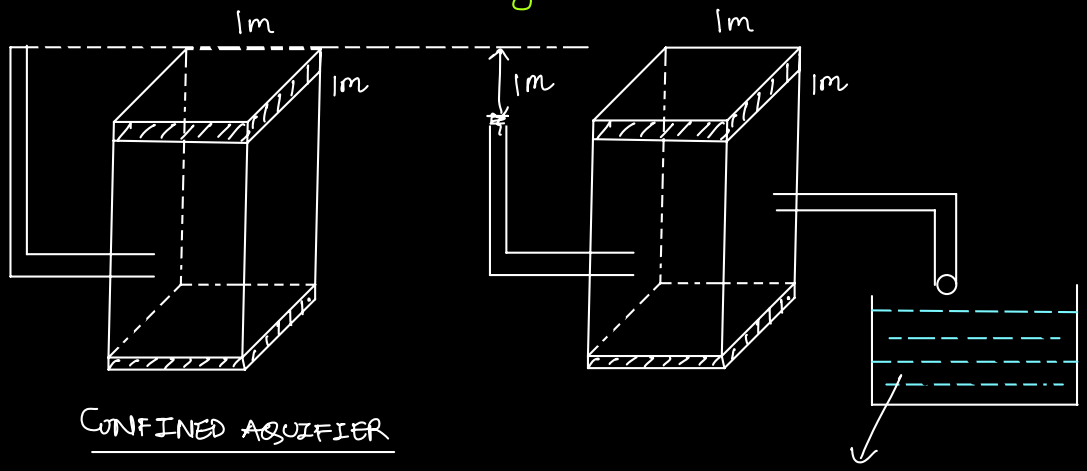
UNITS = $m^3/m^3/m = m^{-1}$ or cm^{-1} or mm^{-1}

$$S_s = (\alpha + \beta \cdot n) \cdot \gamma_w$$

α : Aquifer compressibility

β : Water compressibility

Storage coefficient or storativity (S):



The volume of water extracted freely under the gravity per unit surface area, per unit drop in head is termed as storativity. Storativity or storage coefficient is the water yielding capacity of the confined aquifer. The storage coefficient of the unconfined aquifer is corresponding to its specific yield.

UNIT : $m^3 / m^2 / m \rightarrow$ UNITLESS.

→ Storativity / Storagecoefficient is dimensionless parameter

→ Product of specific storage and thickness of the confined aquifer

$$\text{Storativity} = S_s \times b$$

Specific capacity (Q_s): It is the discharge or the rate of flow per unit drop in hydraulic head.

Units of specific capacity → $m^3 / day / m$

Field methods to determine coefficient of permeability:

To determine permeability of soil in the lab, soil sample is collected from the field or site which has degree of disturbance therefore actual value of K can't be estimated in the lab. In order to determine actual value of permeability field tests can be performed There can be two tests

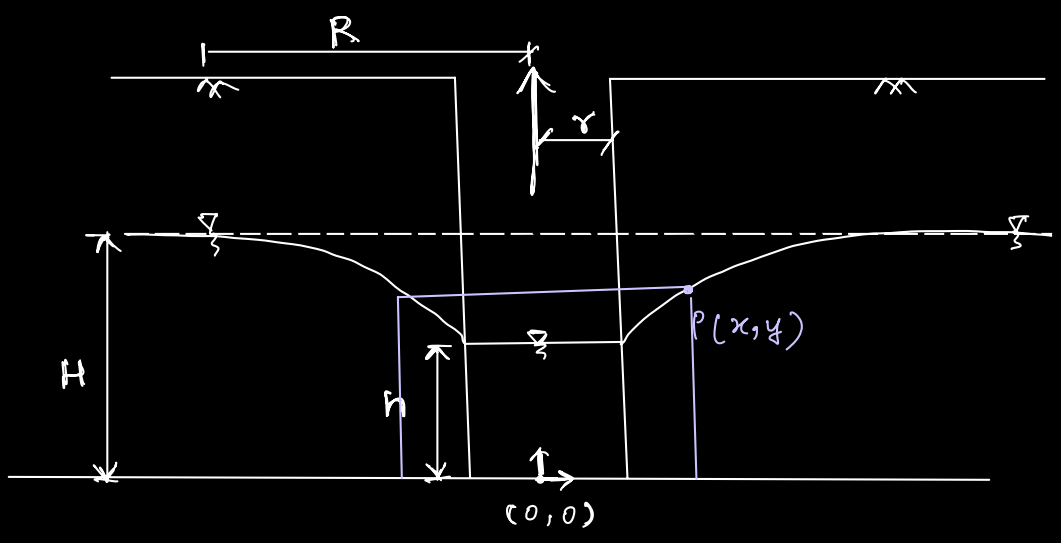
① Pumping out test

② Pumping in test

1. PUMPING OUT: Suitable in case of large area of influence. For ex. Reservoirs

Case I: Well in unconfined aquifer:

* STUDY A: DUPIT'S THEORY:



Discharge passing through any section at a distance x can be given as,

$$Q = K \cdot i \cdot A$$

$$Q = K \cdot \frac{dy}{dx} \cdot (2\pi x \cdot y)$$

$$Q \int_{r}^{R} \frac{dx}{x} = K(2\pi) \int_{h}^{H} y \cdot dy$$

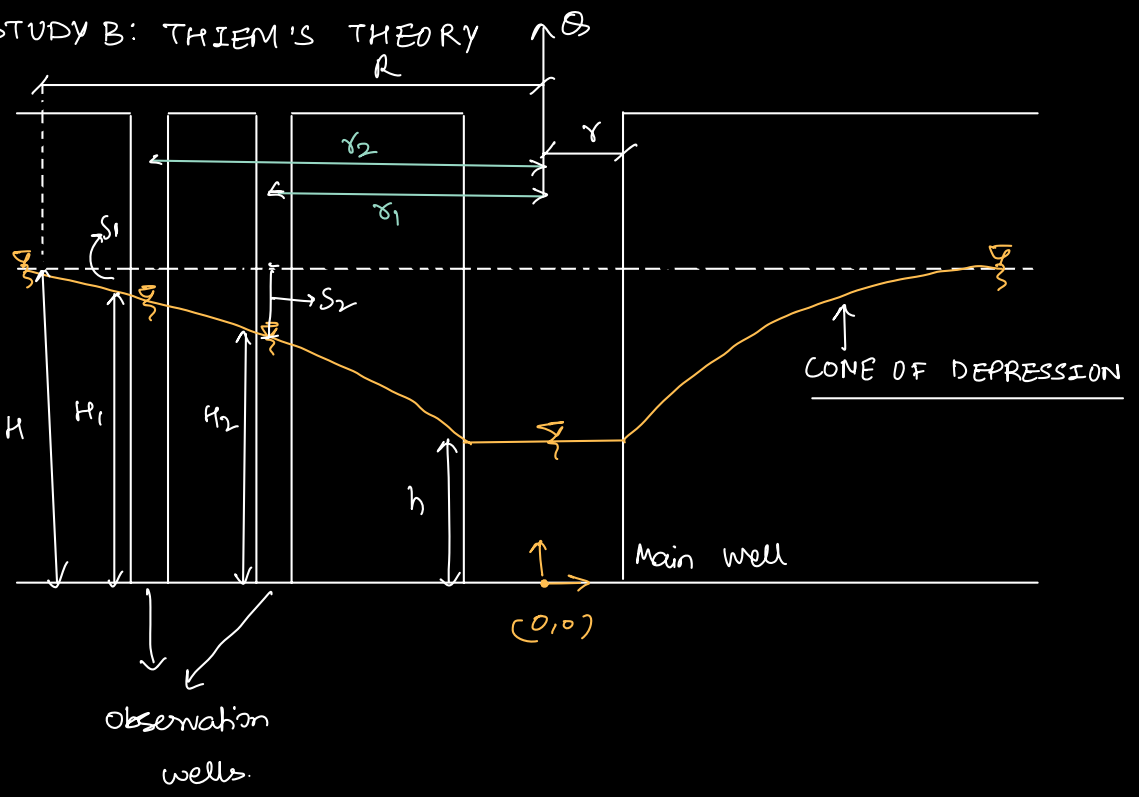
$$Q \cdot [\ln x]_r^R = K \cdot 2\pi \left[y^2/2 \right]_h^H$$

$$Q[\ln R - \ln r] = \pi K \left[\frac{H^2}{2} - \frac{h^2}{2} \right]$$

$$K = \frac{Q \log_{10}\left[\left(R/r\right)\right]}{1.36\left[H^2 - h^2\right]}$$

- K → Hydraulic Conductivity
- Q → Discharge
- R → Radius of influence
- r → Radius of well
- H → Height of GWT above bottom of well
- h → Height of water in well

* STUDY B: THIEM'S THEORY



$$Q = K \cdot i \cdot A$$

$$Q = K \cdot \frac{dy}{dx} \cdot 2\pi x y$$

$$Q \int_{r_1}^{r_2} \frac{dx}{x} = \int_{h_1}^{h_2} K \cdot 2\pi \, y \, dy$$

$$Q \ln\left(\frac{r_2}{r_1}\right) = 2\pi K \frac{(h_1^2 - h_2^2)}{2}$$

$$K = \frac{Q \log\left(\frac{r_2}{r_1}\right)}{1.36\left(h_2^2 - h_1^2\right)}$$

∵ R & H are unknown in Dupit's Analysis

∴ Thiem's analysis came in play.

r_1, r_2 : c/c distances of observation wells from main wells

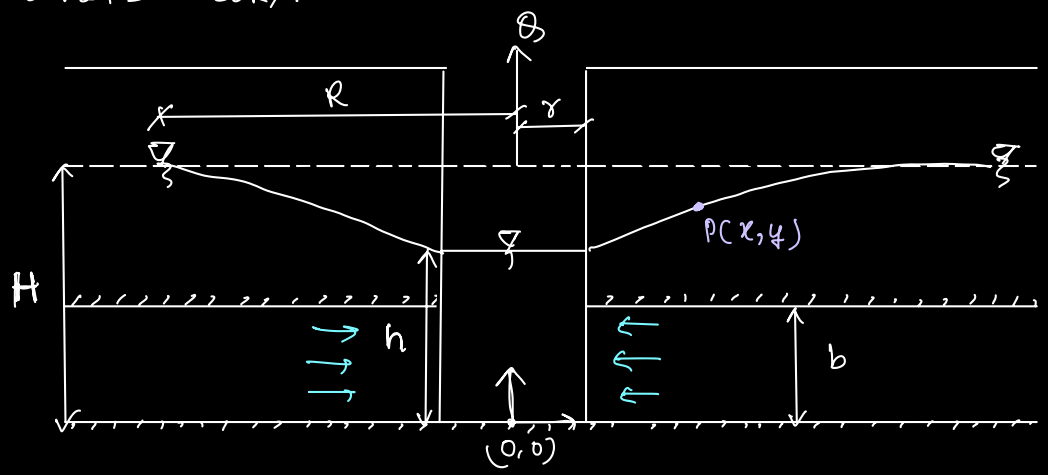
h_1, h_2 : heads in observation wells

S_1, S_2 : are the depressions in observation wells measured from the GWT.

$$\left\{ \begin{array}{l} h_1 = H - S_1 \\ h_2 = H - S_2 \end{array} \right\}$$

Case II : CONFINED AQUIFER (FULLY PENETRATED WELLS) :

1. DUPIT'S THEORY :



$$Q = K \cdot \frac{dy}{dx} \cdot 2\pi x \cdot b \rightarrow \text{fixed}$$

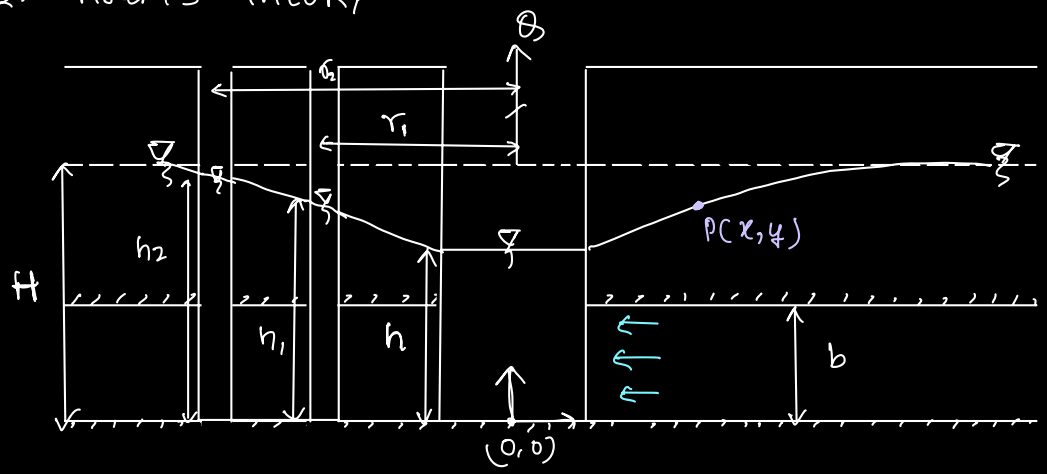
$$Q \int_r^R \frac{dx}{x} = b\,K \cdot 2\pi \int_h^H dy$$

$$Q \ln\left(\frac{R}{r}\right) = b \cdot 2K\pi\,(H-h)$$

$$K = \frac{Q \log_{10}(R/r)}{2.726\,[H-h]}$$

b : Thickness of the confined aquifer.

2. THIEM'S THEORY



$$K = \frac{Q \log_{10}(r_2/r_1)}{2.726\, b (h_2 - h_1)}$$

↳ Derived as earlier.

$$T = Kb = \frac{Q \log_{10}(r_2/r_1)}{2.726\, b (h_2 - h_1)}$$

COEFFICIENT OF TRANSMISSIVITY (T) (only for confined):

It can be defined as the discharge through then unit thickness of confined aquifer.

↳ Coeff of transmissivity is estimated only for confined aquifer & not for unconfined aquifers

↳ It can be defined as the product of coeff. of permeability and thickness of the confined aquifer.

Thus,

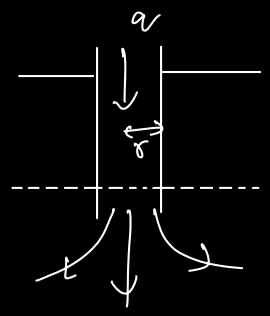
$$T = \frac{Q \log_{10}(R/r)}{2.72(H-h)} \quad \text{or} \quad T = \frac{Q \log_{10}(r_2/r_1)}{2.72\, b (h_2 - h_1)}$$

ii) PUMPING IN: Discharge will be given under constant head

SMALL AREA OF INFLUENCE

* OPEN END TEST: An open end pipe is sunk into the medium and soil is taken out from it. Clean water having the temperature slightly greater than groundwater is pumped into

the medium through a pipe at constant head using a discharge measuring device.



$$K = \frac{q}{(5.5)rh}$$

q : Discharge

h : head under which flow takes place.

r : radius of pipe

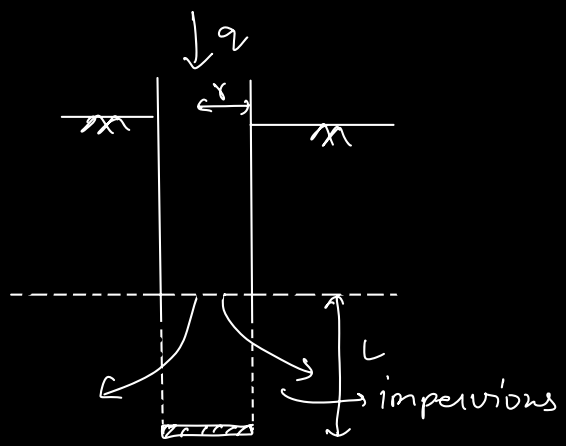
*PACKERS TEST:

↳ $L \geq 10r$

$$K = \frac{q}{2\pi Lh} \log_{10}\left(\frac{L}{r}\right)$$

↳ $r \leq L < 10r$

$$K = \frac{q}{2\pi Lh} \sinh^{-1}\left(\frac{L}{2r}\right)$$



→ pipe perforated to a length L from bottom

Question

Water table drops by 3 m in irrigable land of 50 hectare. If porosity and specific retention are 0.30 and 0.10 respectively, the change in storage in hectare-meter is

(A) 15 **(B) 30**

(C) 45 **(D) 60**

[GATE 1996 : IISc Bangalore]

Solution:

$n = 0.30$

$S_r = 0.10$

$S_y = 0.2$

1 hectare $= 10^4\ m^2$

(Volume = 50 hectare)

Change in storage : $0.2 \times 50 \times 3 = 30$ ha-m

Question

A well of diameter 20 cm fully penetrates a confined aquifer. After a long period of pumping at a rate of 2720 litres per minute, the observations of drawdown taken at 10 m and 100 m distances from the centre of the well are found to be 3 m and 0.5 m respectively. The transmissivity of the aquifer is

(A) 676 m^2/day **(B) 576 m^2/day**

(C) 526 m^2/day **(D) 249 m^2/day**

[GATE 2010 : IIT Guwahati]

Solution

$r = 10cm$

$Q = 2720 \times 10^{-3} \times 24 \times 60$

$Q = 3916.8\ m^3/day$

$r_1 = 10m$, $S_1 = 3m$,

$r_2 = 100m$, $S_2 = 0.5m$,

$$T = \frac{Q \log_{10}(r_2/r_1)}{2.72 \times (h_2 - h_1)}$$

$H - S_2 - H + S_1$

$h_2 - h_1 = S_1 - S_2 = 2.5\,m$

$$T = \frac{3916.8 \times \log_{10}\left(\frac{100}{10}\right)}{2.72 \times 2.5} = 576\ m^2/day.$$ ans

Assumptions and limitations of Dupeit's Analysis:

(i) Aquifer is Homogeneous, Isotropic and of infinite aerial extent

(ii) Darcy's law is valid and flow is laminar

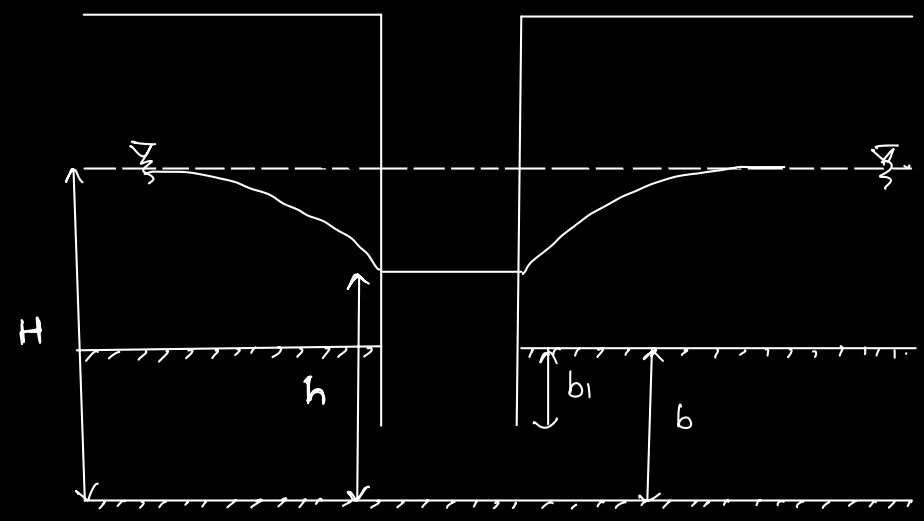
(iii) Velocity of flow is directly proportional to the tangent of the hydraulic gradient

(iv) Well penetrates and recieves the water from the full depth / thickness of the aquifer.

v) The flow is horizontal (radial flow) and uniform everywhere at the vertical section

vi) Coefficient of Transmissivity is constant every where in the aquifer.

Partially penetrating inverted Artesian well



In case of partially penetrating well the flow pattern is no more horizontal (Radial), changes in flow pattern is observed.

The discharge through partially penetrating well can be given as Q=

$$Q_p = Q \cdot G$$

$$Q_p = \frac{2.72\, Kb(H-h)}{\log_{10}(R/r)} \times G.$$

Q_p : Discharge through partially penetrating well

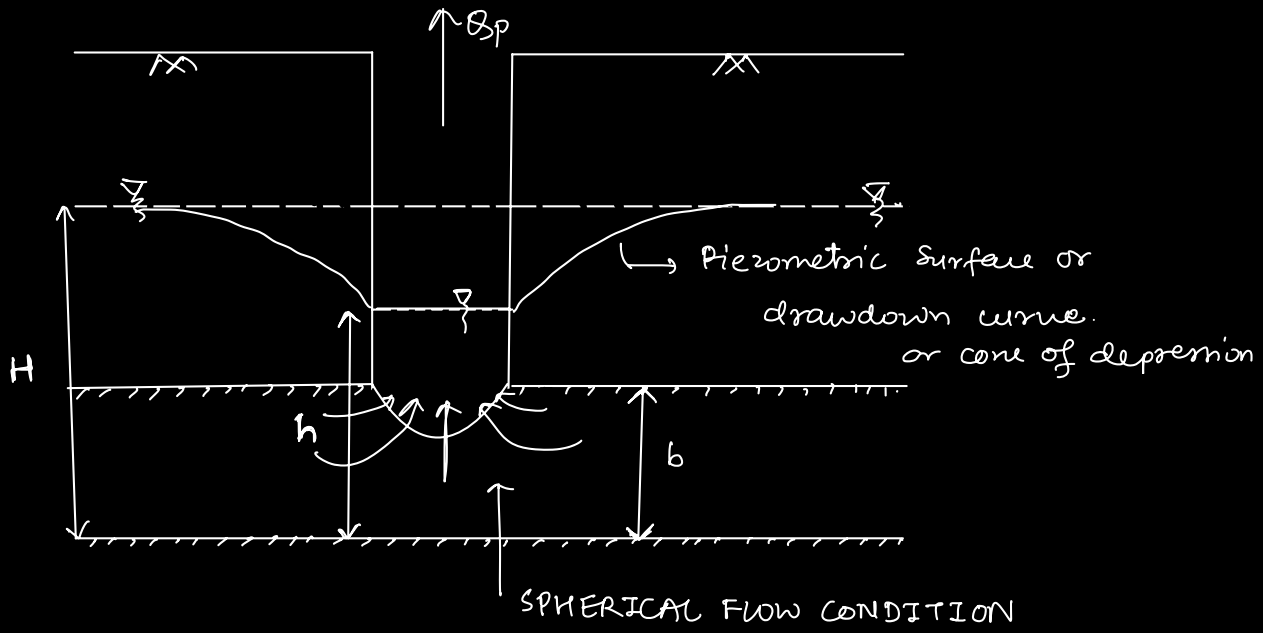
G : Correction factor

$$G = \frac{b_1}{b}\left(1 + 7\sqrt{\frac{r}{2b_1}}\cos\frac{\pi b_1}{2b}\right)$$

↳ Given by Kozeny.

Spherical flow in the well : when bottom of the well is located at the top layer of the artesian aquifer then spherical flow is observed in the well.

In the previous case of partially penetrating well the depth of penetration was b_1 whose value is zero ($b_1 = 0$). In this case spherical flow is observed.



The discharge in case of spherical flow can be given as

$$Q_s = 2\pi r K(H-h) \quad - \text{eqn } (1)$$

Let 'Q' is the discharge of fully penetrating well in which radial flow occurs.

$$Q = \frac{2\pi Kb(H-h)}{2.303\log_{10}\left(\frac{R}{r}\right)} = \frac{\left(\frac{Q_s}{r}\right)(b)}{2.303\log_{10}\left(\frac{R}{r}\right)}$$

$$Q_s = 2.303\left(\frac{r}{b}\right)\log_{10}\left(\frac{R}{r}\right)Q \quad - \text{eq}(2)$$

$$\boxed{Q_s < Q}$$

Radial Flow gives more discharge than spherical flow. i.e penetrate at full depth.

Interference among the wells: when two or more wells are located closely then they interfere to each other. The interference is observed in their drawdown curves considering the effect of interference, the composite drawdown curve can also be plotted.

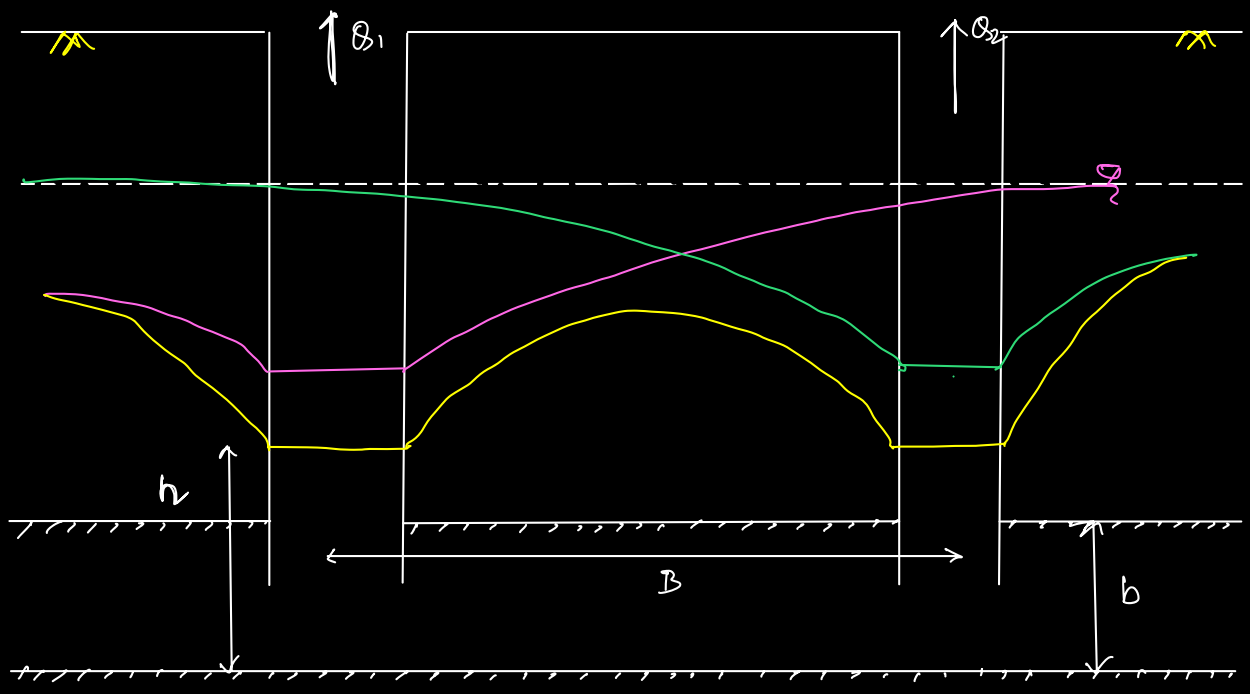
Let two wells are closely spaced at a c/c distance of B. from the principle of superposition the total drawdown in the composite drawdown curve is the sum of the drawdowns of the individual wells.

Let the two wells of equal radius 'r' and equal drawdowns (H-h) are closely spaced with c/c distance of B are fully penetrating the Artesian Aquifers then the discharge in the wells can be given as:

$$Q_1 = Q_2 = \frac{2.72\,Kb(H-h)}{\log_{10}\left(\frac{R^2}{r\cdot B}\right)} \quad \text{— equation (1)}$$

The discharge of a single well without any interference can be given as.

$$Q = \frac{2.72\,Kb(H-h)}{\log_{10}\left(\frac{R}{r}\right)}$$



$$\because \frac{R}{B} >> 1$$

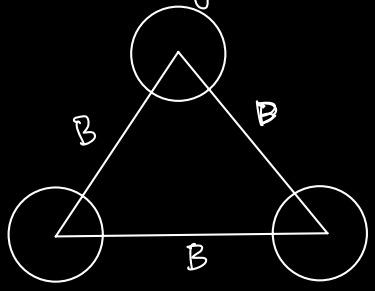
$$\therefore \frac{R}{r} < \frac{R}{r} \cdot \frac{R}{B}$$

$$\therefore Q > (Q_1 = Q_2)$$

But $Q < (Q_T = Q_1 + Q_2)$

Note: from the above expression it is clear that the discharge in the single well of without interference (Q_1 would more than the discharge of individual wells with interference but the total discharge ($Q_T = Q_1 + Q_2$) from the wells of the interference will be more than discharge of a single well (Q_1 of without interference.

SPECIAL CASE: let three wells are located in a pattern of equilateral triangle and closely spaced with c/c distance of B.



let radius (r) and drawdown (H-h) are same for all the three wells, considering interference the discharge can be given as:

$$Q_1 = Q_2 = Q_3 = \frac{2.72\, K b\, (H-h)}{\log_{10}\left(\frac{R}{r}\cdot\frac{R}{B}\cdot\frac{R}{B}\right)}$$

$$Q_1 = Q_2 = Q_3 = \frac{2.72\, K b\, (H-h)}{\log_{10}\left(\frac{R^3}{rB^2}\right)} \quad \text{— eqn (1)}$$

let Q is the discharge from the single well of without interference then,

$$Q = \frac{2.72\, K b (H-h)}{\log_{10}\left(\frac{R}{r}\right)} \longrightarrow \text{NO INTERFERENCE}$$

On comparing eqn (1) & eqn (2)

$$Q > (Q_1 = Q_2 = Q_3)$$

$$Q < (Q_T = Q_1 + Q_2 + Q_3)$$

$$\boxed{Q < (Q_T = 3Q_1)}$$

Note: Interference among the wells can be seen in case of unconfined Aquifers also.

Case I: Interference among two wells.

$$Q_1 = Q_2 = \frac{1.36\, K (H^2 - h^2)}{\log_{10}\left(\frac{R}{r}\cdot\frac{R}{B}\right)} = \frac{1.36\, K (H^2-h^2)}{\log_{10}\left(\frac{R}{r}\cdot\frac{R}{B}\right)}$$

$$Q > (Q_1 = Q_2)$$

$$Q < (Q_1 + Q_2)$$

Case II: Interference among three wells

$$Q_1 = Q_2 = Q_3 = \frac{1.36\, K (H^2-h^2)}{\log_{10}\left(\frac{R^3}{rB^2}\right)}$$

Question

Q. 60 cm diameter well is being pumped at a rate of 1360 litres/minute. Measurements in a nearby test well were made at the same as follows. At a distance of 6 m from the well being pumped, the drawdown was 6 m, and at 15 m the drawdown was 1.5 m. The bottom of the well is 90 m below the ground water table.

Solution

$r = 0.3m$

$Q = 1360 \times 10^{-3} \times 24 \times 60 = 1958 m^3/day$

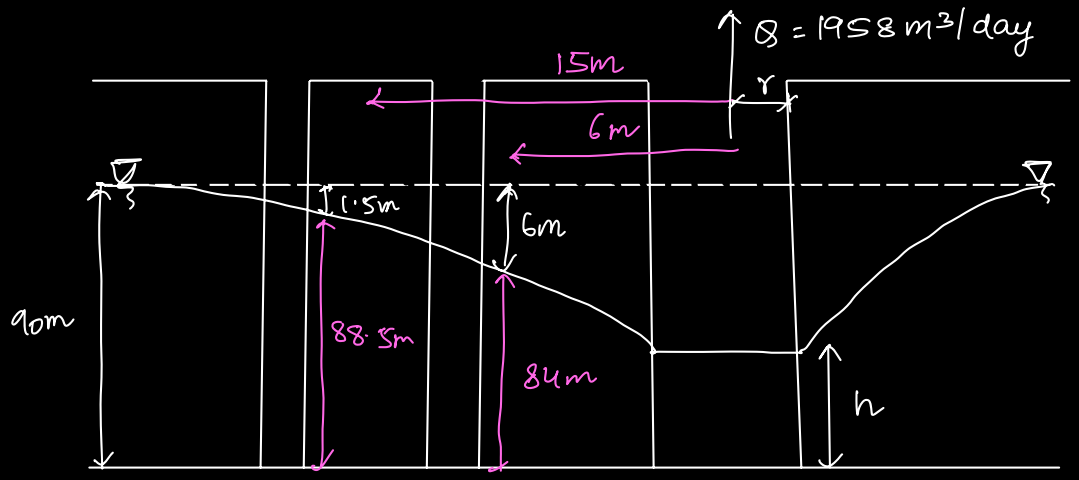
$= 1.36 \ m^3/min$

$r_1 = 6m$, $r_2 = 15m$

$S_1 = 6m$, $S_2 = 1.5m$

$H = 90m$

$h_1 = 84m$, $h_2 = 88.5m$



(A) Find out the coefficient of permeability.

(B) If all the observed points were on the Dupuit curve, what was the drawdown in the well during pumping?

(C) What is the sp. Capacity of the well?

(D) What is the rate at which water can be drawn from this well?

UNCONFINED AQUIFER

A) $$K = \frac{Q \log_{10}\left(\frac{r_2}{r_1}\right)}{1.36\left(h_2^2 - h_1^2\right)}$$

$$= \frac{1.36 \log_{10}\left(\frac{15}{6}\right)}{1.36\left(88.5^2 - 84^2\right)}$$

$$= 5.126 \times 10^{-4} \ m/min$$

B) Dupit's curve,

$$K = \frac{Q \log_{10}\left(\frac{R}{r}\right)}{1.36\left(H^2 - h^2\right)}$$

$$5.126 \times 10^{-4} = \frac{1.36}{1.36} \frac{\log_{10}\left(R/r_1\right)}{\left(H^2 - h_1^2\right)}$$

$$90^2 - h_1^2 = \frac{\log_{10}\left(R/r\right)}{5.126 \times 10^{-4}}$$

$$90^2 - 84^2 = \frac{\log_{10}\left(R/6\right)}{5.126 \times 10^{-4}}$$

$$R = 20.5m$$

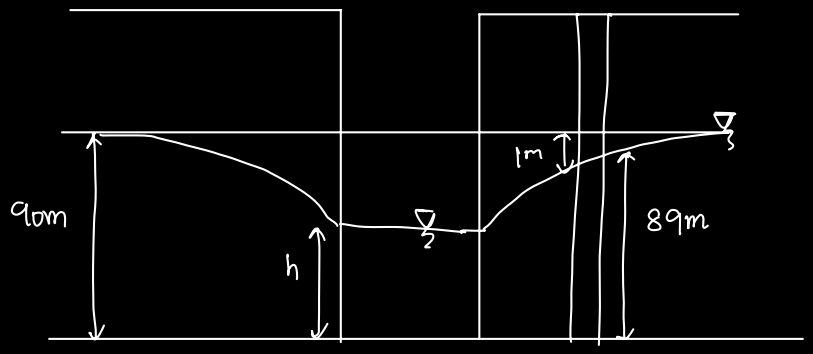
$$K = \frac{Q \log_{10}\left(\frac{R}{r}\right)}{1.36(H^2 - h^2)}$$

$$5.126 \times 10^{-4} = \frac{1.36}{1.36} \frac{\log_{10}\left(\frac{20.5}{0.3}\right)}{90^2 - h^2}$$

$$(h = 67.23m)$$

Drawdown $= H - h = 90 - 67.23m = 22.77m.$ Ans

C) Specific capacity :



$$K = \frac{Q \log_{10}\left(\frac{R}{r}\right)}{1.36(H^2 - h^2)}$$

$$Q = \frac{1.36K(H^2 - h^2)}{\log_{10}(R/r)}$$

$$Q_{sp} = \frac{1.36K(H^2 - (H-1)^2)}{\log_{10}\left(\frac{R}{r}\right)}$$

$$Q_{sp} = \frac{1.36 \times 5.126 \times 10^{-4}[90^2 - 89^2]}{\log_{10}\left(\frac{20.5}{0.3}\right)}$$

$$Q_{sp} = 0.068 \ m^3/min/m.$$

$$Q_{sp} = 68.02 \ lit/min/m.$$

D Maximum rate at which water can be drawn from the well.

$$Q = \frac{1.36K(H^2 - h^2)}{\log_{10}\left(\frac{R}{r}\right)} \qquad \{h \to 0, \ Q \to Q_{max}\}$$

$$Q_{max} = \frac{1.36KH^2}{\log_{10}\left(\frac{R}{r}\right)} = \frac{1.36 \times 5.126 \times 10^{-4} \times 90^2}{\log_{10}\left(\frac{20.5}{0.3}\right)}$$

$$= 3.07 \ m^3/min.$$

→ जब कुएं में पानी खत्म हो जाए तब Aquifier से तेजी से पानी आता है.

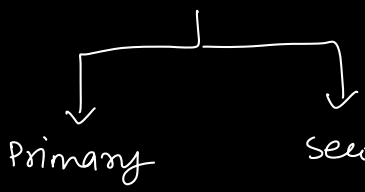
#COMPRESSIBILITY

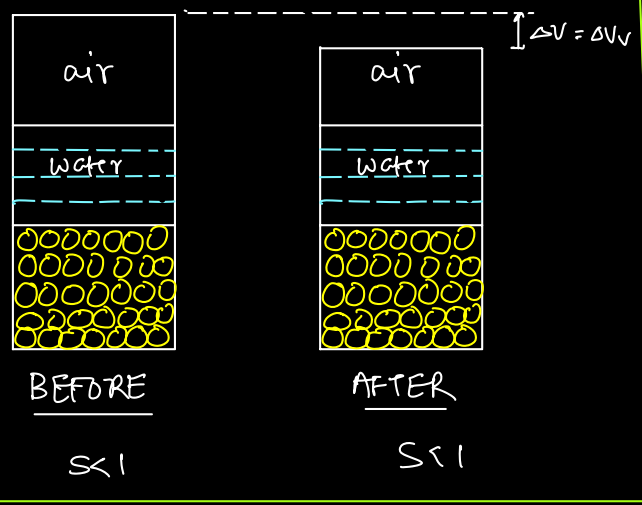
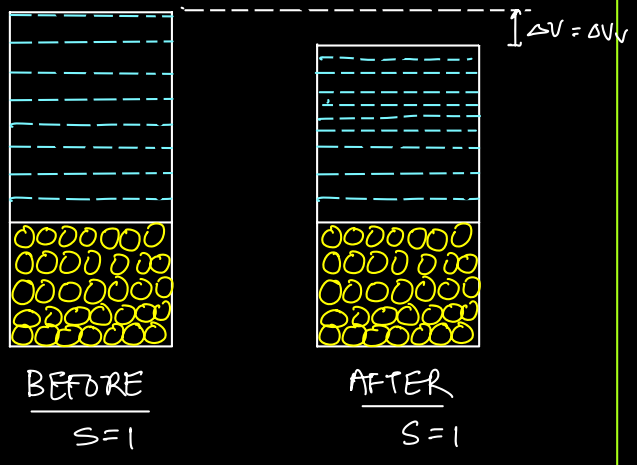
→ Compressibility is the process of reducing the volume of soil [means volume of voids]

→ Compressibility of soil occurs in two stages

(i) Compaction &

(ii) Consolidation



COMPACTION	CONSOLIDATION
1. Instantaneous process of reducing the volume due to expulsion of pore air.	1. It is the time dependent process of reducing the volume of soil due to expulsion of pore water.
2. 'S' remains less than 1	2. S always remains equal to 1
3. Compaction can be carried out artificially by the use of rollers, vibrators & Rammers.	4. Consolidation occurs on account of a load placed on the soil.
	

COMPACTION

→ It is the process of densification of given soil by the application of mechanical energy using Rammers, vibrators & Rollers

→ Compaction reduces the volume of soil, volume of voids, void ratio, permeability etc.

→ Compaction increases the density, effective stress, shear strength, bearing capacity.

→ Degree of compaction is measured in the terms of dry unit wt of compacted soil.

CHARACTERISTICS OF COMPACTION

The characteristics of compaction can be determined by performing the following tests:

(i) Standard Proctor test

(ii) Modified Proctor test

(iii) Indian Standard light compaction test.
(equivalent to standard Proctor test)

(iv) Indian standard heavy weight test
(equivalent to standard Proctor test)

[For compaction tests use IS 2720 Code]

CHARETERISTICS OF COMPACTION	STANDARD PROCTOR TEST	MODIFIED PROCTOR TEST	IS light compaction	IS heavy compaction
i) Volume of Mould	$\frac{1}{30}$ ft^3 (944cc)	$\frac{1}{30}$ ft^3	1000 cm^3	1000 cm^3
ii) Weight of hammer	5.5 lb (2.495 kg)	10 lb	2.6 kg	4.9 kg
(iii) Height of free fall	12 inches (304mm)	18 inches	310m	450mm
(iv) No of layers	3	5	3	5
(v) No of blows per layer.	25	25	25	25

The energy imparted in Indian standard light compaction per unit volume of soil can be given as

$$E = \frac{mgh \cdot N}{V} = \frac{2.6 \times 9.81 \times 310 \times 10^{-3} \times 75}{1000 \times 10^{-6}}$$

$$E = 593.01 \times 10^3 \ \frac{N}{m^2}$$

$\because 1 \frac{J}{m^3} = 1 \frac{N}{m^2}$

$\because 1 \frac{KN}{m} = 1 \frac{KJ}{m^3}$

$$\boxed{E = 593.01 \ \frac{KJ}{m^3}}$$

Similary the energy imparted in Indian standard heavy compaction per unit volume can be given as,

$$E = \frac{mgh \times N}{V} = \frac{4.9 \times 9.81 \times 450 \times 10^{-3} \times 125}{1000 \times 10^{-6}}$$

$$\boxed{E = 2703.88 \ \frac{KJ}{m^3}}$$

$$\frac{IS\ HC}{IS\ LC} = \frac{2703.88}{593.01} = 4.55$$

Note: Energy imparted in heavy compaction is 4.55 times energy imparted in light compaction

Procedure for compaction test.

↳ In order to perform the compaction test 20Kg of soil is collected and it is air dried

↳ Take 2.5 to 3Kg of this dry soil and it is mixed with water content. For sand - add 7%, For clay add 10% of water content

↳ After adding the water, the soil is mixed and kept in the air tighted container for measuring for sufficient time. Now

↳ Now the matured soil sample is filled in the given

mould of $1000\,cm^3$ in No of layers and each layer is compacted 25 times with the hammer of standard weight.

↳ Now the weight of the compacted soil filling the mould can be determined

W_1: weight of Empty Mould

W_2: weight of compacted soil

$$W = W_2 - W_1$$

↓

(weight of compacted soil.)

↳ The volume of compacted soil can be taken as the volume of mould as $1000cm^3$

↳ Now determine bulk unit weight of compacted soil as $\gamma = \frac{W}{V}$

↳ The compacted soil is taken out from the mould and a representative sample of compacted soil is taken from the middle portion of the soil, and tested for water content by oven drying machine.

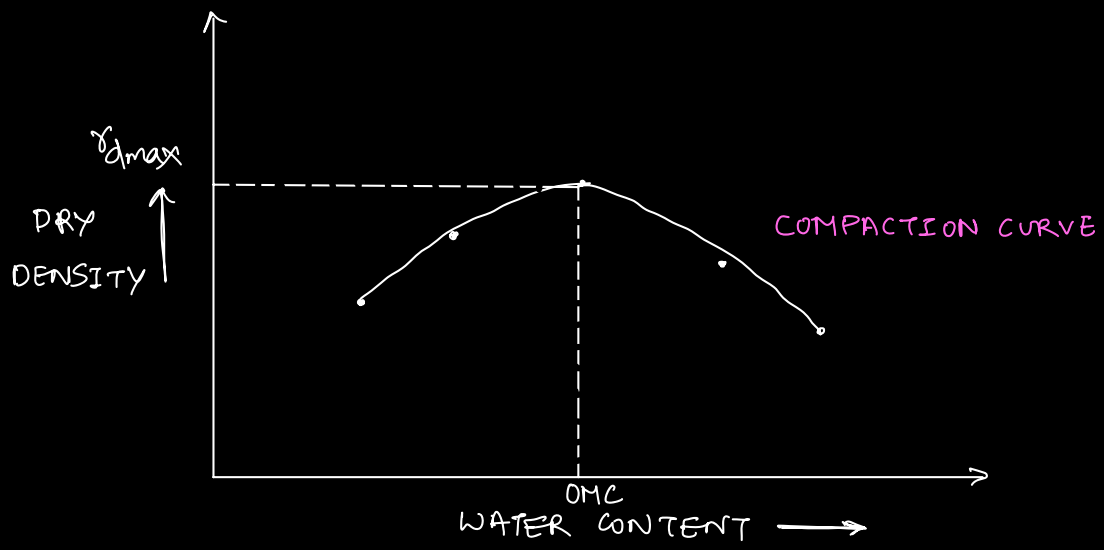
↳ Now calculate γ_d, which is a measure of compaction,

$$\gamma_d = \frac{\gamma}{(1+w)}$$

↳ Repeat the test number of times for fresh soil specimen with increasing value of water content as compared to previous one.

↳ At every time the water content and corresponding dry density of the compacted soil is measured.

↳ Plot the compaction curve taking water content on the X-axis and γ_d on the Y-axis.



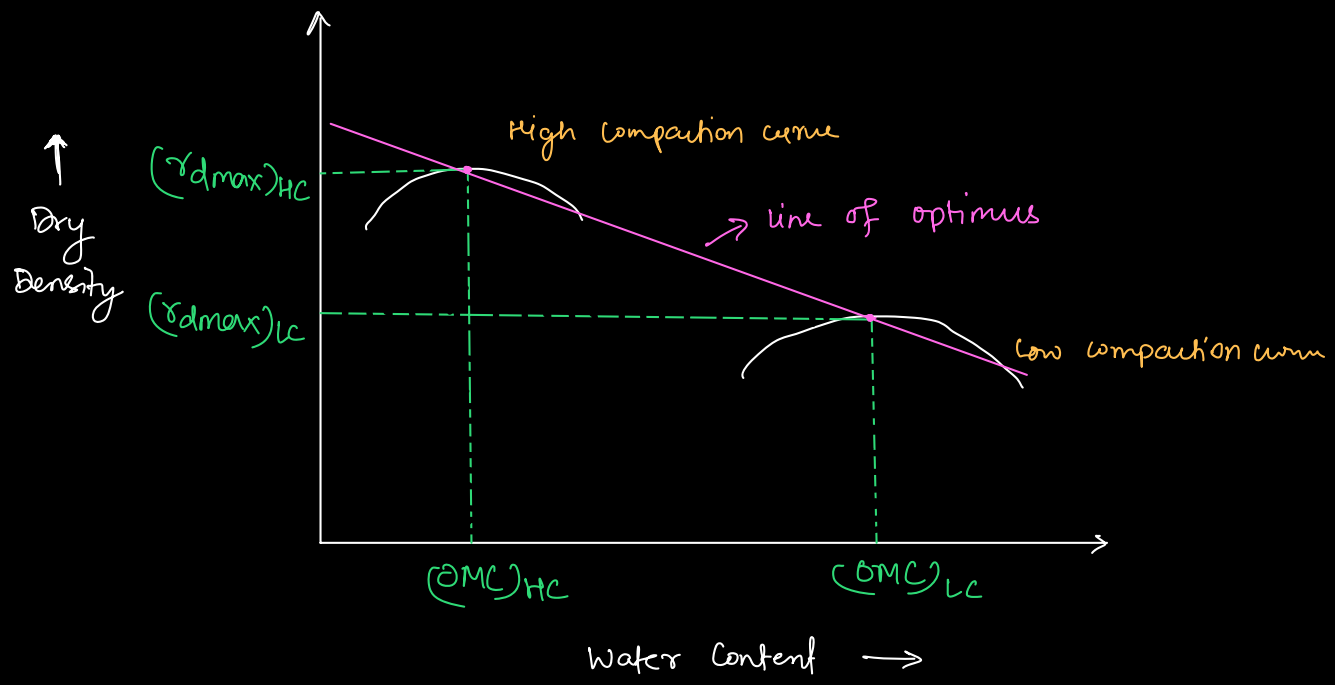
→ The objective of performing compaction curve is to plot the Compaction curve and determination of that value of water content corresponding to which we get maximum dry density. That water content is termed as optimum moisture content.

Note: For the given soil γ_{dmax} & OMC are not constant they can be changed by changing compaction efforts

Note: If LWC and HWC both are performed on the same soil then,

$$(\gamma_{dmax})_{Heavy} > (\gamma_{dmax})_{light}$$

$$(OMC)_{Heavy} < (OMC)_{light}$$



Air void line: A line which shows the relationship between dry density and water content of the compacted soil at a constant percentage air voids (n_a) is termed as Air void line

$$\gamma_d = \frac{(1-n_a)\, G\gamma_w}{1+wG}$$

During compaction if air is completely removed then,

$$V_a = 0, \quad n_a = \frac{V_a}{V} = 0 \quad ; \quad \gamma_d \rightarrow \gamma_{dth}$$

$$\gamma_{dth} = \frac{(1-0)\, G\gamma_w}{1+wG}$$

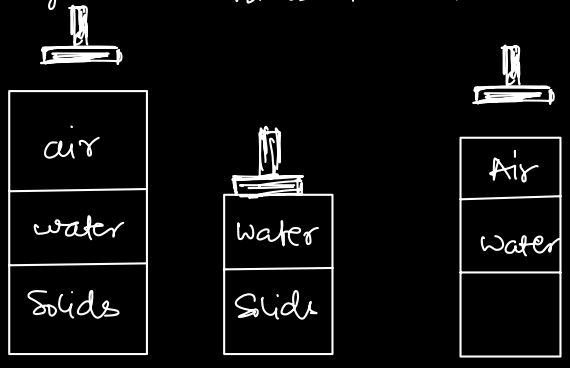
$$\gamma_{dth} = \frac{G\gamma_w}{1+wG}$$

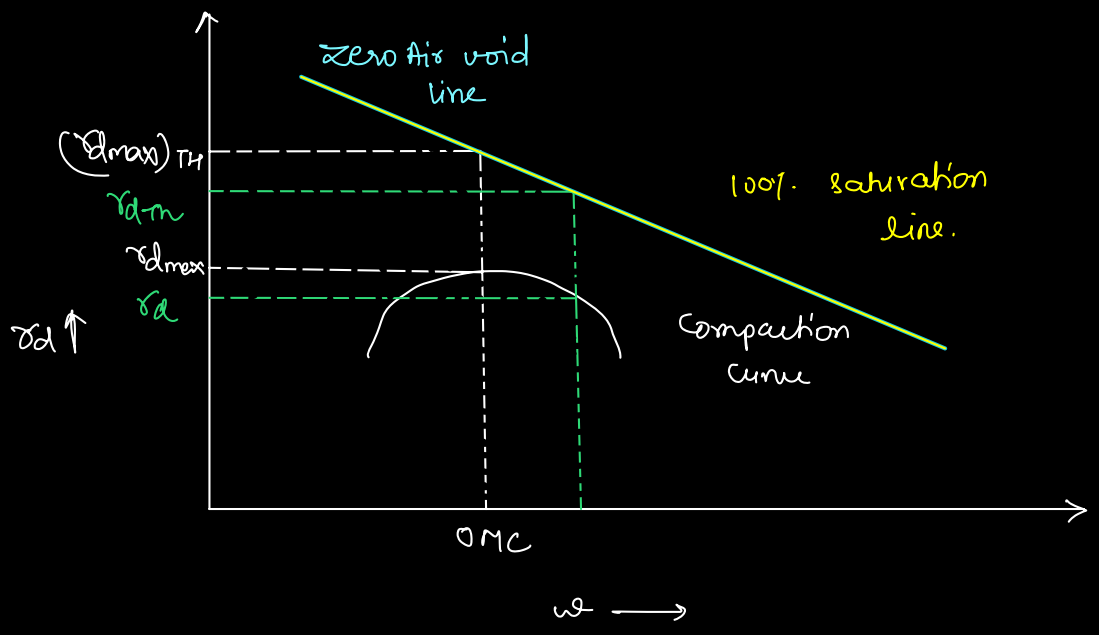
→ This is the equation of zero air void line.

→ Zero air void line is the line which shows the relationship between dry density and water content at zero percent of air voids ($n_a = 0$).

Note:- During compaction, air can not be removed completely that means practically $V_a = 0$, $n_a = 0$ are not possible in the compaction. Therefore the dry density calculated considering $n_a = 0$, is called theoretical dry density and is represented as γ_{dth}.

Note: The complete air from the soil cannot be removed because after every blow of hammer when it is lifted some amount of air enters the soil.





Saturation line: A line which shows the relation between dry density and water content of the compacted soil at constant degree of saturation is termed as saturation line. The equation of saturation line can be given as

$\because \gamma_d = \frac{G\gamma_w}{1+e}$ $\quad\because e_s = Gw$

$\therefore \gamma_d = \frac{G\gamma_w}{1+\frac{Gw}{S}}$ ⟶ saturation line.

Let during compaction DOS = 100% then

$\because S = 1$

$\therefore \gamma_d = \frac{G\gamma_w}{1+Gw}$] 100% Saturation line.

This is practically not possible, coz air enters when we lift the hammer, thus DOS becomes less than 100%. Therefore if γ_d is estimated for 100% DOS. then it is termed as theoretical dry density (γ_{dTH}).

A line which shows the relationship between dry density & water content at 100% DOS is termed as 100% Saturation line. The equation of 100% saturation line. is given above.

Note: From the equation of zero air void line & 100% Sat. line it is clear that both the lines are identical & coincides each other as we have seen zero air void line & 100% saturation line are identical, that does not mean any x% of air void line will coincide the (100-x)% of saturation line.

For ex: 20% air void line will not coincide the 80% saturation line.

Field Compaction control: The field compaction control consists of the determination of the following two important properties of soil.

(i) Water content

It can be determined using any of the following methods

(i) Calcium Carbide Method

(ii) Proctor needle method

(ii) Dry unit weight

$$\therefore \gamma_d = \frac{\gamma}{1+w}$$

$w \rightarrow$ water content already we have determined in the previous step

'γ' can be determined by any of the following methods:

1) Core cutter method

2) Sand replacement method

3) Rubber balloon method.

Relative Compaction: It can be estimated for both cohesive & cohesionless soil.

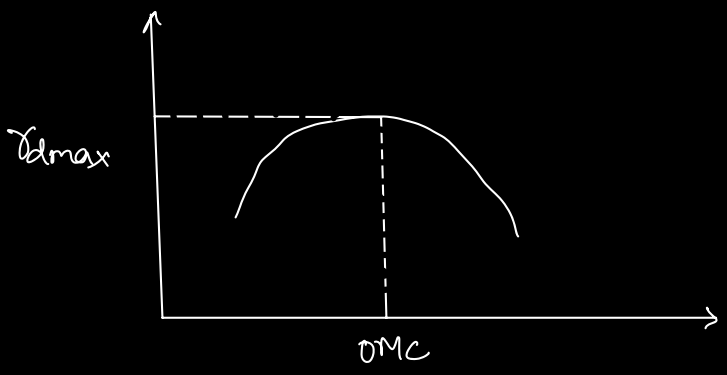
→ It can be defined as the ratio of dry density of compacted soil obtained from the field to maximum dry density

of the compacted soil obtained from the lab.

$$R_c = \frac{(\gamma_d)_{field}}{(\gamma_d)_{Lab}} \times 100.$$

Factors affecting the compaction:

(i) WATER CONTENT: From the compaction curve it is clear that if water content is increased below the OMC the dry density increases whereas beyond the OMC if water content is increases dry density decreases.



The concept or logic behind it can be explained by following two theories:

(i) "Theory of lubrication" by Proctor
(ii) "Lambe's Double layer theory".

THEORY OF LUBRICATION:

- According to this theory, at low water content soil is stiff or Hard as a result it offers more resistance against the compaction
- Therefore at lower water content less dry density is achieved in the compaction
- But when a water content is increased then a film of water is formed surrounding the soil solids acting as lubricants.
- Due to this lubrication they can easily roll or move over each other means compaction becomes easier as a result 'γd' of soil increases which means larger

value of γ_d is achieved this time.

- when water content reaches to the OMC, the effect of lubrication is maximum, as a result maximum dry density is achieved in the compaction
- But when water content is increased beyond the OMC the effect of lubrication remains same it doesn't increase after the OMC. As a result in the given fixed volume of soil water occupies more space which results the decrease in the dry density of soil.

(That means in the fixed volume of mould amount of water is which replaces the soil solids from the mould.

$\therefore \gamma_w < \gamma_s$

$\therefore$ less dry density is achieved in the compaction

LAMBE'S DOUBLE LAYER THEORY:

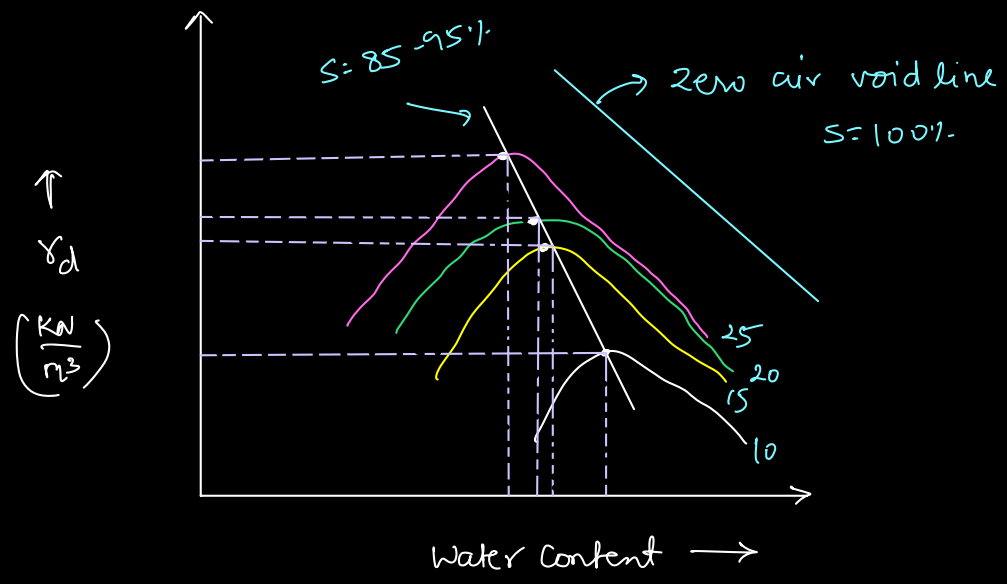
It is observed that at low water content attraction force is present between the particles due to the presence of Vander Wall's forces, whereas at high water content force of repulsion is present b/w the particles due to the developement of diffused double layer.

When the soil is compacted at low water content then net force b/w the particles is attractive because at low water content thickness of diffused layer is very less (not fully developed) as a result loose resistance is offered by the soil against the compaction therefore less compactness or less dry density is achieved But when the water content is increased & compacted, thickness of double layer expands) as a result net force between the particles is repulsive due to which particles move or roll over each other that means soil can be easily compacted now. It leads to the higher dry density or

Compactness. when water content reaches to the OMC the full expansion of double layer is observed. Therefore maximum dry density is achieved at the OMC. Beyond the OMC if water content is increased then it doesn't lead any expansion in the double layer as a result dry density is reported to be less because in the given soil water occupies more space and replace the solid particles.

(ii) AMOUNT OF COMPACTION / EFFORTS:

- With the higher compaction efforts the maximum dry density (γ_{dmax}) is reported to be higher and optimum moisture content (OMC) is reported to be lower.



- As the compaction effort is increased, maximum dry density γ_{dmax} is increased but there is no direct & linear relation between them.
- Because the maximum dry unit weight does not go on increasing as the compaction effort is increased
- The margin of increase in maximum dry density becomes smaller and smaller on the dry side of OMC while on wet side of OMC it hardly increases.
- For example you can see the figure in which the margin is

initially 5 KN/m³ then it reduces to the 2 KN/m³ then 1 KN/m³

- It is also observed that with increase in water content the influence of compaction on increment of dry density decreases as shown in figure.
- It is also observed that on increasing the compaction effort the compaction curve shifted to TOP-LEFT.

#LINE OF OPTIMUS: If the peak points of all the compaction curves are joined by a line, it is termed as line of optimus which is nearly parallel to zero air void line.

Note: At zero air void line, the degree of saturation at every point is 100%. Thus line of optimus is a practical line, the DOS is nearly equal to 85-95% at every point on the line of optimus.

(iii) TYPES OF SOIL: Compaction also depends on type of soil as follows

(i) Well graded coarse grained soil with some percentage of finer will give higher value of maximum dry density. If % of finer increases then it tends to reduce the maximum dry density which can be observed in the compaction curve number ①, ② & ③.

→ From the soil classification we know that

% of finer is <5% in SW [Curve ①]

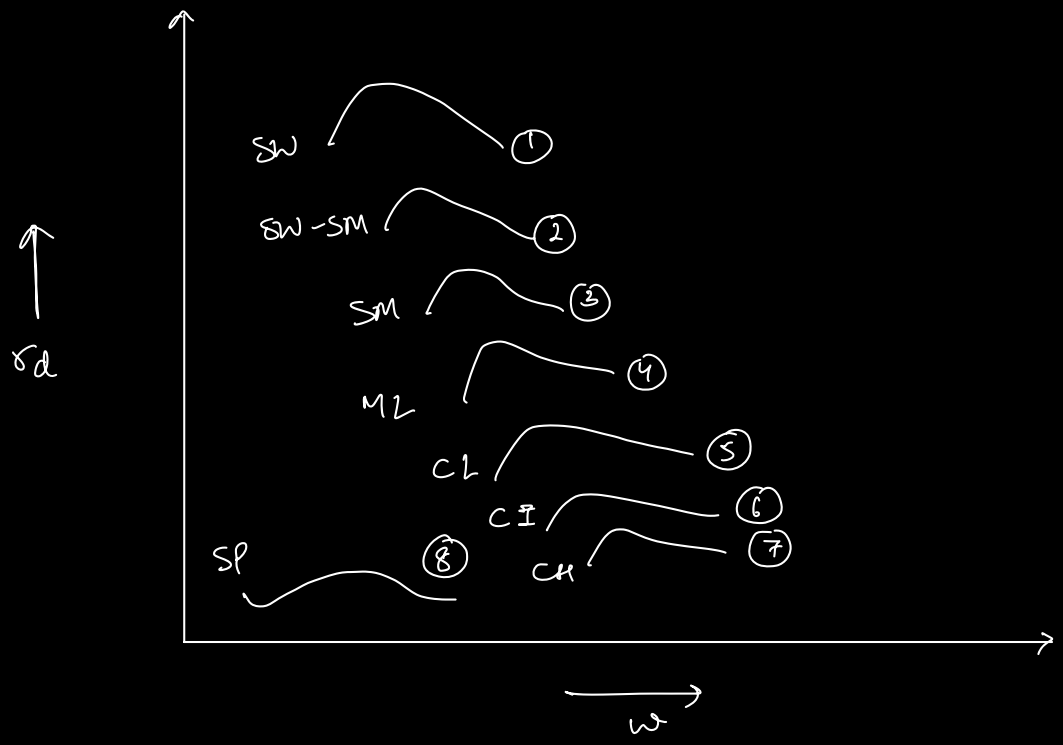
% of finer is 5-12% in SW-SM [Curve ②]

% of finer >12% in SM [Curve ③]

(ii) From the compaction curve 1 & 8 it is clear that poorly graded CGS shows least value of MDD as compared to well graded CGS.

(iii) As plasticity increases it tends to decrease the maximum dry density. It can be observed among ③, ④, ⑤, ⑥, ⑦

(iv) Cohesive soils have high value of OMC than cohesionless soil.



Compaction curve for pure sand:

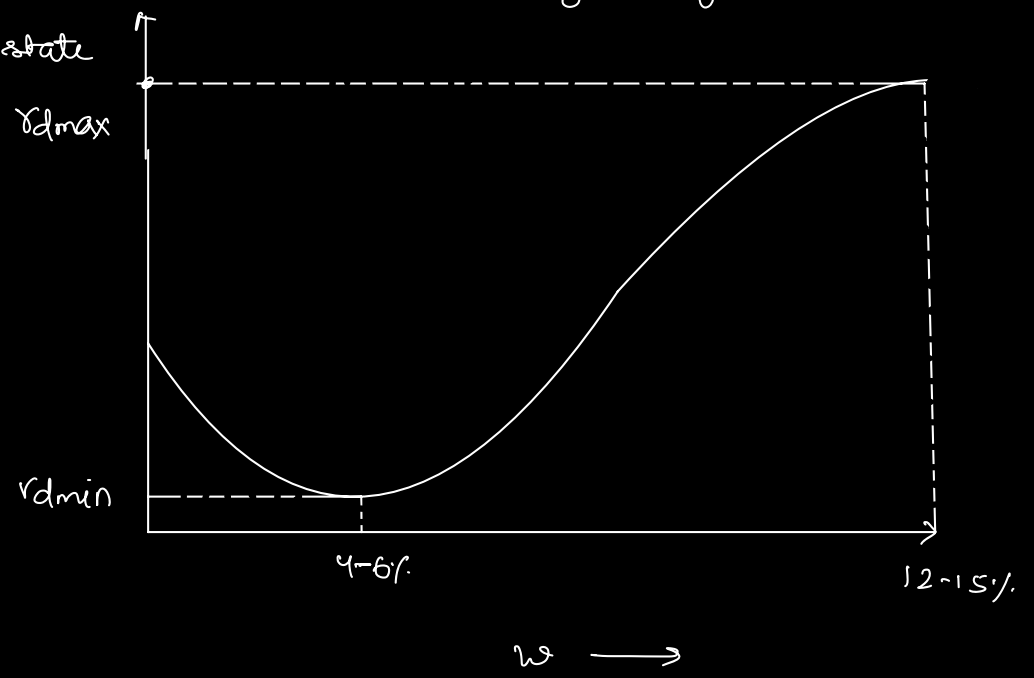
When sand contains some % of fines it's compaction curve is generally inverted v shaped. But when we talk about the pure sand the trend is some what different.

→ If sand is compacted at 4-6% water content then minimum dry density is obtained due to bulking of sand

→ When the water content is increased beyond 6%, cohesion decreases as a result the clusters of honeycomb starts breaking down which results in the decrease in the volume of soil which tends to increase the dry density of soil.

→ once the water content reaches to 12-15% (in coarse sand) the meniscus is completely destroyed & all the clusters of honeycomb get breakdown and reaches to the dense state At this state degree of saturation reaches to 100% and maximum dry density is obtained in the compaction.

Therefore it is suggested that the compaction of sand should be done in either at completely dry state or at fully saturated state



Note: In case of sand or coarse grained soils SSA is small therefore it doesn't tend to adsorb more water therefore double layer is not performed. Therefore Lambe's double layer theory is not applicable in case of sand Therefore we don't obtain inverted V-shaped graph in case of pure sand.

V) METHOD OF COMPACTION:

The compaction is greatly affected by method or manner of compaction - the various factors are as follows

(i) Weight of equipment used for compaction.

(ii) Manner of compaction - Impact, static, kneading etc.

(iii) Time and area of contact between equipment & soil.

EQUIPMENT USED:

A) ROLLERS

(i) Smooth wheeled roller:

→ A large steel drum is rolled over the soils to compact them

→ It is suitable for the Compaction of weathered rocks, cohesionless soil, boundary soil and crushed rocks & gravelly soils

→ It is used in the construction of highways for the compaction of surface dressing

→ offers large contact pressure.

(ii) SHEEP FOOT ROLLER:

→ It has kneading action

→ This kneading action breaks the lumps of the soil

→ It is suitable for cohesive soils [clay and silt] only

→ suitable for cohesionless soils

→ used in compaction of the core of the earth dam

→ Soil compacted by sheep foot roller has better orientation of particles (dispersed structure) which shows less permeability of the soil.

(iii) PNEUMATIC TYRED WHEEL ROLLER:

→ It offers kneading action and contact pressure both

→ It is suitable for cohesive and cohesionless soils both

→ by the use of kneading action, the flocks of the lumpy soil can be broken.

→ used in the compaction of sub-base course, Base course, embankment.

(iv) GRID ROLLER

→ It offers large contact pressure but very less kneading action. Therefore it is not suitable for fine grained soil like clay & silts

→ It is suitable for weathered rocky soils, well

graded coarse grained soils, gravelly soils etc.

Note: It can be used sometimes for fine grained soils especially when they are wet.

(B) VIBRATORS:

→ It is modified or the advanced version of the smooth wheeled roller with vibration effect.

→ Not suitable for cohesive soils.

→ Used in the compaction of shallow foundations embankments etc.

C) RAMMER or DROPING WEIGHT:

→ offers impact to the soil in compaction.

→ This process of compaction is termed as "tamping"

→ It can be done manually

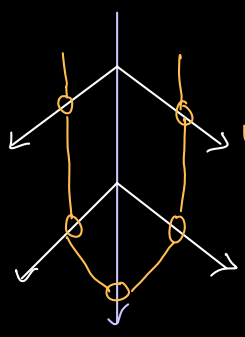
→ It can be used for all the types of soil.

→ Used when area is confined

D) FROG HAMMER:

It can be called the advanced version of rammer which is electrically operated. Suitable for cohesionless soils. Generally used to compact the soils in the corners of the field where big size roller or vibrators cannot reach.

VIBROFLOATATION:

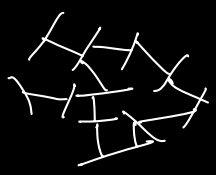


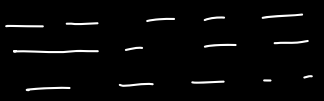
(VIBROFLOATATION)

→ In this method, we provide vibration to the sand along with the water jets with high pressure. Now in the bore hole good quality sand can be added which shall be settle down under the self weight After that, vibrofloat is slowly lifted & taken out.

→ This method is suitable for the improvement of poorer sand or cohesionless soil.

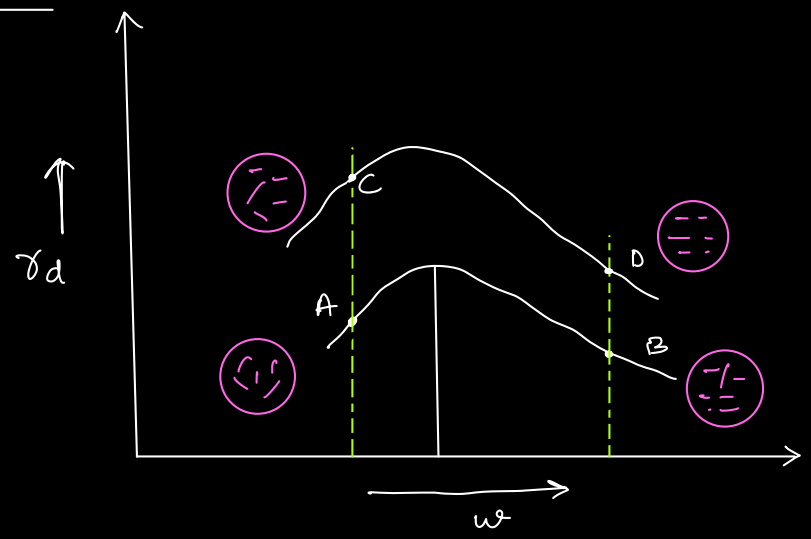
FLOCCULATED STRUCTURES	DISPERSED STRUCTURE
↳ more random orientation	↳ nearly parallel orientation
↳ void ratio is more	↳ void ratio is less
↳ permeability is more	↳ permeability is less
↳ very poor orientation	↳ orientation is improved
↳ obtained at dry side of Compaction	↳ obtained at wet side of Compaction





Effect of Compaction on the Engineering properties:

1. Soil structure:



- For a given water content if Compaction effort is increased the soil tends to becomes more oriented specially

on dry side of optimum.

That means the soil structure at point "c" is more oriented than the point "A".

- Structure at point D is more oriented than point B
- Structure at point A is more random oriented than the point "B". Same for point "C" and "D".

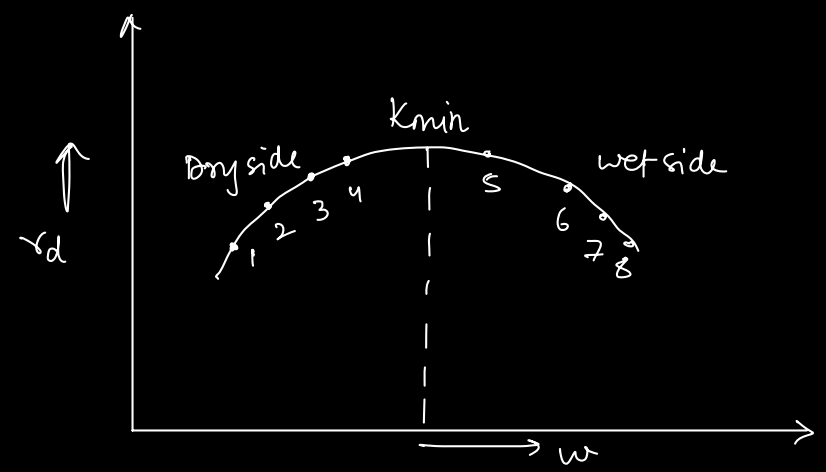
2. Permeability :

→ For given compaction efforts, permeability decrease sharply with increase in water content on dry side of optimum.

→ Beyond the OMC, the permeability may show a slight increase but remains smaller than the permeability of the dry side of optimum

→ The sharp decrease in the permeability on dry side of optimum is observed due to the improvement in orientation of particles on increasing the water content as a result void ratio decreases and it tends to decrease the permeability.

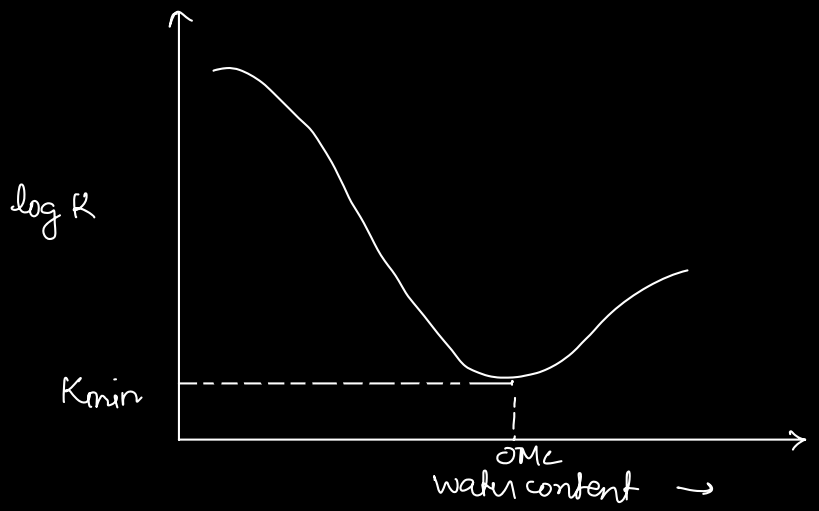
→ The slight increase in permeability on the wet side of optimum is due to the effect of a decrease in dry unit weight which is more dominating than the effect of improved orientation.



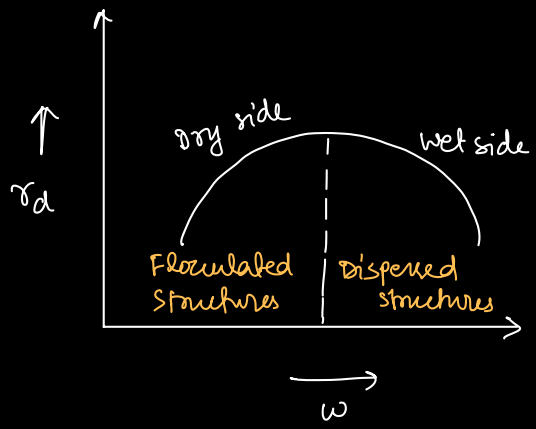
$$K_1 > K_2 > K_3 > K_4 > K_{min}$$

$$K_5 < K_6 < K_7 < K_8$$

$$(K_1, K_2, K_3, K_4) > (K_5, K_6, K_7, K_8)$$



3. Compressibility



→ At relative low stress levels, a soil compacted wet of optimum is more compressible than the flocculated soil at the dry side of optimum.

→ Because the strong inter-particle bonds in a flocculated structures do not allow the particles to be displaced

→ whereas at high stress levels, the opposite phenomenon is observed because once the bonds are broken, it under goes large decrease in volume

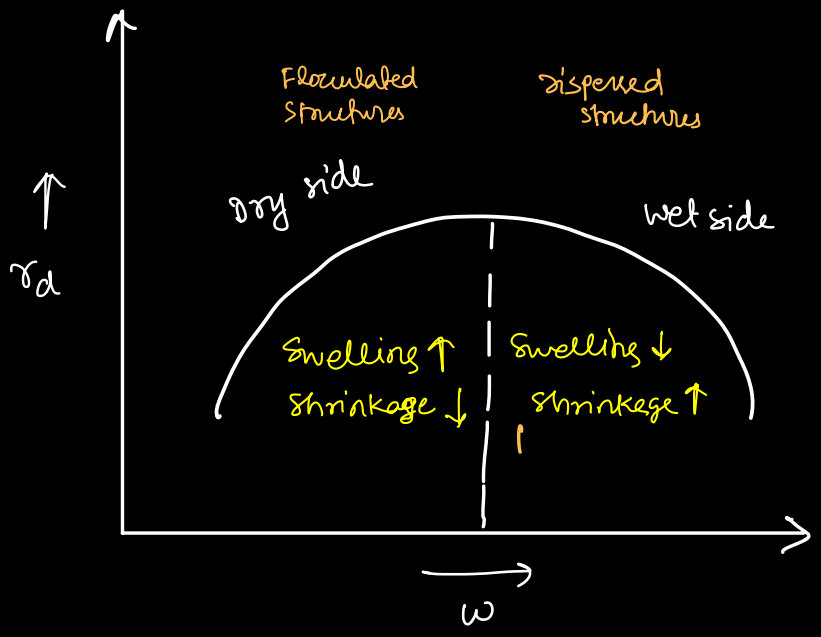
④ Swelling

A soil on the dry side of optimum has a higher water deficiency and a more random particle arrangement. Therefore, because of its large void space, it can

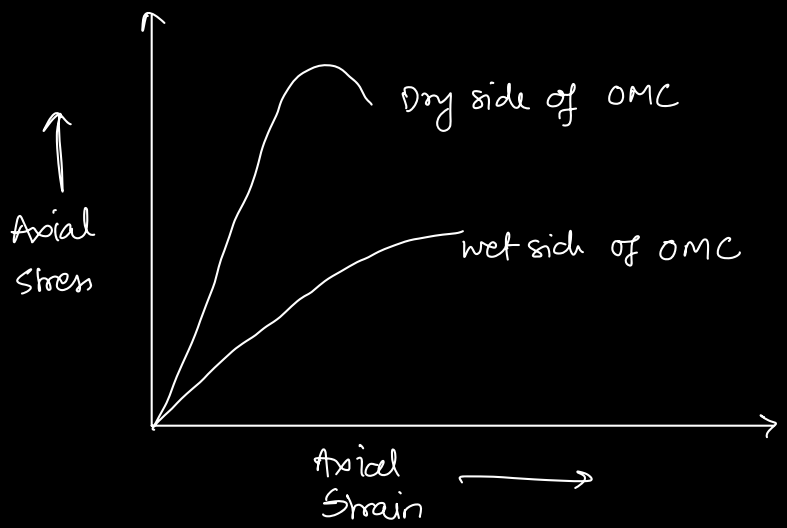
imbibe more water, therefore large swelling is observed on dry side of optimum

5. Shrinkage

- Soils compacted on the wet side of OMC tends to exhibit more shrinkage upon drying than those compacted on dry side of optimum



6. Stress - strain Relationship.



→ Soils compacted on dry side of OMC, with their strong bond, resist the formation upon application of axial stress, therefore stress-strain curves increase rapidly to the peak and then fall on breakage of bonds

→ Soils compacted wet of optimum have a flatter σ-ε curve & without any fall.

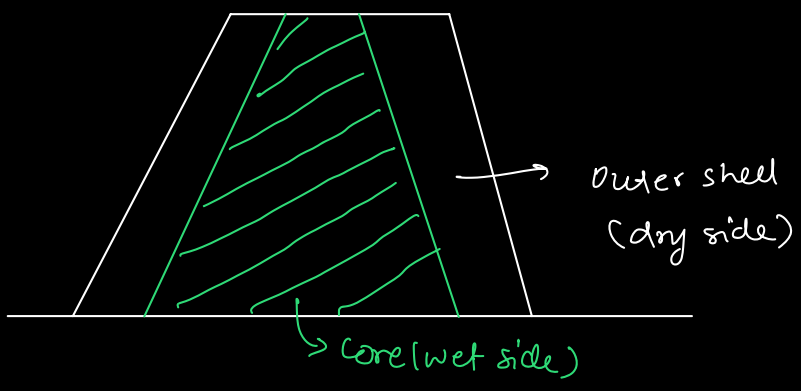
7. Construction pore water pressure:

- Soil compacted on wet side will have higher PWP compared to soils compacted on dry side which initially have less (maybe negative) pore water pressure.

Note: On the above discussion it should be clear that even through compaction at OMC produces maximum dry density but it is not always desired to do so.

Depending upon requirement, the soil should be compacted either wet or dry side of optimum.

Projects	Compaction	Remark
1. Core of earth Dam (composite)	Wet of Optimum	To reduce permeability & to prevent cracking of core due to swelling
2. Homogeneous earth dam & outer shell of composite earth dam.	Dry of Optimum	To have stronger soil and to prevent buildup of high PWP.
3. Subgrade of pavement.	Wet of Optimum	To limit the volume change due to swelling



CONSOLIDATION

Processes or steps involved in the compressibility of soil

i) Compaction of air
ii) Expulsion of air
} COMPACTION

iii) Compression of water
iv) Expulsion of water
} PRIMARY CONSOLIDATION

v) Compression soil solids
vi) Plastic readjustment of soil solids
} SECONDARY CONSOLIDATION.

The total compressibility can be expressed in the terms of total settlement as follows.

$$S_t = S_i + S_p + S_s$$

S_t : Total settlement or total volume change

S_i : Immediate settlement

S_p : Primary consolidation settlement

S_s : Secondary consolidation settlement. (creep settlement)

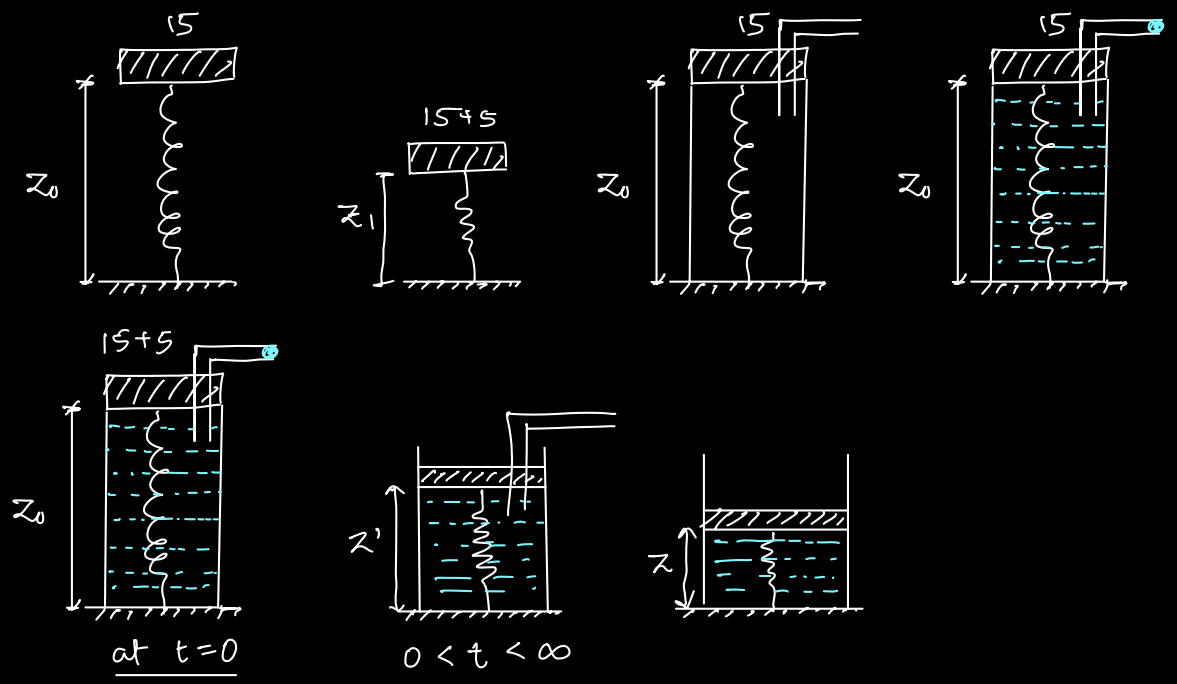
→ Pore water & soil solids are considered to incompressible thus there compression can be neglected in the process of compressibility.

Primary Consolidation	Secondary Consolidation
i) After completion of compaction when DOS becomes 1 then primary consolidation starts	i) It is also a time dependent process of reducing the volume of soil due to a) plastic readjustment of solids b) removal of highly viscous adsorbed water
ii) In the primary consolidation the volume of soil reduces due to expulsion of pore	

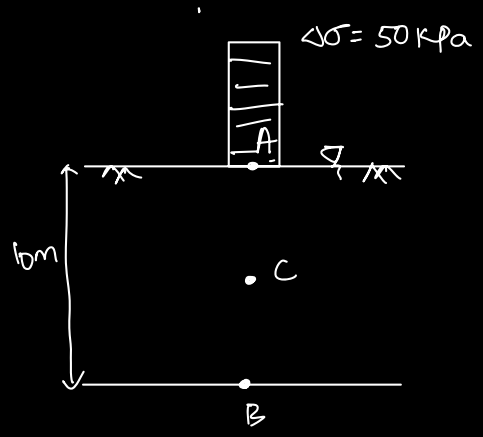
water (iii) The DOS remains ∞ throughout the primary consolidation (iv) Theoretically ∞ time is required in the completion of primary consolidation	(ii) takes place after completion of primary consolidation (iii) In the secondary consolidation the decrease in the volume of the soil is observed at constant stress level. iv) It is also known as creep settlement v) It is higher for soft clay, soft organic soils, peat but in case of stiff clay, preconsolidated clay it becomes less significant

PRIMARY CONSOLIDATION

[Explained with the help of spring analogy by terzaghi



Note: with the help of spring dashpot system, the mechanism of primary consolidation can be explained.



$\gamma_{sat} = 20 KN/m^3$

$\gamma_w = 10 KN/m^3$

At $t = 0$,

$\sigma_A = 0 + 50 \quad u_A = 0 + 50 \quad \bar{\sigma}_A = 0$

$\sigma_C = 100 + 50 \quad u_C = 50 + 50 \quad \bar{\sigma}_C = 50$

$\sigma_B = 200 + 50 \quad u_B = 100 + 50 \quad \bar{\sigma}_A = 100$

$(0 < t < \infty)$

$\sigma_A = 0 + 50 \quad u_A = 0 + 0 \quad \bar{\sigma}_A = 0 + 50$

$\sigma_C = 100 + 50 \quad u_C = 50 + 40 \quad \bar{\sigma}_C = 50 + 10$

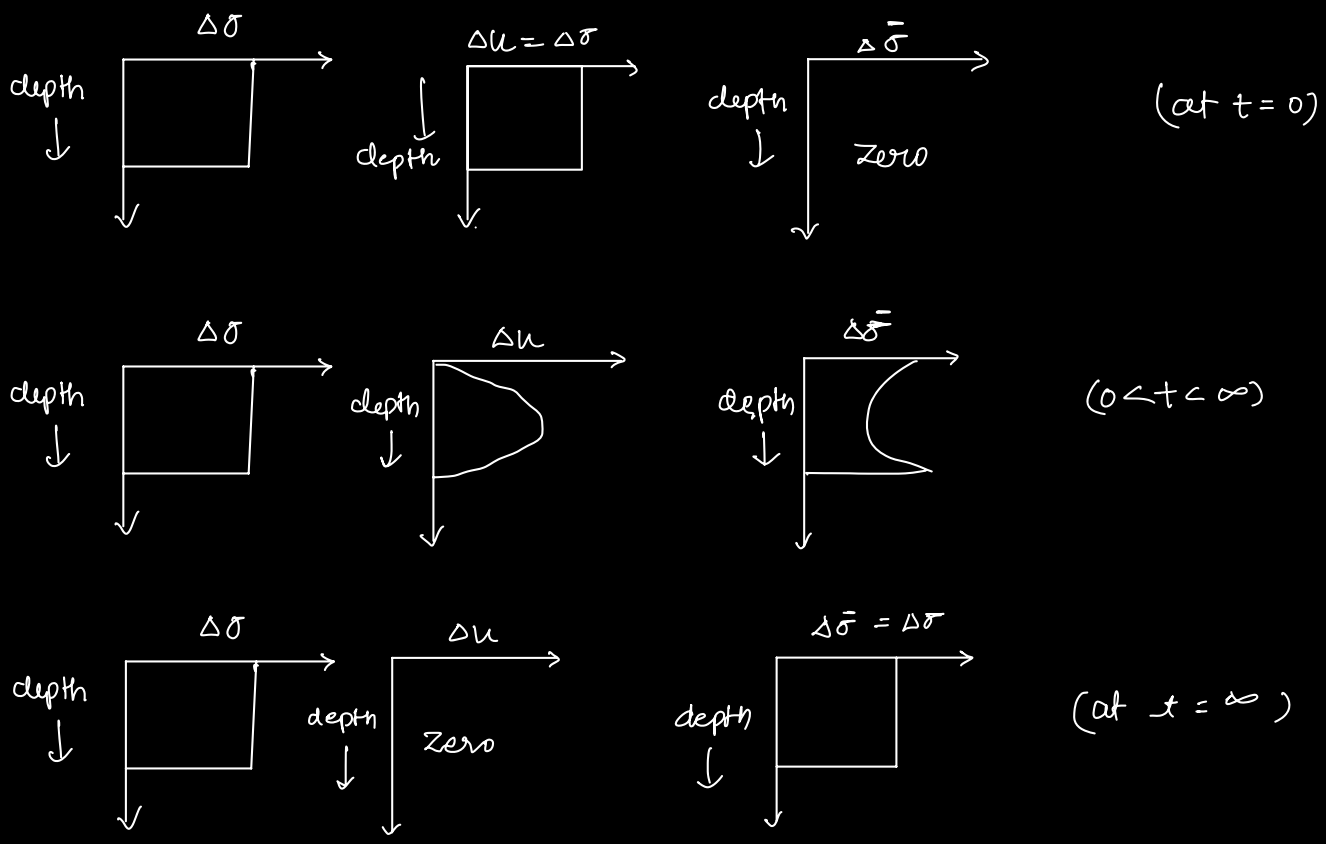
$\sigma_B = 200 + 50 \quad u_B = 100 + 0 \quad \bar{\sigma}_B = 100 + 50$

at $t = \infty$

$\sigma_A = 0 + 50 \quad u_A = 0 + 0, \quad \bar{\sigma}_A = 0 + 50$

$\sigma_C = 100 + 50 \quad u_C = 50 + 0, \quad \bar{\sigma}_C = 50 + 50$

$\sigma_D = 200 + 50 \quad u_B = 100 + 0, \quad \bar{\sigma}_B = 100 + 50.$

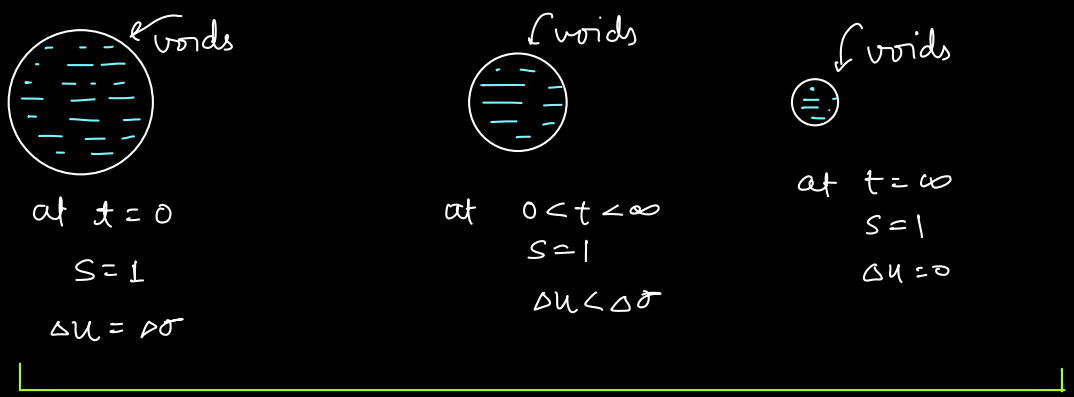


→ In the fully saturated soil the surcharge $\Delta\sigma$ is applied as a result the excess pore water pressure Δu is developed of equal magnitude immediately at $t = 0$

→ As the time passes this excess pwp starts dissipating due to expulsion of pore water.

→ After ∞ time the excess pore water pressure is completely removed.

→ When the drainage of pore water pressure stops that is the end of primary consolidation.



NOTE: Degree of saturation remains 1 throughout the process of consolidation.

Compressibility of the soil depends upon the following factors:

a) Type of soil

b) Role of stress history

c) Boundary conditions (oneway or two way drainage)

d) Effect of stress.

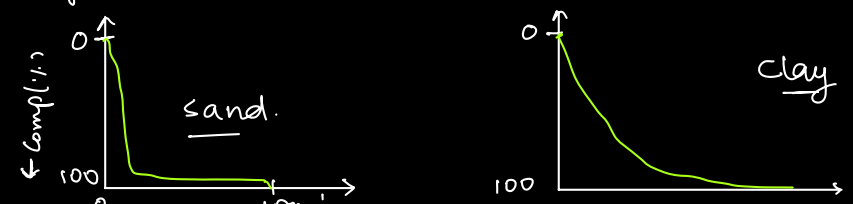
Effect of soil type on Compressibility:

(1) Rate of consolidation: It means the rate of expulsion of pore water or the rate of settlement (compression)

→ Rate of consolidation depends upon the permeability

→ For sands, K ↑↑, rate ↑↑, time ↓

→ For clays, K ↓↓, rate ↓↓, time ↑



time → 10⁴min 0 10⁴min

(Rate of Consolidation)$_{SAND}$ > (Rate of Consolidation)$_{CLAY}$.

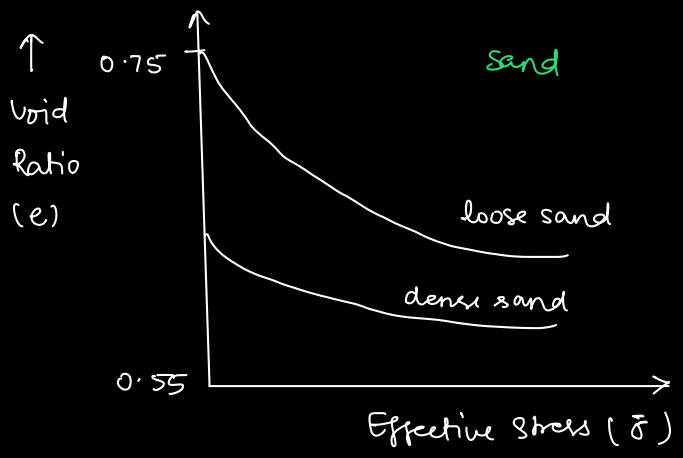
(iii) Total Consolidation or compressibility

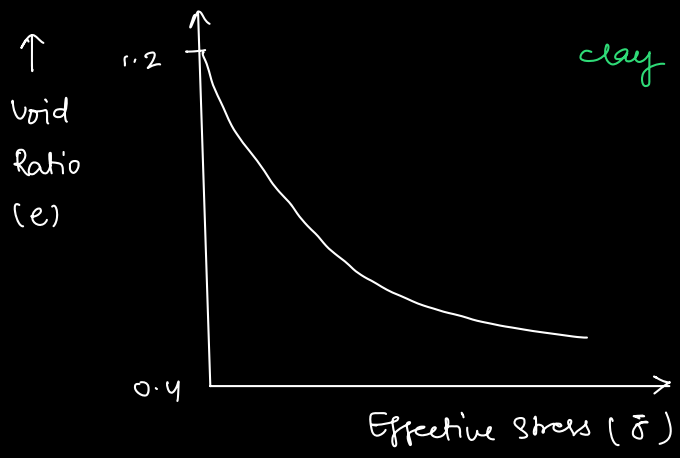
→ It depends upon the available voids in the soil.

→ If the void ratio of the soil is more then consolidation settlement will be more.

→ If the void ratio is less then the consolidation settlement will be less.

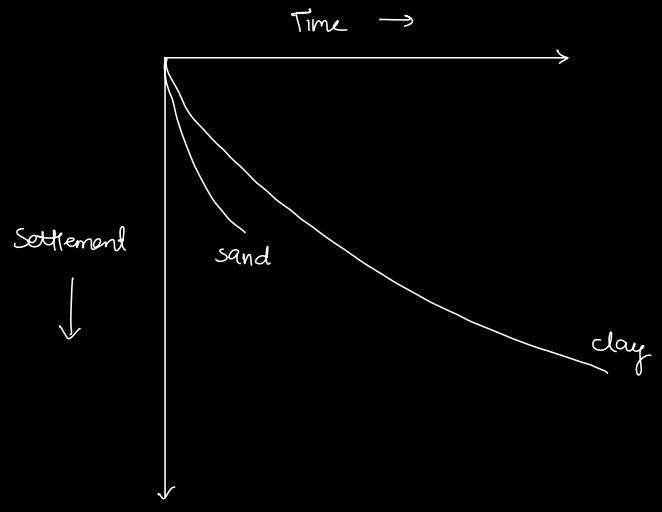
→ For sands : void ratio is less as compared to clay





Note: Clay is more compressible than sand.

Note: Loose sand is more compressible than dense sand.



Note: The consolidation study is more important and significant for clay because of its high compressibility

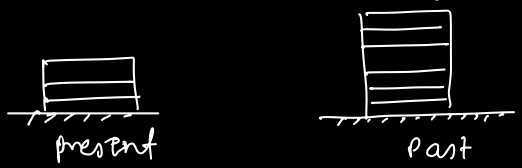
Role of stress History:

Soil tend to remain the effects of stress changes that has taken place in their geological history, in the form of their structure. If the given soil is subjected to the stress level greater than the stress level of its past condition then breakdown of soil structure happens and large settlement is observed.

- if $(\bar{\sigma}_{past})$ is less than $(\bar{\sigma})_{present}$, the soil is termed as NCC (Normally consolidated clays)

- In the present condition soil can be termed as NCC (Normally consolidated clay)

- if $(\bar{\sigma}_{past})$ is greater than $(\bar{\sigma})_{present}$, the soil is termed as OCC (over consolidated clays or pre consolidated clay.



- In the present condition soil can be termed as OCC (over consolidated clay)

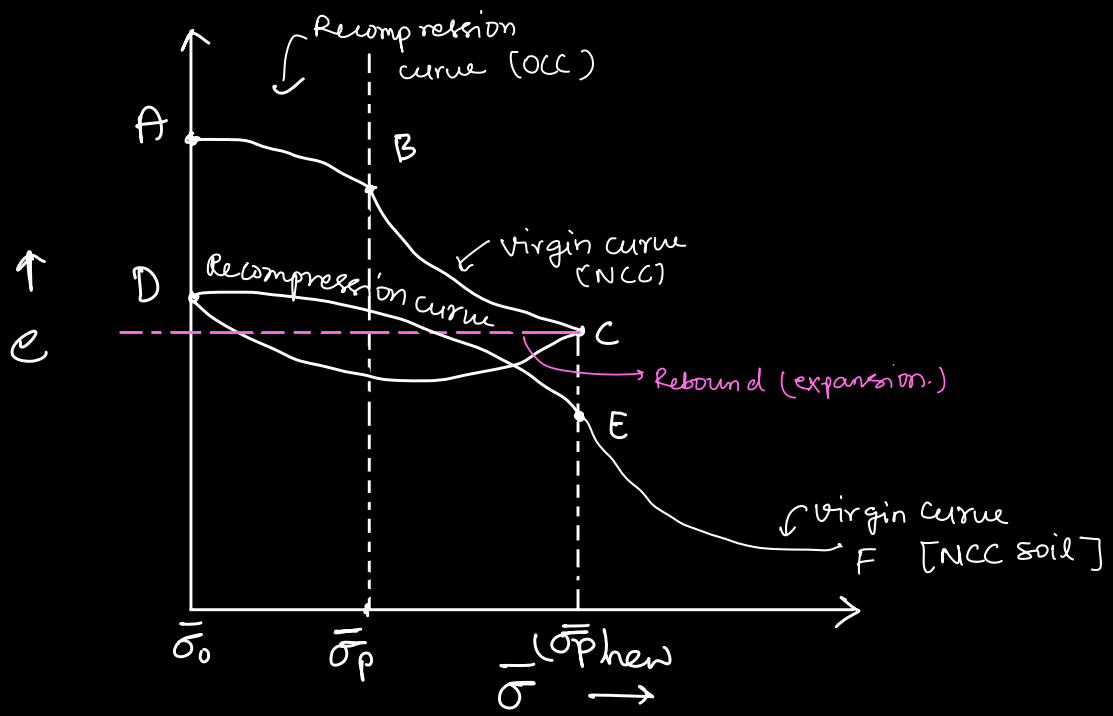
Overconsolidation Ration (OCR): It can be defined as the ratio of effective stress of the past to the effective stress of present

$$OCR = \frac{\bar{\sigma}_{past}}{\bar{\sigma}_{present}}$$

- For NCC soils OCR ≤ 1 & For OCC soils, OCR > 1.

NCC Soil	OCC soil
i) $(\bar{\sigma})_{present} > \bar{\sigma}_{(past)}$	i) $\bar{\sigma}_{past} \geq \bar{\sigma}_{present}$
ii) void ratio is more	ii) void ratio is less
(iii) Looseness is more	(iii) Denseness is more
(iv) "more compressible"	(iv) "less compressible"
(v) Consolidation settlement is more	v) Consolidation settlement is less (or very less)

(vi) OCR ≤ 1 vii) OCR > 1



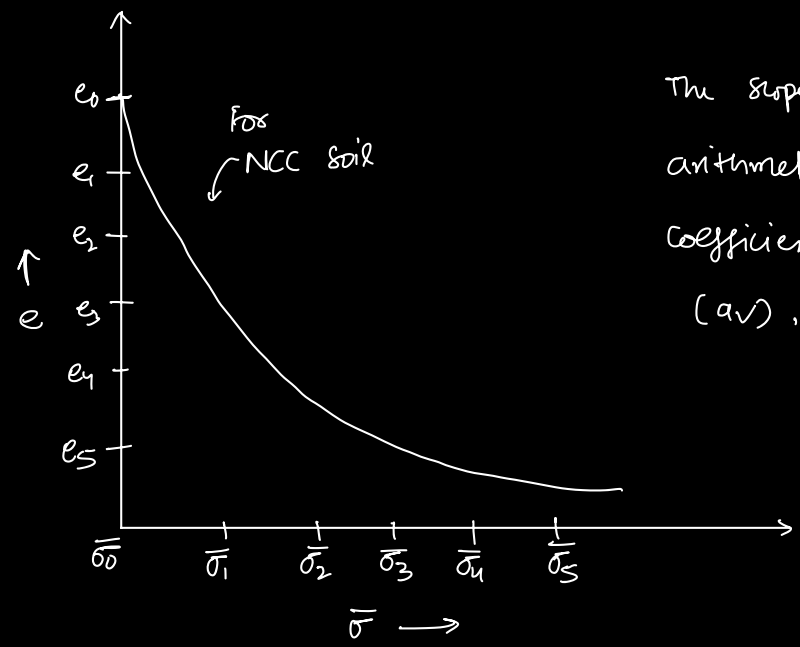
AB, DE : Recompression curve [OCC] : CONVEX

BC, EF : Virgin curves [NCC] : CONCAVE

CD : Rebound curve / swelling curve / expansion curve / unloading curve

DE : Reloading curve / Recompression curve.

Virgin curve on arithmetic scale:



The slope of the virgin curve on arithmetic scale is called coefficient of compressibility (a_v).

$$a_v = -\left(\frac{e_0 - e_1}{\bar{\sigma}_0 - \bar{\sigma}_1}\right)$$

$$a_v = \frac{e_0 - e_1}{\bar{\sigma}_1 - \bar{\sigma}_0} = \frac{\Delta e}{\Delta \bar{\sigma}}$$

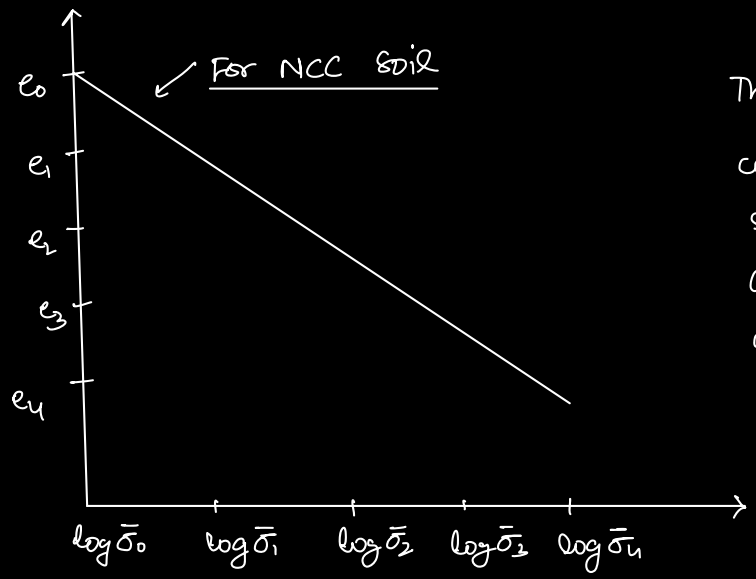
{UNIT : m^2/kN}

→ a_v is the slope of the virgin curve which shows the decrease in the void ratio on increasing effective stress

→ The slope of virgin curve on the arithmetic scale is changing (decreasing) on increasing the effective stress. Thus value of a_v is changing. Thus a_v is less significant

→ To find the stable parameter which does not change we plot the $e - \bar{\sigma}$ plot on the semi-log scale taking void ratio on arithmetic scale and effective stress on log scale

Virgin curve on semi-log scale:



The slope of the virgin curve on the semi-log scale is termed as coefficient of compression or compression index [C_c].

$$C_c = \frac{-(e_1 - e_0)}{\log_{10}\bar{\sigma}_1 - \log_{10}\bar{\sigma}_0}$$

$$C_c = \frac{(e_0 - e_1)}{\log_{10}\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_0}\right)} = \frac{\Delta e}{\log_{10}\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_0}\right)}$$

$$C_c = \frac{\Delta e}{\log_{10}\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)}$$

$$C_c = \frac{\Delta e}{\log_{10}\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)} \rightarrow \text{UNITLESS}$$

Empirical relationships for C_c

i) For undisturbed soil (medium to low sensitivity) :

$$C_c = 0.009 (w_L - 10)$$

ii) Remoulded soil (medium to high sensitive):

$$C_c = 0.007 (w_L - 10)$$

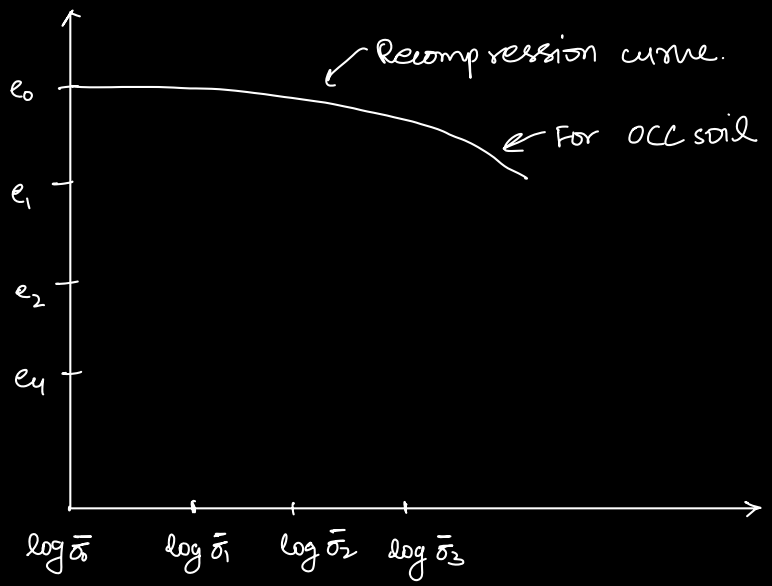
or

$$C_c = 0.007 (w_L - 7)$$

(iii) For organic soils:

$C_c = 1.15 \times 10^{-2} \cdot w_n$ (Natural water content)

Recompression curve on semi-log scale:



The slope of recompression curve on semilog scale is termed as coefficient of recompression or Recompression index.

Recompression index.

$$C_r = \frac{-(e_0 - e_1)}{\log \bar{\sigma}_1 - \log \bar{\sigma}_0} = \frac{(e_0 - e_1)}{\log \left(\frac{\bar{\sigma}_1}{\bar{\sigma}_0}\right)}$$

$$C_r = \frac{\Delta e}{\log \left(\frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}_0}\right)} \rightarrow \underline{\text{UNITLESS}}$$

→ C_c & C_r represent the compressibility of NCC & OCC soil respectively. The change in void ratio on increasing the effective stress will be more for NCC soil therefore "C_c" is greater than "C_r". That indicates, NCC soil is more compressible than OCC soils.

$$C_r = \left(\frac{1}{5^{th}} \text{ or } \frac{1}{10^{th}}\right) \text{ of } C_c$$

Note:- For NCC soil, a_v and C_c both represent the compressibility of soil. The C_c is more stable and significant parameter than a_v because it almost remains same for every value of stress.

Factors responsible for preconsolidation pressure on the soil($\bar{\sigma_p}$):

(i) Large structure (buildings) in past and small structure in present.

(ii) Erosion, weathering, disintegration of top layer of the soil.

(iii) Rise in ground water table.

(iv) drying of soils to high temperature and No rains

v) Drying of soil due to plant's life

Estimation of the preconsolidation pressure for OCC soils: (proposed by A. Casagrand).

→ first draw the e-log$\bar{\sigma}$ graph for the given soil

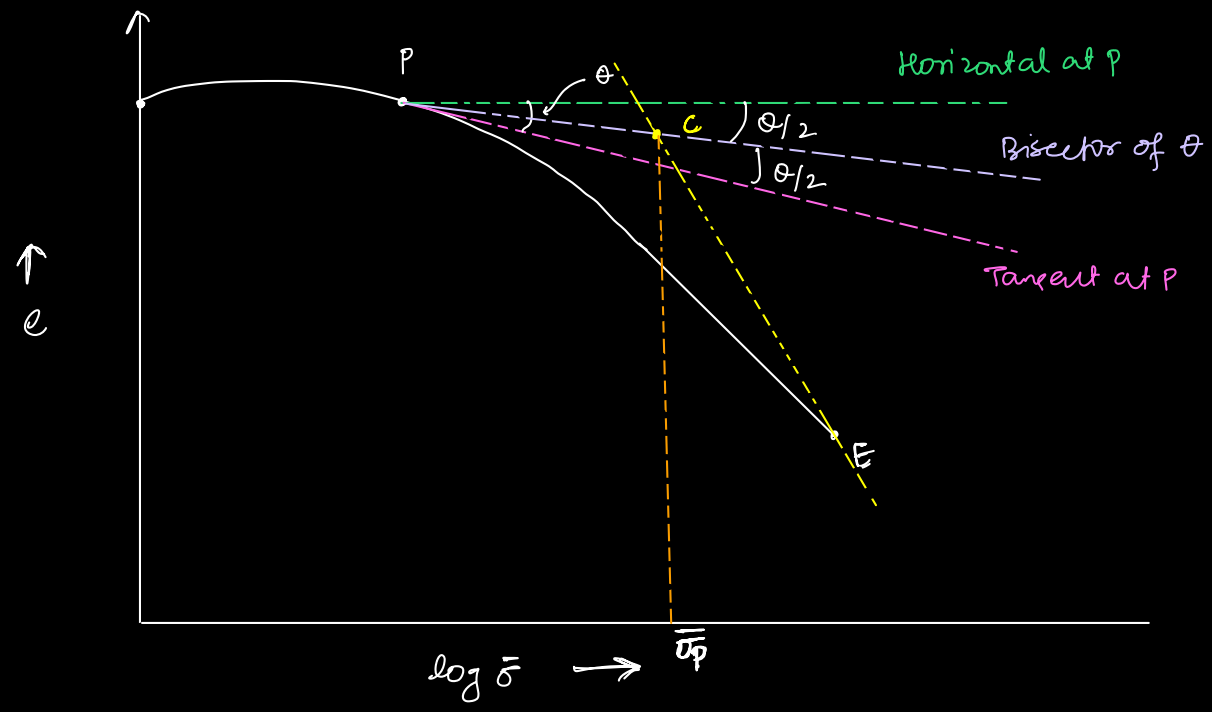
→ Take a point (P) on the recompression curve where radius is minimum & curvature is maximum

→ Draw the bisector of θ which is subtended between the Horizontal and tangent at the point P.

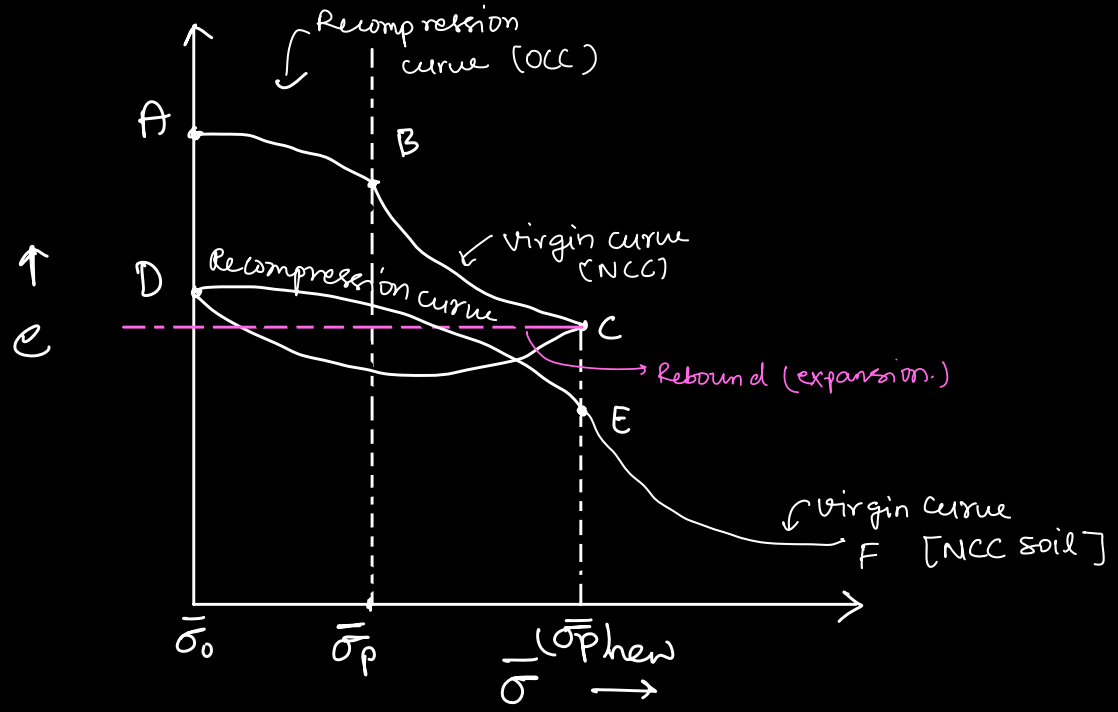
→ Extend the recompression till it becomes linear and then draw a linear line AB which intersect the bisector at

any point C.

→ Take the vertical projection of point C to determine preconsolidation pressure. ($\bar{\sigma}_p$)



Swelling index:



→ The curve 'CD' is termed as swelling curve or rebound curve or expansion curve or unloading curve

→ The slope of the swelling curve is termed as coefficient of swelling or swelling index.

$$C_s = \frac{\Delta e}{\log_{10}\left(\frac{\bar{\sigma}_0 + \Delta\sigma}{\bar{\sigma}_0}\right)} \qquad \text{unitless}$$

Terzaghi's one dimensional consolidation theory:

Assumptions:

i) Soil is homogeneous, isotropic & semi Infinite (Soil sample is laterally restrained.

ii) Darcy's law is valid. (Soil is fully saturated and flow is laminar.

(iii) The flow is unidirectional (ie. vertical)

(The flow is one dimensional but it can be one way or two way)

ONE WAY: when expulsion of drainage occurs from either top or bottom

TWO WAY: When expulsion / drainage of water occurs from top & bottom (SD - SD)

(iv) The change in the volume of soil is observed in the direction or depth of soil along which water is being drained out.

(Because sample is laterally restrained)

v) "Hydrodynamic lag" is considered and "Plastic - lag" is ignored

Hydrodynamic lag: time for primary consolidation

Plastic lag: time for secondary consolidation.

Note: In terzaghi's one dimensional consolidation theory primary consolidation has been considered & secondary consolidation has been removed.

vi) There is unique relation between change in void ratio and $\Delta\bar{\sigma}$ which is independent of time. For small increment

in stress, $a_v = \frac{\Delta e}{\Delta \bar{\sigma}}$ can be considered to be constant

(That means a unique linear relationship is considered between "e" and "$\Delta\sigma$" for small increment in the effective stress which doesn't exist in case of secondary consolidation)

- According to terzaghi's one dimensional consolidation theory,

$$\frac{\partial u'}{\partial t} \propto \frac{\partial u^2}{\partial t^2}$$

u': pore water pressure

$\frac{\partial u'}{\partial t}$: rate of expulsion of pore water.

$\frac{\partial^2 u'}{\partial t^2}$: change in excess pore water pressure along the path.

$$\therefore \frac{\partial u'}{\partial t} \propto \frac{\partial^2 u'}{\partial t^2}$$

$$\frac{\partial u'}{\partial t} = c_v \frac{\partial^2 u'}{\partial t^2}$$

C_v: coefficient of consolidation [cm^2/s]

$$C_v = \frac{K(1+e_0)}{a_v \cdot \gamma_w} \longrightarrow \text{eqn A}$$

Note: C_v shows the rate of consolidation.

Note: whereas a_v, C_c, C_r shows the amount of consolidation or compressibility relatively.

from the equation (A)

$$C_v = \frac{K}{\frac{a_v}{(1+e_0)} \cdot \gamma_w}$$

$$C_v = \frac{K}{m_v \cdot \gamma_w}$$

$(K = C_v \cdot m_v \cdot \gamma_w)$ m/s or cm/s

m_v: coefficient of volume compressibility

$$\left\{ m_v = \frac{a_v}{1+e_0} = \frac{\Delta e}{\Delta \bar{\sigma}(1+e_0)} \quad m^2/kN \right\}$$

∵ due to expulsion of water, change in volume of soils is observed

$$\Delta V = \Delta V_v$$

$$\Delta V = \Delta e \cdot V_s$$

$$\Delta e = \frac{\Delta V}{V_s} = \frac{\Delta V}{\frac{V}{1+e_0}}$$

$$\Delta e = \frac{(1+e_0) \cdot \Delta V}{V}$$

Thus, $\frac{\Delta V}{V} = \frac{\Delta e}{(1+e_0)}$

$$\therefore m_v = \frac{\Delta e}{\Delta \bar{\sigma}(1+e_0)} = \frac{\left(\frac{\Delta V}{V}\right)}{\Delta \bar{\sigma}}$$

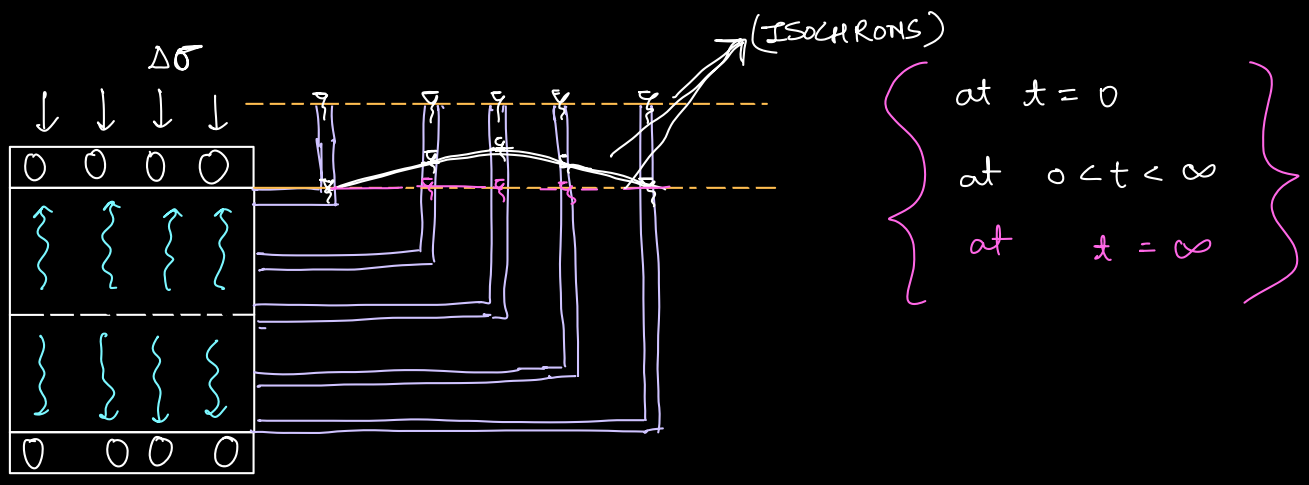
$$m_v = \frac{\left(\frac{\Delta V}{V}\right)}{\Delta \bar{\sigma}} \quad m^3/kN$$

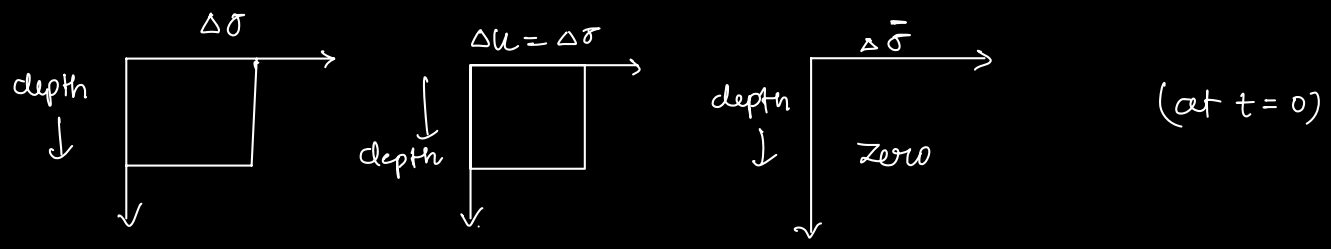
$K = C_v \cdot m_v \cdot \gamma_w$

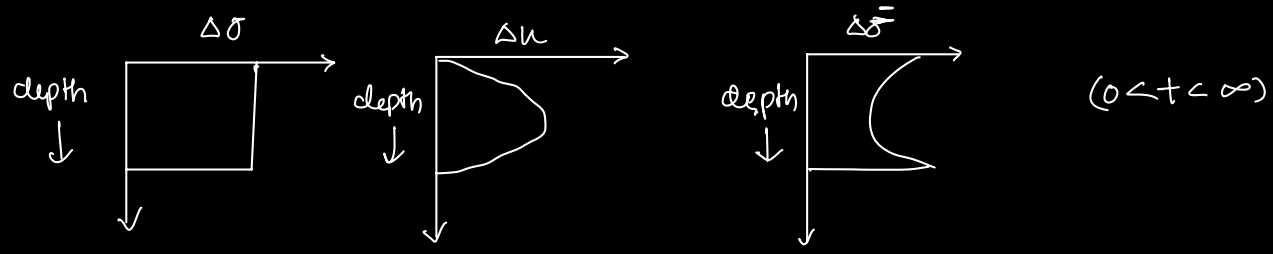
↳ This is one of the indirect method to determine the coefficient of permeability using consolidation test

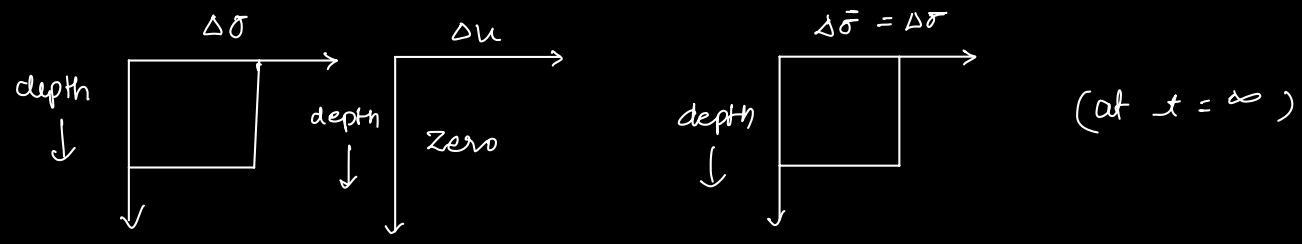
Types of soil	Suitable test of permeability
Gravel & Sand	Constant head test
Silt & clay	Falling head test
clay	Consolidation test

Experimental setup of Terzaghi's one dimensional theory.









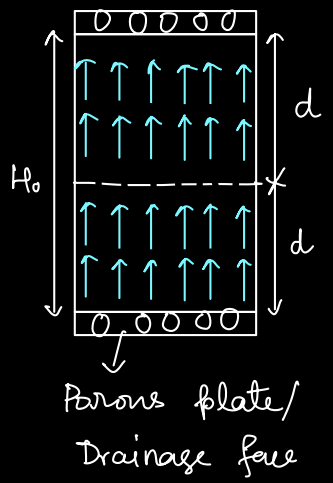
Note: From the above experiment terzaghi has given his one dimensional consolidation equation.

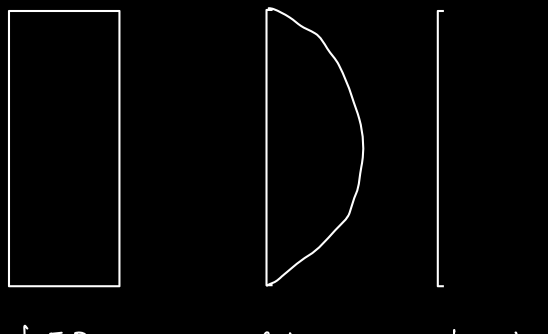
$$\frac{\partial u'}{\partial t} = c_v \frac{\partial^2 u'}{\partial t^2}$$

$$\left\{ \frac{\partial u'}{\partial t} = \frac{K(1+e_0)}{a_v \cdot \gamma_w} \frac{\partial^2 u'}{\partial t^2} \right\}$$

Two way & one way drainage system.

(i) Two way drainage:





$$d = \frac{H_0 - \Delta H}{2}$$

$\because \Delta H << H_0$

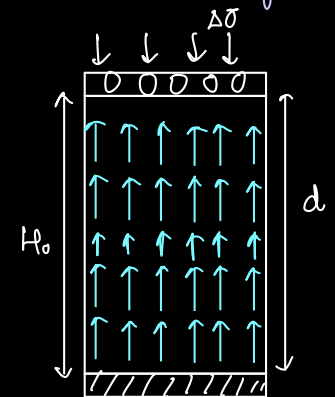
$\therefore \Delta H \rightarrow$ ignoring

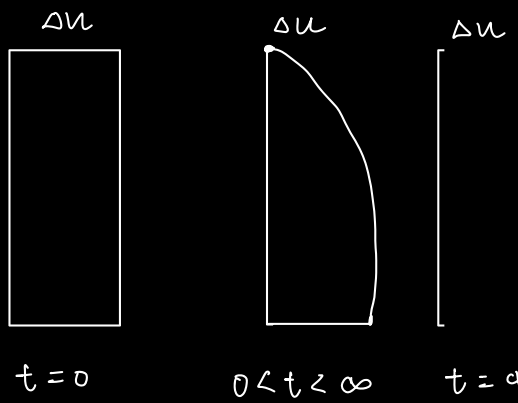
$$d = \frac{H_0}{2}$$

d: the longest distance travelled by water called drainage path

H_0: Initial height of soil sample

(ii) One way drainage

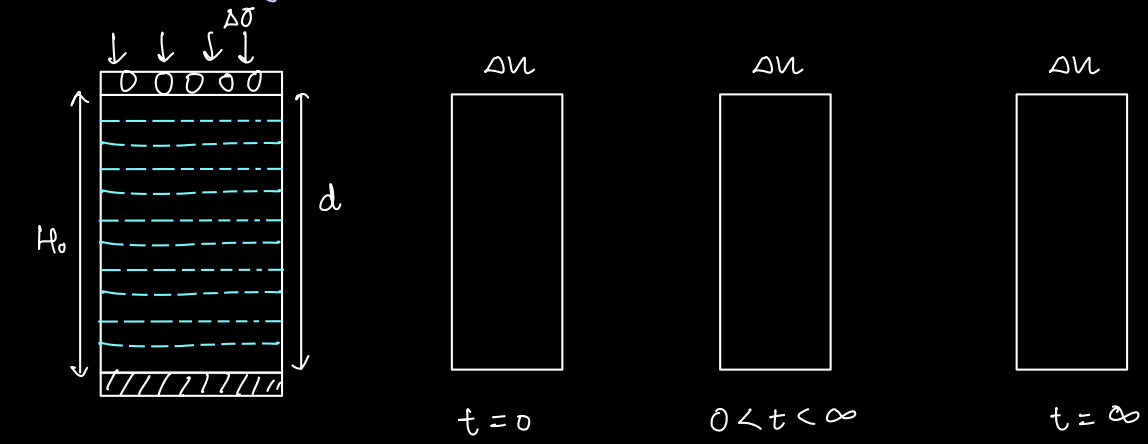




$d = H_0$

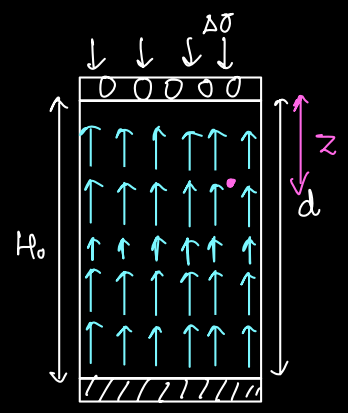
↳ This is the largest distance travelled by water.

(iii) No drainage



Non-dimensional Parameter of Terzaghi's Analysis:

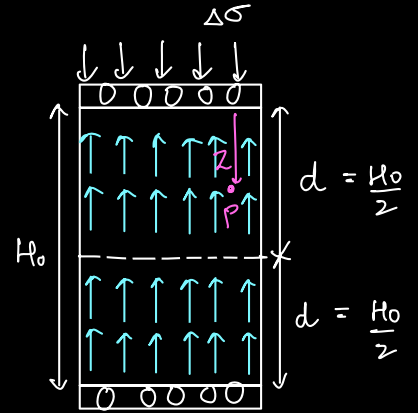
i) Drainage path ratio (Z): It can be defined as the ratio of the distance travelled by the water to reach drainage face to the longest drainage path.



ONE WAY

$$\{ Z = \frac{z}{d} \}$$

$$Z = \frac{z}{H_0} \quad \text{(unitless)}$$



TWO WAY

$$Z = \frac{z}{d} = \frac{2z}{H_0}$$

$$Z_{\text{TWO WAY}} = 2 Z_{\text{ONE WAY}}$$

Note: In two drainage the longest drainage path is half of the longest drainage path of oneway drainage whereas drainage path ratio in total drainage becomes double of one way drainage.

(ii) Degree of Consolidation (U):

It shows how much percentage of primary consolidation is completely at any time 't'.

Case(i): Based on settlement:

$$\% U = \frac{\Delta h}{H} \times 100$$

H: ultimate settlement at the end of primary settlement

Δh: settlement at any time t. $(0 < t < \infty)$

Case ii): Based on void ratio

$$\% U = \frac{e_0 - e}{e_0 - e_\infty} \times 100$$

e_0: Initial void ratio at any time, $t = 0$

e_∞: final void ratio at the end of primary consolidation.

e: Natural void ratio at any time, $0 < t < \infty$

Case iii): Based on pore water pressure.

$$\% U = \frac{\Delta u_i - \Delta u_t}{\Delta u_i - \Delta u_f} \times 100$$

$$\% U = \frac{\Delta u_i - \Delta u_t}{\Delta u_i} \times 100$$

Δu_i: Excess pwp at $t = 0$

Δu_t: Excess pwp at $0 < t < \infty$

Δu_f: Excess pwp at $t = \infty$

(iii) Time factor :

→ It is the factor which correlates the degree of consolidation & time required to complete that.

→ It is denoted by T_v and it is given as

$$T_v = \frac{C_v t}{d^2} \quad \text{(unitless)}$$

C_v : coefficient of consolidation.

d : drainage path

$d = \frac{H_o}{2}$ (Two way drainage)

$d = H_o$ (one way drainage)

t : time required to complete the given degree of consolidation.

— from the solution of Terzaghi equation, the approximate relation between, degree of consolidation and time factor (T_v) can be given as:

- For $U \leq 60\%$,

$$T_v = \frac{\pi}{4} U^2 \quad (U \text{ is in fraction})$$

- For $U > 60\%$

$$T_v = 1.781 - 0.933 \log_{10}(100 - U\%) \quad (U \text{ in } \%)$$

or

$$T_v = -0.933 \log_{10}(1 - U) - 0.0852 \quad (U \text{ in fraction})$$

$$\infty = \frac{C_v t}{d^2}$$

$(t = \infty)$

Thus we assume that the primary consolidation is over at 90%.

Note: For two different soil, the time factor (T_v) can be same for the same degree of consolidation (U).

Soil (1) Soil (2)

Let % U be same in both the soil

$$(T_v)_{soil_1} = (T_v)_{soil_2}$$

$$\frac{C_{v_1} t_1}{d_1^2} = \frac{C_{v_2} t_2}{d_2^2}$$

$$\frac{K_1 \cdot t_1}{\gamma_w \cdot m_{v_1} \cdot d_1^2} = \frac{K_2 \cdot t_2}{\gamma_w \cdot m_{v_2} \cdot d_2^2}$$

$$\boxed{\frac{K_1 \cdot t_1}{m_{v_1} \cdot d_1^2} = \frac{K_2 \cdot t_2}{m_{v_2} \cdot d_2^2}}$$

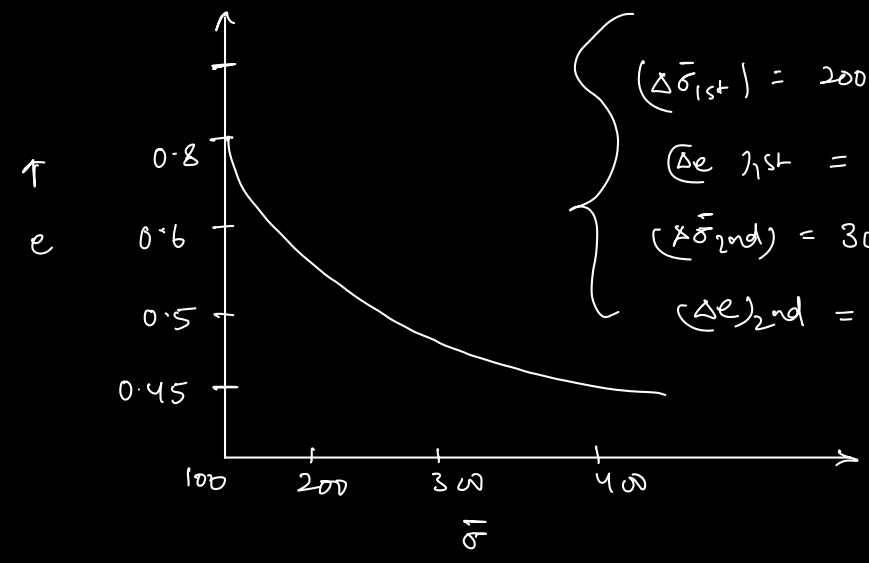
Note: The above three non-dimensional parameters are obtained from the solution of Terzaghi's one dimensional consolidation equation using Fourier series expansion.

Note: C_v shows the time rate of consolidation. it is the function of the stress increment.

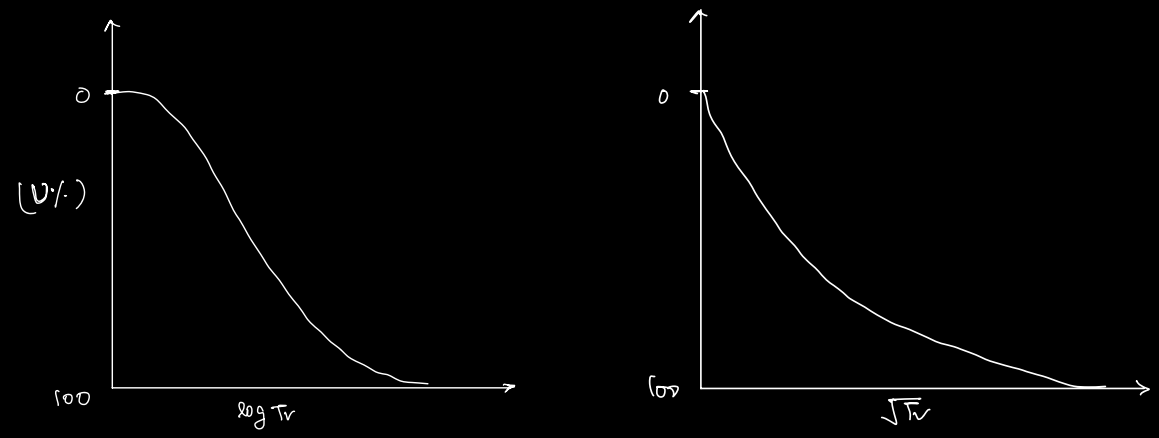
$$C_v = \frac{K(1+e_0)}{a_v \cdot \gamma_w} = \frac{K}{m_v \cdot \gamma_w}$$

That means if consolidation pressure is increased, C_v increases (because water drains at a faster rate) & a_v decreases (because the volume of voids decreases)

Note: The effect of T_s in effective stress can be seen on the compressibility of the soil. As we can see increase in effective stress, leads to decrease in void ratio, by decrease in void ratio decreases on equal increment of the effective stress as shown in figure.



Note: The time factor T_v can be calculated either mathematically or graphically.



Consolidation test in lab (Oedometer test):

→ The consolidation parameter that controls the magnitude in the soil is the coefficient of compressibility (a_v).

→ The time of rate of consolidation is controlled by the coefficient of consolidation (C_v)

→ we are intersested in determination of a_v & C_v. along with the estimation of a_v & C_v, the coefficient of permeability and swelling chaveeteristic can also be determined.

→ In this oedometer test, one dimensional consolidation with double drainage (two way drainage) system is carried out

→ This oedometer is also termed as consolidometer which consists of loading mechanism and container known as consolidation cell.

→ The soil is filled and porous stones are placed at top & bottom

→ The size of the specimen can be taken as

Diameter - 65 mm to 110 mm

Height - 15 mm to 30 mm

→ The soil sample is loaded & usually the applied load is kept for 24 hours, After 24 hours, before giving the increment in the load measure the compression (settlement) of the soil sample using dial gauge.

→ The stress commonly used are 25, 50, 100, 200, 400, 800 kN/m^2.

→ After the consolidation under the last stress increment is over, the specimen is unloaded in two or three stages and soil is allowed to swell.

→ The soil sample is taken out from the consolidation cell and it is oven dried to take the final moisture content & volume of solids

→ Void ratio - Effective stress relationship

In this test for every stress level equilibrium void ratio is required to be calculated using.

(i) Heights of solid method

(ii) change in void ratio method.

Note: Height of solid method can be used for unsaturated soil mass.

whereas

change in void ratio can be used for fully saturated soils only

* Change in void ratio method:

Suitable for fully saturated soil

$$e_s = wG$$

$$e_f = w_f \cdot G$$

$$\because \quad \frac{\Delta V}{V} = \frac{\Delta e}{1+e}$$

$$\frac{A \cdot \Delta H}{A \cdot H} = \frac{\Delta e}{1+e}$$

$$\frac{\Delta H}{H} = \frac{\Delta e}{1+e}$$

$$\Delta e = \frac{\Delta H}{H}(1+e)$$

$$\Delta e = \frac{\Delta H}{H}(1+e_f)$$

Applied stress	Dial Gauge Readings	ΔH	H	$\Delta e = \frac{\Delta H}{H}(1+e)$	e
0	a	–	H_0		$e_0 = e_1 + \Delta e_1$
25	b	$\Delta H_1 = b - a$	$H_1 = H_0 - \Delta H_1$	Δe_1	$e_1 = e_2 + \Delta e_2$
50	c	$\Delta H_2 = c - b$	$H_2 = H_1 - \Delta H_2$	Δe_2	$e_2 = e_3 + \Delta e_3$
100	d	$\Delta H_3 = d - c$	$H_3 = H_2 - \Delta H_3$	Δe_3	$e_3 = e_4 + \Delta e_4$
200	e	$\Delta H_4 = e - d$	$H_4 = H_3 - \Delta H_4$	Δe_4	$e_4 = e_5 + \Delta e_5$
400	f	$\Delta H_5 = f - e$	$H_5 = H_4 - \Delta H_5$	Δe_5	$e_5 = e_6 + \Delta e_6$
800	g	$\Delta H_6 = g - f$	$H_6 = H_5 - \Delta H_6$	Δe_6	$e_6 = e_f - \Delta e_7$
0	h	$\Delta H_7 = h - g$	$H_7 = H_6 + \Delta H_7$	Δe_7	e_f

→ Now draw the curve between e v/s $\bar{\sigma}$ and then a_v, can be calculated

→ We can plot the curve on the semilog scale and also between e v/s log $\bar{\sigma}$ to estimate the compression index (C_c) and preconsolidation pressure

Question

Q. Following results were recorded from a consolidation test on a specimen of saturated clay each load being maintained constant for 25 hour before the addition of the next increment.

Applied stress (kN/m^2)	0	50	100	200	400	800	0
Sample thickness (mm)	19.70	19.30	19.15	18.85	18.56	18.36	18.90

The water content of the sample at the end of the test was 28 percent and the specific gravity of the soil particles was 2.72.
Calculate the void ratio at the end of the each stress range and plot the $e-\bar{\sigma}$ curve. Compute the coefficient of volume compressibility, m_v for an effective stress range of 150-350 kN/m^2 from the $e-\bar{\sigma}$ curve.

Solution

$\frac{\Delta H}{H} = (1+e)$ → $\frac{\Delta e}{\Delta \bar{\sigma}}$ → $\frac{a_v}{1+e}$

$\bar{\sigma}$	$\Delta\bar{\sigma}$	H	ΔH	Δe	e	a_v $\times 10^{-4}$	m_v $\times 10^{-4}$ (m^2/kN)
0	-	19.70	-		0.837	-	-
50	50	19.30	-.40	-0.037	0.80	7.4	4.11
100	50	19.15	-.15	-0.014	0.786	2.8	1.56
200	100	18.85	-.30	-.028	0.758	2.8	1.59
400	200	18.56	-.29	-0.027	0.731	1.35	0.78
800	400	18.36	-.20	-0.019	0.712	.475	0.27
0		18.90	+.54	0.05	0.762	-	-

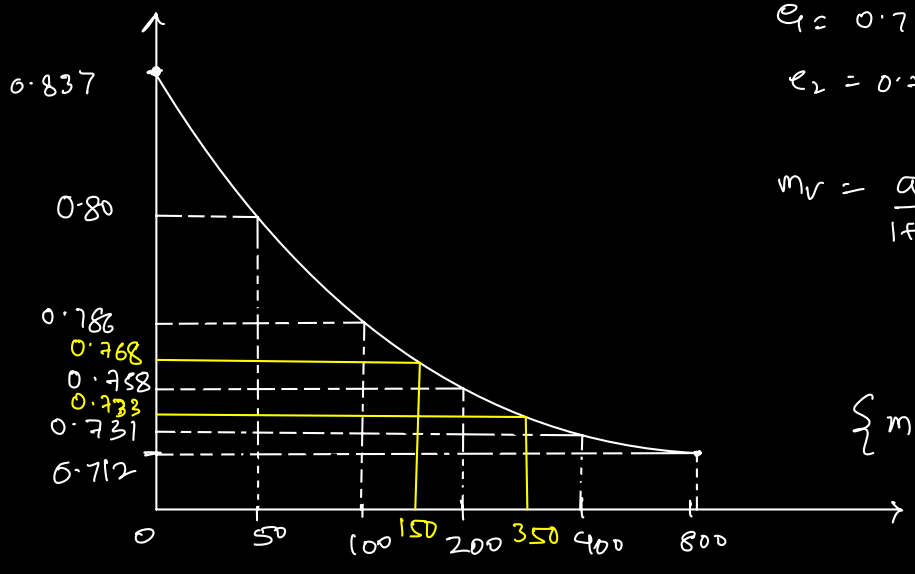
$$e_f = w_f \cdot G$$

$$e_f = 0.28 \times 2.72$$

$$= 0.762$$

$$\Delta e = \frac{\Delta H}{H}(1+e_f)$$

$$\Delta e = \frac{.54}{18.90}(1+0.762) = 0.05$$



$e_1 = 0.768$ $\quad \bar{\sigma}_1 = 150$ kPa

$e_2 = 0.733$ $\quad \bar{\sigma}_2 = 350$ kPa

$$m_v = \frac{a_v}{1+e_0} = \frac{\Delta e}{\Delta\bar{\sigma}(1+e_0)}$$

$$= \frac{0.768 - 0.733}{(350-150)(1.768)}$$

$$\{m_v = 0.99 \times 10^{-4}\ m^2/kN\}$$

Determination of Cv in the consolidation test:

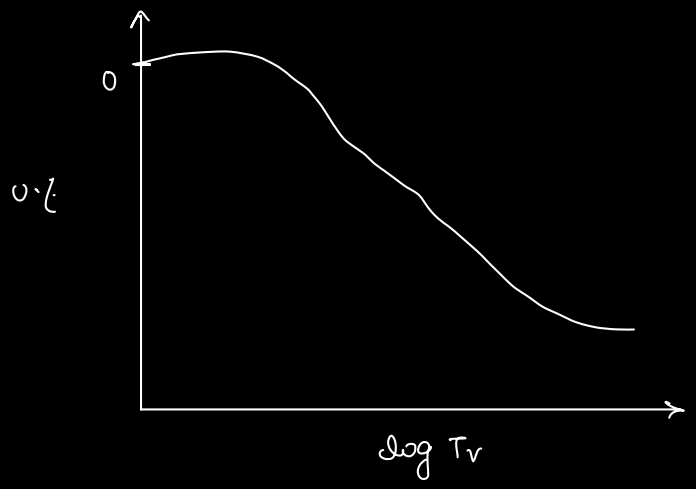
There are two methods to determine Cv.

(i) Logarithm of time fitting method
[CASAGRANDE]

(ii) Square root of time fitting method.

(i) logarithm of time fitting method:

→ In this method the theoretical characteristic curve between % U v/s log Tv is used to determine the coefficient of consolidation Cv.



→ In cassagrand's logarithm time fitting method Cv is calculated corresponding to the 50% degree of consolidation.

$$T_v = \frac{C_v t}{d^2}$$

$$T_v = \frac{\pi}{4} U^2$$

$$C_v = \frac{T_{v50} \cdot d^2}{t_{50}}$$

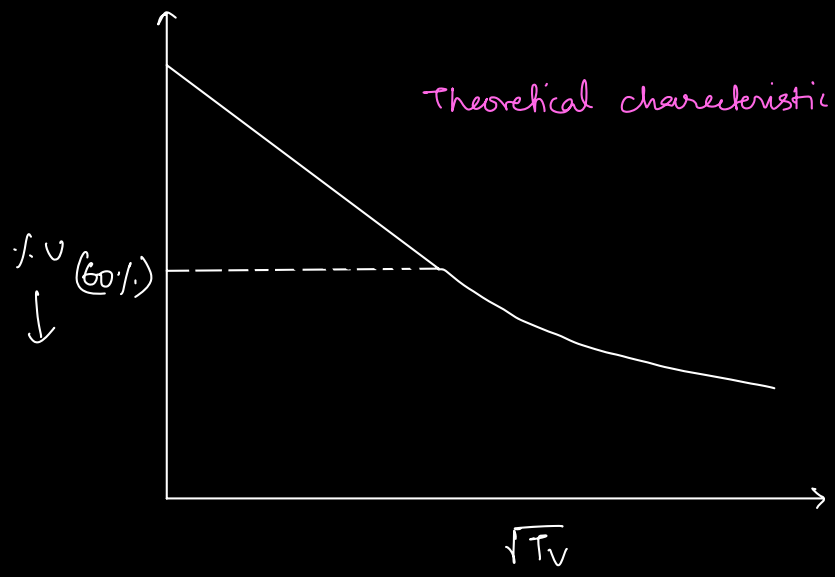
$$C_v = \frac{0.196 \cdot d^2}{t_{50}}$$

d: longest drainage path

t_{50}: time required for completion of 50% consolidation

(iii) Square root of time fitting method:

In this method the theoretical characteristics curve between % U vs $\sqrt{T_v}$ is used to determine the coeff of consolidation, Cv.



- In taylor's square root of time fitting method, the Cv is calculated corresponding to the 90% Degree of consolidation.

$$T_v = \frac{C_v t}{d^2}$$

$$T_{v90} = 1.781 - 0.933 \log_{10}(100 - V\%)$$

$$= 1.781 - 0.933 \log_{10}(10)$$

$$= 1.7981 - 0.933$$

$$T_{v90} = 0.8651$$

$$C_v = \frac{0.8651 \times d^2}{t_{90}}$$

d: longest drainage path

t_{90}: time required for completion of 90% degree of consolidation.

Determination of coefficient of permeability using consolidation test

$$K = C_v \cdot m_v \cdot \gamma_w$$

for each stress increment permeability K can be determined

Total settlement (St):

$$S_t = S_i + S_p + S_s$$

S_i: Immediate Settlement (Elastic settlement)

S_p: Primary Consolidation settlement

S_s: Secondary Consolidation settlement.

Primary consolidation settlement:

$$\Delta V = \Delta V_v$$

$$\Delta V = e \cdot V_s$$

$$\Delta V = \Delta e \cdot \frac{V}{1+e_0}$$

$$\left\{ \frac{\Delta V}{V} = \frac{\Delta e}{1+e_0} \right\}$$

$$\frac{A \cdot \Delta H}{A \cdot H_0} = \frac{\Delta e}{1+e_0}$$

$$\Delta H = \frac{\Delta e}{1+e_0} \cdot H$$

Case(i): When change in void ratio Δe is given

$$\Delta H = \frac{\Delta e}{1+e_0} \cdot H_0$$

Case(ii): When C_c is given

$$\because \quad C_c = \frac{\Delta e}{\log_{10}\left(\frac{\bar{\sigma_0} + \Delta\bar{\sigma}}{\bar{\sigma_0}}\right)}$$

$$\Delta e = C_c \cdot \log_{10} \cdot \left(\frac{\bar{\sigma_0} + \Delta\bar{\sigma}}{\bar{\sigma_0}}\right)$$

$$\Delta H = \frac{C_c}{1+e_0} \log_{10}\left(\frac{\bar{\sigma_0} + \Delta\bar{\sigma}}{\bar{\sigma_0}}\right) \cdot H_0$$

$$\Delta H = \frac{C_c \cdot H_0}{1+e_0} \log_{10}\left(\frac{\bar{\sigma_0} + \Delta\bar{\sigma}}{\bar{\sigma_0}}\right)$$

(Case iii) when m_v is given:

$$m_v = \frac{a_v}{1+e_0} = \frac{\Delta e}{\Delta \bar{\sigma}(1+e_0)}$$

$$\Delta e = m_v \cdot \Delta \bar{\sigma} (1+e_0)$$

put in equation:

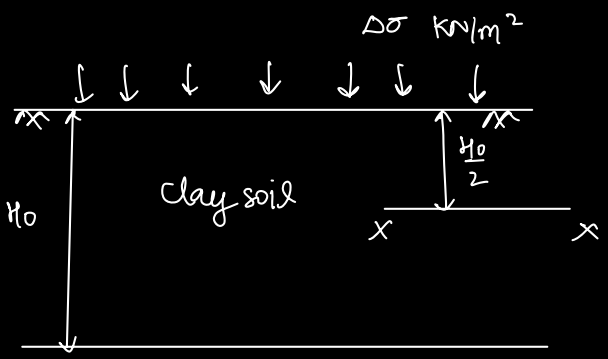
$$\boxed{\Delta H = m_v \cdot \Delta \bar{\sigma} \cdot H_0}$$

Rules to estimate Consolidation settlement:

(i) Consolidation settlement shall be calculated in the compressible soils only like clay.

(ii) Consolidation settlement can be calculated on the middepth of compressible soil.

(iii) when the uniform pressure or surcharge acts over the large area (infinite area) then load distribution is not required.



$$\Delta H = \frac{C_c H_0}{1+e_0} \cdot \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)$$

H_0: Thickness of Compressible soils

e_0: Initial void Ratio

$\Delta\bar{\sigma}$: applied surcharge ($\Delta\sigma$)

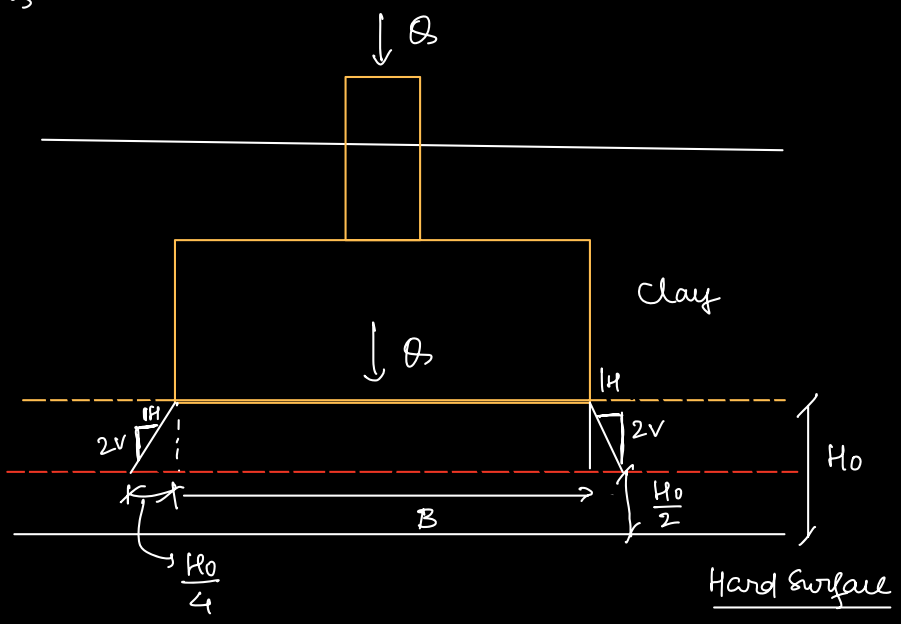
$\bar{\sigma}_0$: Effective overburden pressure at the mid depth of the compressible soils

(iv) load dispersion is required when the load is acting over a small area.

(Note: In case of footing load dispersion is required)

The load dispersion can be done with the slope of 2V : 1H

v) The consolidation settlement shall be calculated below the base of the footing.



$$\left\{ \Delta H = \frac{C_c \cdot H_0}{1+e_0} \log_{10}\left(\frac{\bar{\sigma_0} + \Delta\bar{\sigma}}{\bar{\sigma_0}}\right) \right\}$$

$$\Delta\bar{\sigma} = \frac{Q}{(L + H_0/2)(B + H_0/2)} \quad \} \text{ [Rectangular Footing]}$$

$$\Delta\bar{\sigma} = \frac{Q}{\left(B + \frac{H_0}{2}\right)^2} \quad \} \text{ (Square footing)}$$

Question

Q.1 **The results of a consolidation test on an undisturbed soil, sampled at a depth of 10 m below the ground level are as follows:**

Saturated unit weight : 16 kN / m³

Pre-consolidation pressure : 90 kPa

The water table was encountered at the ground level. Assuming the unit weight of water as 10 kN / m³, the over-consolidation ratio of the soil is

(A) 0.67 **(B) 1.50**

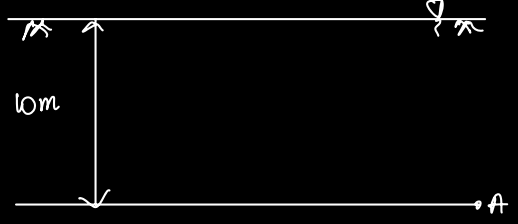
(C) 1.77 **(D) 2.00**

[GATE 2016 : IISc Bangalore]

Solution:

$\gamma_{sat} = 16\ KN/m^3$

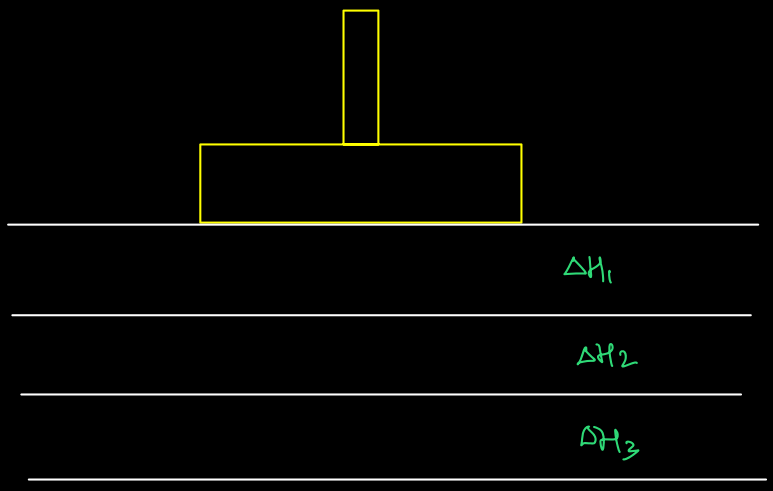
$\bar{\sigma_p} = 90\ KPa$



$\bar{\sigma_A} = 60\ KPa$

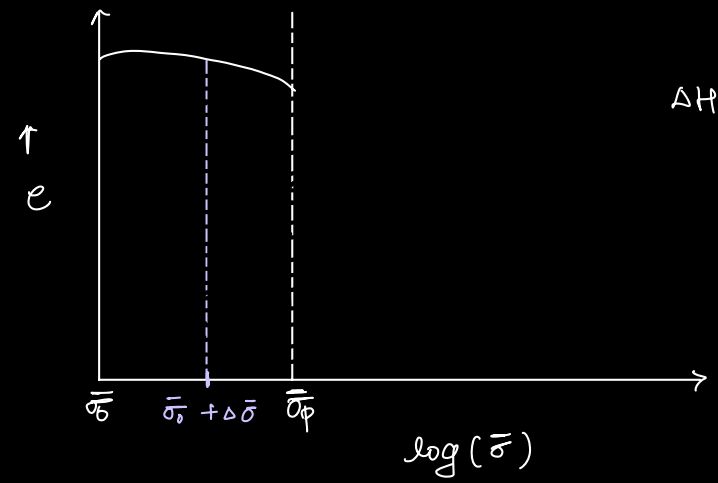
$$OCR = \frac{90}{60} = \frac{3}{2} = 1.5$$

Note: When there are more than 1 layer of compressible soils then total consolidation settlement can be given as the sum of consolidation settlements at the middepth of individual layers.



$$\Delta H = \Delta H_1 + \Delta H_2 + \Delta H_3$$

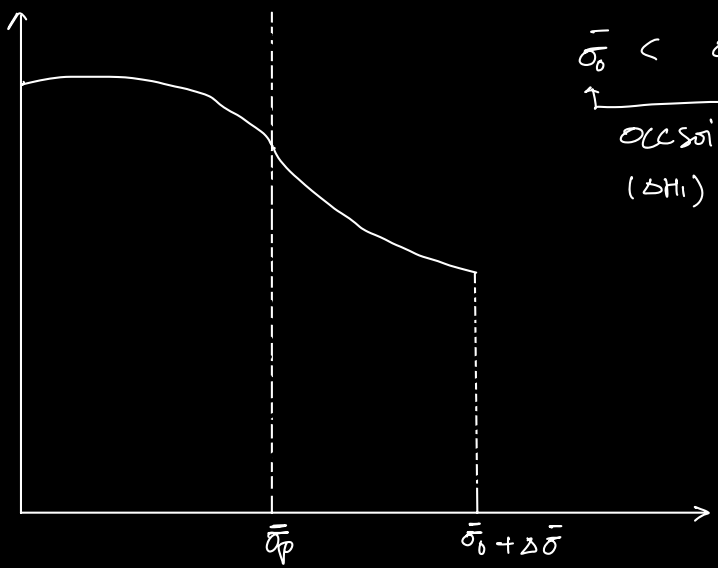
CASE 1) when $\bar{\sigma_o} + \Delta\bar{\sigma} < \bar{\sigma_p}$

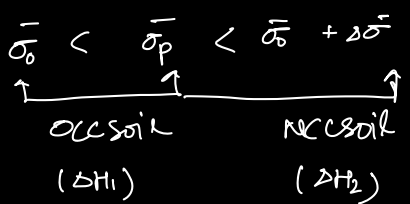


$$\Delta H = \frac{C_r \cdot H_o}{1+e_o} \log_{10}\left(\frac{\bar{\sigma_p}}{\bar{\sigma_o} + \Delta\bar{\sigma}}\right)$$

CASE 2) when $\bar{\sigma_o} < \bar{\sigma_p} < \bar{\sigma_o} + \Delta\bar{\sigma}$

→ In this case settlement shall be computed in two stages





$$\Delta H_1 : \quad \frac{C_R H_0}{1+e_0} \log_{10}\left(\frac{\bar{\sigma}_p}{\bar{\sigma}_0}\right)$$

$$\Delta H_2 : \quad \frac{C_c H_1}{1+e_1} \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_p}\right)$$

$$\{ H_1 : \quad H_0 - \Delta H_1 \}$$

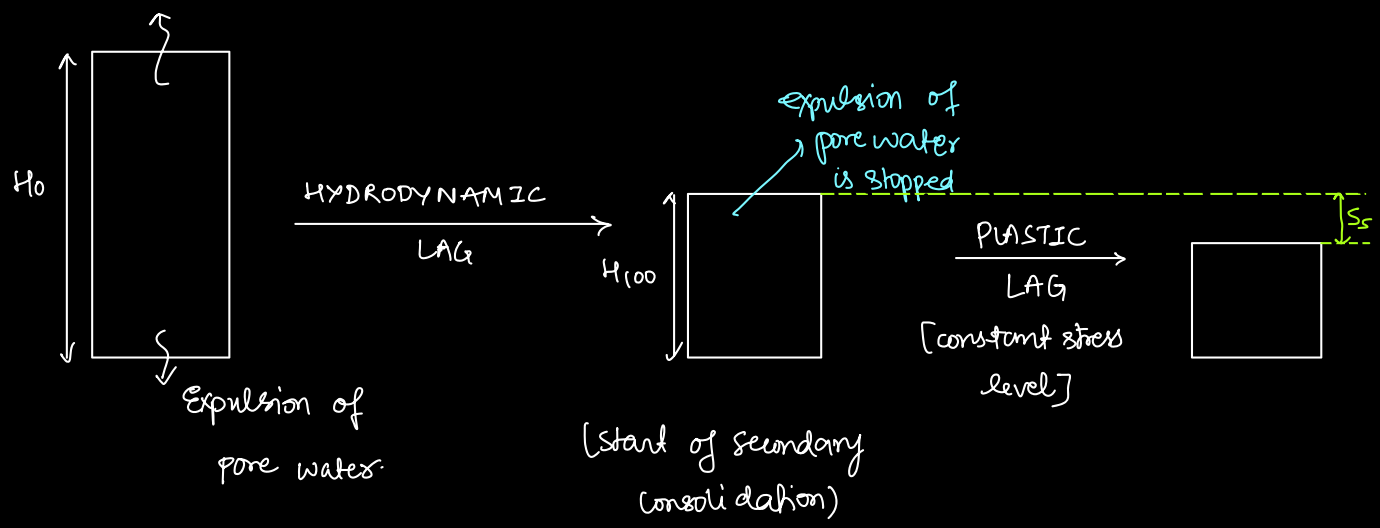
$\because$ OCC soil is less compressible

$$e_1 \simeq e_0 \quad \& \quad H_1 \simeq H_0$$

$$\Delta H_2 : \quad \frac{C_c H_0}{1+e_0} \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_p}\right)$$

$$\boxed{\Delta H = \Delta H_1 + \Delta H_2}$$

SECONDARY CONSOLIDATION SETTLEMENT [S_s]:



S_s → Due to plastic readjustment of soil solids

S_s → Due to removal of highly viscous adsorbed water

Note: Secondary consolidation settlement is also termed as CREEP SETTLEMENT.

$$C_\alpha = \frac{\Delta e}{\log_{10}\left(\frac{t}{t_{100}}\right)} = \frac{\Delta e}{\log_{10}\left(\frac{t_{100}+\Delta t}{t_{100}}\right)}$$

$$\because \quad \Delta H = \frac{\Delta e}{1+e_0} \cdot H_0$$

$$\therefore \quad \Delta H = \frac{\Delta e}{1+e_{100}} \cdot H_{100} \quad \text{(Start of secondary consolidation)}$$

$$\Delta H = S_s = \frac{C_\alpha \cdot H_{100}}{1+e_{100}} \log_{10}\left(\frac{t_{100}+\Delta t}{t_{100}}\right)$$

t_{100} : the time at which primary

e_{100} : final void ratio at the end of primary consolidation.

H_{100} : The height of compressible soil at the end of primary consolidation

or

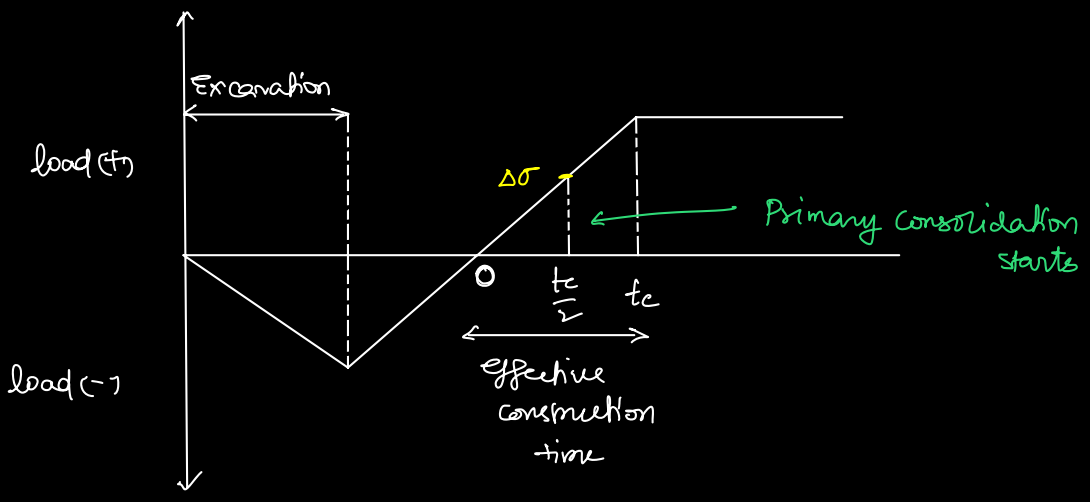
Initial height for secondary consolidation.

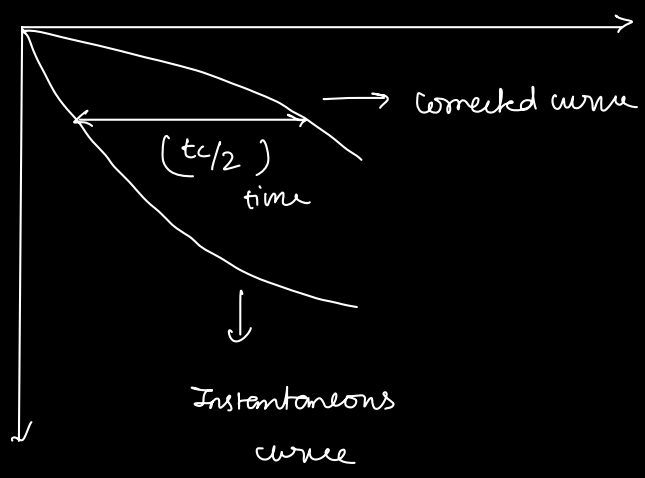
$$H_{100} = H_0 - S_p$$

S_p: Primary consolidation settlement

C_α: Secondary compression index

* Correction for construction period.



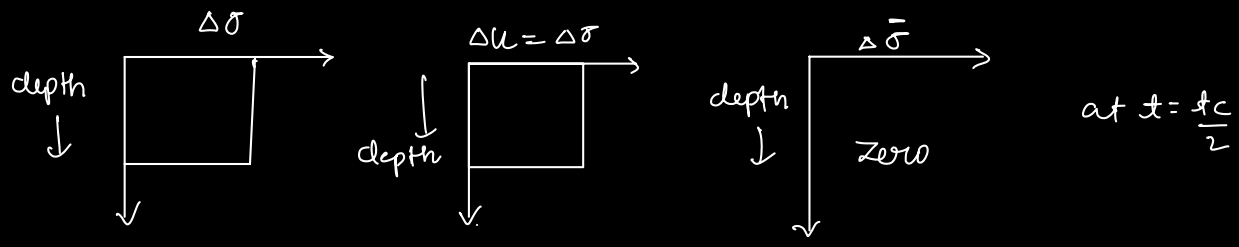


→ It is assumed that the structural load that is applied instantaneously produces the consolidation in clay.

→ But in the construction of any structure extends for several months, sometimes even years

→ The consolidation can be assumed to start from the middle of the effective construction period

→ Thus the below mentioned steps are assumed to start from the time $t = \frac{t_c}{2}$

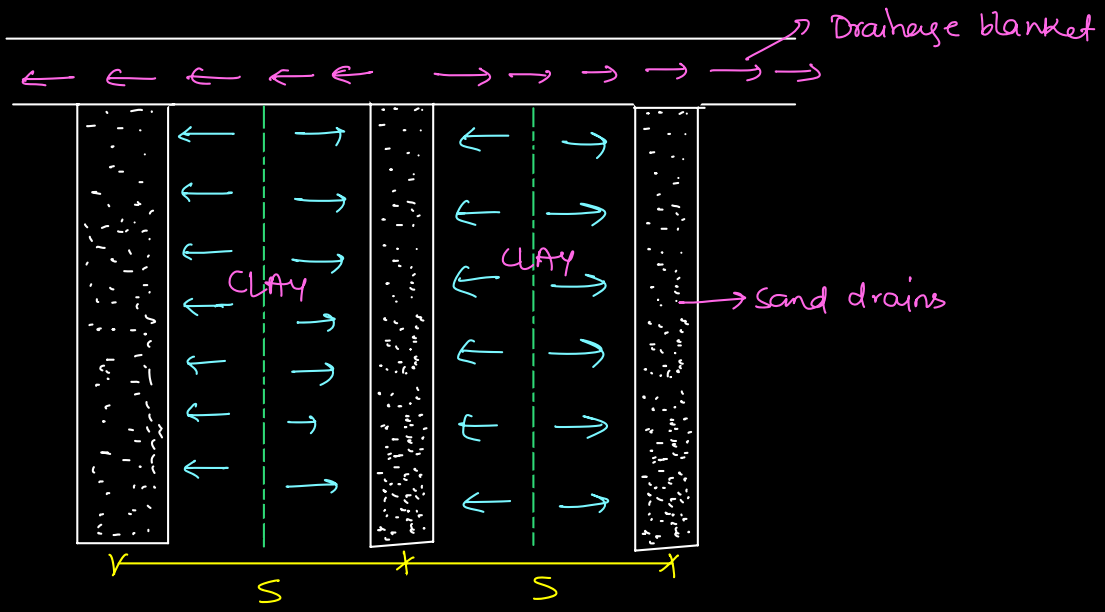


Note: If consolidation is assumed to be started instantaneously then the consolidation pressure is overestimated.

Vertical Sand drains :

- The principle of the functioning of sand drain is based on the consolidation theory of radially & vertically drained clay system. The slow rate of consolidation of saturated clay of low permeability may be accelerated by means of vertical sand drains which provide radial drainage, resulting in shorting of the drainage path.

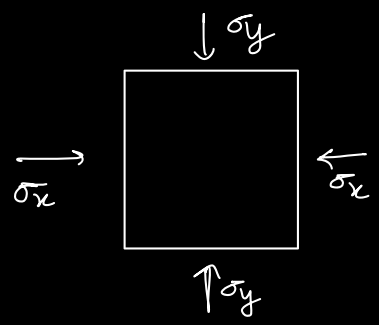
- The magnitude of consolidation settlement is uneffective after providing the vertical sand drains. They are used to increase the rate of consolidation
- Sand drains are mostly used when embankment are to be constructed over highly compressible clay layers. Sand drains are provided to accelerate the rate of consolidation so that embankment can be brought into service much sooner.
- The diameter of sand drains vary from 450 to 650 mm
- In general c/c spacing between sand drains vary from 1.8 to 4.5m
- Terzaghi recommended that 80% consolidation is brought about during the time of construction of the embankment itself.
- Sand drains have been used upto the depth of 30m



SHEAR STRENGTH OF SOIL

The shear strength of the soil can be defined as the maximum value of shear stress that can be applied in the soil without causing any shear failure.

Analysis of stress in biaxially stressed element



let, $\sigma_y > \sigma_x$

then $\sigma_y = \sigma_1$

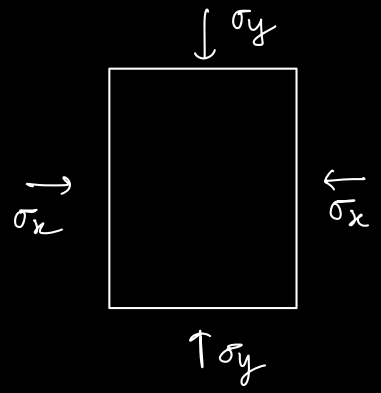
$\sigma_x = \sigma_3$

PRINCIPLE PLANE: The plane of zero shear stress are termed as principle plane. There are total three principle plane.

(i) Major Principle plane

(ii) Intermediate principle plane

(iii) Minor principle plane.



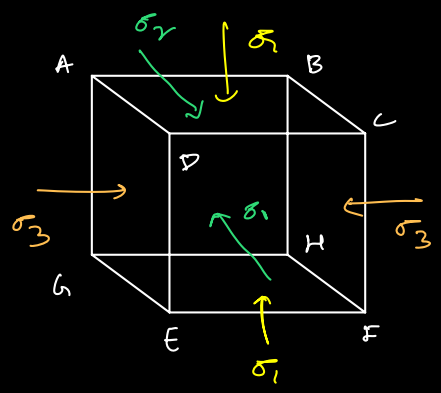
$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^{\nearrow 0}}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2} = \sigma_y$$

$$\sigma_3 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^{\nearrow 0}}$$

$$\sigma_3 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_y - \sigma_x}{2}\right)$$

$$\boxed{\sigma_3 = \sigma_x}$$



→ Major principle plane (ABCD) will carry only major principle stress (σ_1)

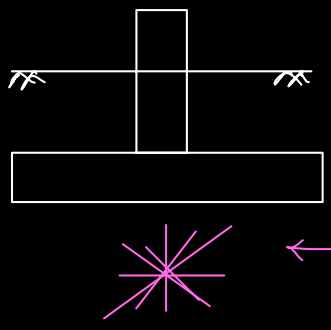
→ Intermediate principle plane (CDEF) will carry only intermediate principle stress (σ_2)

→ Minor principle plane (ACEG) will carry only minor principle stress (σ_3)

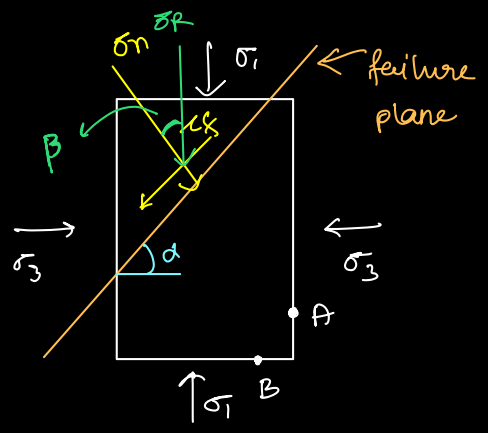
Note: In geotechnical engineering ($\sigma_1 = \sigma_3$)

Note: All principle planes are orthogonal to each other

Note: Principle planes will carry only the normal stress, no shear stress



← Through any given element ∞ planes can pass. Among them 3 planes are the principle planes of zero shear stress. Among these n number of planes, the failure plane can be identified. The failure plane is the plane which carries the maximum value of Normal stress (σ_{nf}) and Shear stress (τ_f) at the time of failure



Major principle plane → B → Horizontal.

Minor principle plane → A → Vertical

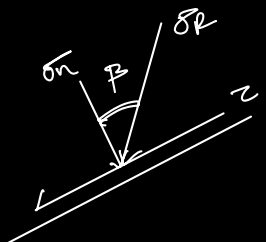
α: Angle of failure plane.

σ_n: Normal stress on the failure plane.

τ: Shear stress on the failure plane.

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\alpha$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\alpha.$$

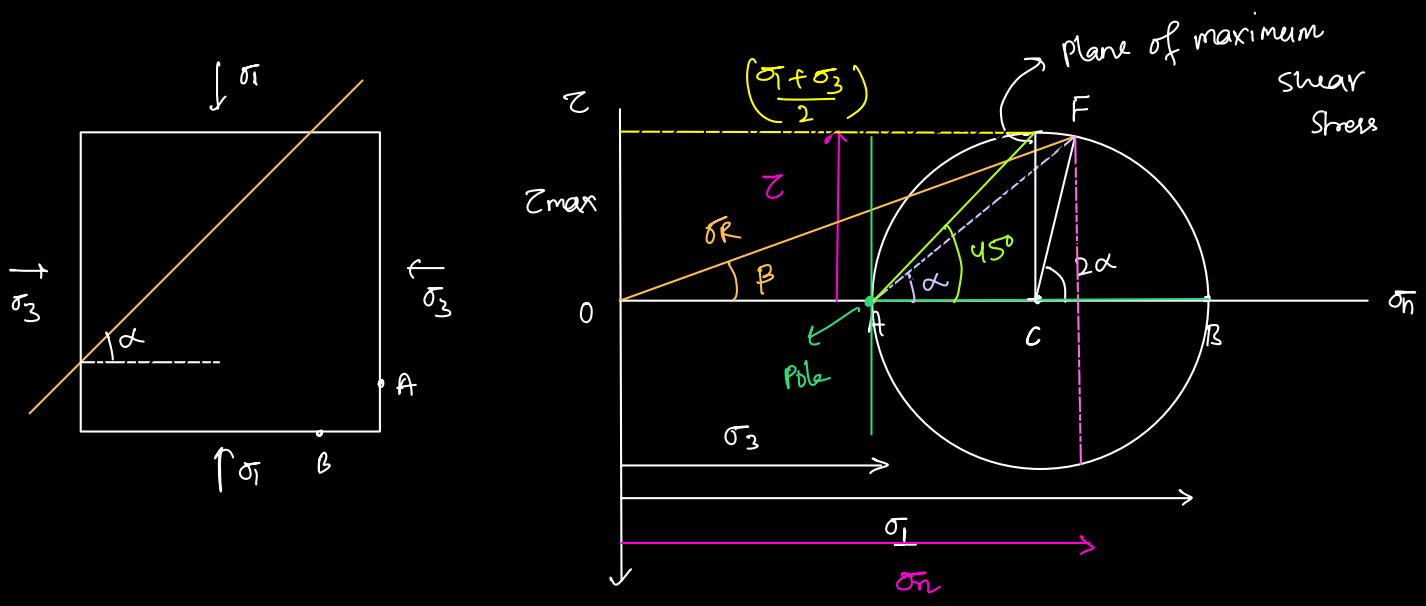


σ_R: Resultant stress

$$\sigma_R = \sqrt{\sigma_n^2 + \tau^2}$$

$$\left\{ \beta = \tan^{-1}\left(\frac{\tau_f}{\sigma_n}\right) \right\}$$

β: angle of obliquity



Pole: Any point in the Mohr's circle which is considered as the origin of all the planes is termed as "Pole"

Note: The max shear stress equals the radius of Mohr's circle

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

Note: At the plane of maximum shear stress, the normal stress

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2}$$

Note: The plane of maximum shear stress makes an angle 45° with major principle plane.

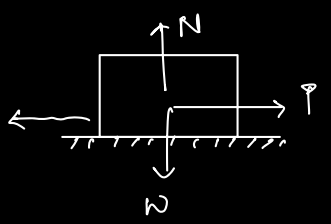
Soil derives its shear strength:

(i) Interlocking between the particles:

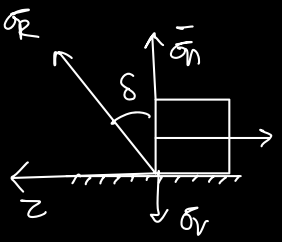
} Due to action of rolling friction

interlocking is observed. It is observed in case of CGS (such as sand & gravel).

(ii) Friction resistance:



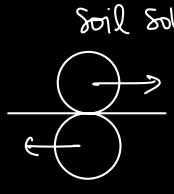
$F = \mu N$



$\tan\delta = \frac{\tau}{\sigma_n}$

$\tau = \sigma_n \tan\delta$

$\delta \rightarrow$ friction angle

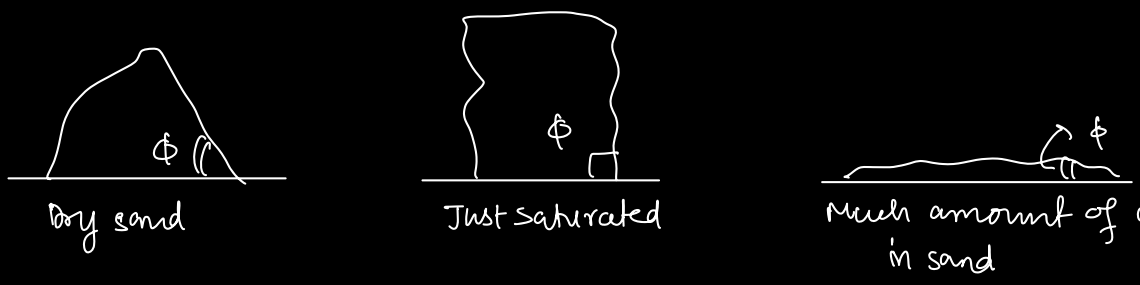


$\tau = \sigma_n \tan\phi$

ϕ: Angle of internal friction

→ Frictional resistance is responsible for the shear strength of the soil. Frictional resistance occurs due to the occurance of sliding friction.

Note: Angle of internal friction can be estimated approximately as.



→ this friction resistance (ϕ) is observed in gravels, sands & silts.

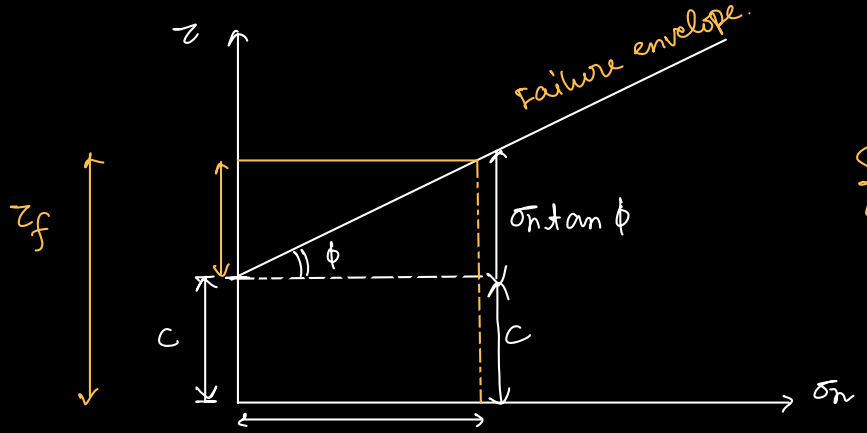
(iii) Intermolecular Adraction Force: It acts in the form of cohesion which can be denoted by "c" and expressed in KN/m^2, Kg/cm^2, N/mm^2, MPa etc.

- Cohesion is observed in case of fine grain soils
- Cohesion is not observed in sand & gravels.

COHESIVE SOILS
- CLAY: due to cohesion only ; $\phi = 0$
  - ↳ frictionless soil ($\phi = 0$)
- SILT: due to cohesion and frictional resistance
  - ↳ $c - \phi$ soil

COHESIONLESS SOILS.
- SAND: Due to frictional resistance & interlocking.
- GRAVEL: Due to frictional resistance & interlocking.

MOHR - COULOMB CRITERIA: [For $c-\phi$ soil] (Easy)



$$\{\tau_f = \bar{\sigma}_n \tan\phi + c\}$$

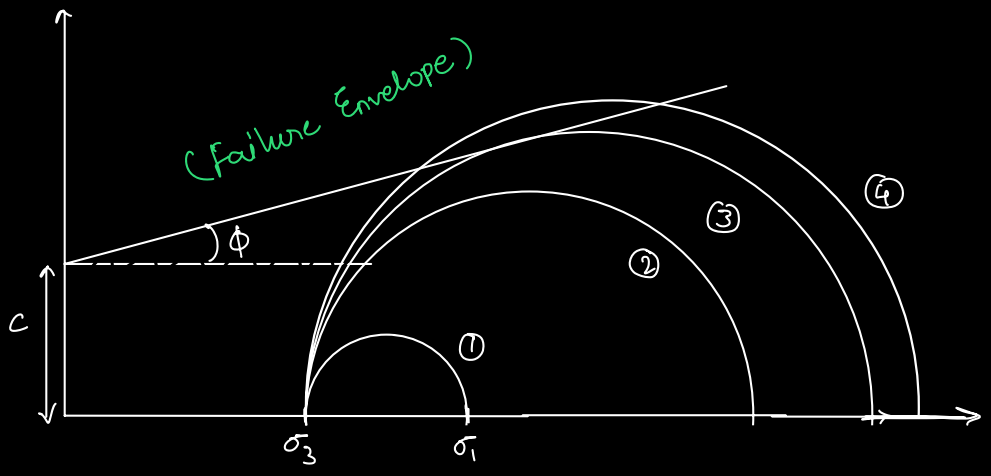
σ_n

→ The shear strength of the soil can be measured in terms of cohesion & frictional resistance (sliding friction). The equation of shear strength given by Mohr-Coulomb can be given as

$$\tau_f = c + \sigma_n \tan\phi$$

τ_f : shear strength of soil

c, ϕ : shear strength parameters



→ The soil elements for which Mohr's circle ① & ② are drawn are safe and stable

→ But the soil ③ is critical whereas soil ④ is unsafe

For Mohr's circle ① & ② ; $\tau < \tau_f$: elastic state of equilibrium

For Mohr's circle ③ ; $\tau = \tau_f$; plastic state of equilibrium

For Mohr's circle ④ ; $\tau > \tau_f$: unstable & unsafe / sheared condition.

→ Mohr coulomb equation for shear strength can be given as

$$\tau_f = C + \sigma_n \tan\phi$$

τ_f : shear strength for total stress analysis

c : total cohesion

σ_n : Total Normal stress

ϕ : total angle of internal friction.

→ Modified Mohr's Coulomb Equation:

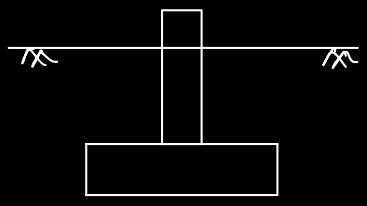
$$\{\tau_f = c' + \bar{\sigma}_n \tan\phi'\}$$

τ_f: Shear strength corresponding to effective stress Analysis

c': Effective cohesion

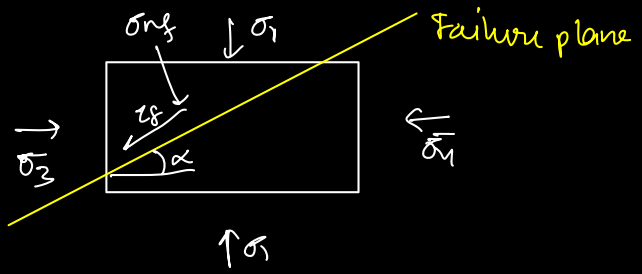
ϕ': Effective angle of internal friction

$\bar{\sigma}_n$: Effective normal stress $(\bar{\sigma}_n = \sigma_n - u)$



Among all these planes, there will be one plane at which failure occurs. That termed as failure plane or critical plane.

* N - Number of planes can pass through any given soil element.



From the Mohr Coulomb's failure criteria,

$$\tau_f = c' + \bar{\sigma}_{nf} \tan\phi'$$

$$\tau_f = c' + \left(\frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cos 2\alpha\right)\tan\phi'$$

$$\tau = \left(\frac{\sigma_1-\sigma_3}{2}\right)\sin 2\alpha$$

The failure is observed on that plane where difference between shear strength (τ_f) and shear stress (τ) becomes minimum.

$$\frac{\partial}{\partial\alpha}(\tau_f - \tau) = 0$$

$$\frac{\partial}{\partial\alpha}\left(c' + \left[\left(\frac{\sigma_1+\sigma_3}{2}\right) + \left(\frac{\sigma_1-\sigma_3}{2}\cos 2\alpha\right)\right]\tan\phi' - \left(\frac{\sigma_1-\sigma_3}{2}\right)\sin 2\alpha\right) = 0$$

$$0 + 0 + \left(\frac{\sigma_1-\sigma_3}{2}\right)(-\sin 2\alpha)(2)\tan\phi' - \left(\frac{\sigma_1-\sigma_3}{2}\right)\cos 2\alpha \cdot 2 = 0$$

$$(\sigma_3-\sigma_1)\sin 2\alpha \tan\phi' - (\sigma_1-\sigma_3)\cos 2\alpha = 0$$

$$(\sigma_1 - \sigma_3)[\sin 2\alpha \tan\phi' + \cos 2\alpha] = 0$$

$$\therefore \quad \sin 2\alpha \tan\phi' + \cos 2\alpha = 0$$

$$\tan\phi' + \cot 2\alpha = 0$$

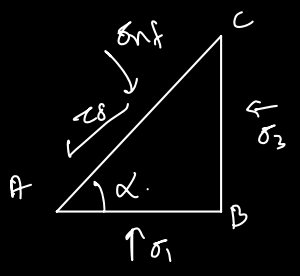
$$\cot 2\alpha = -\tan\phi'$$

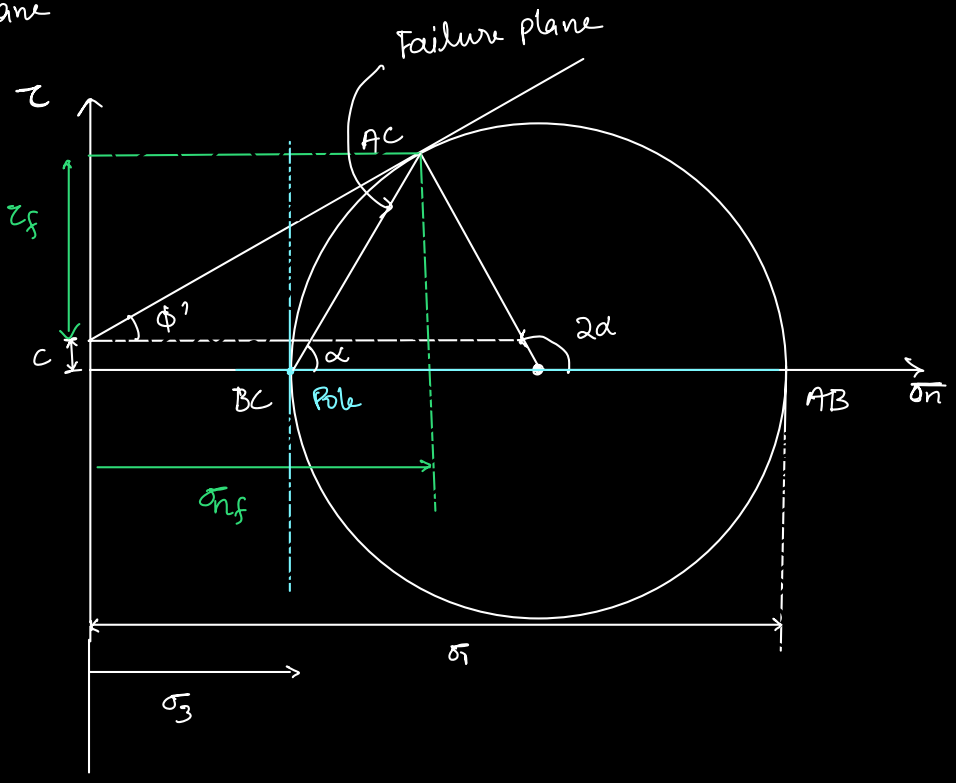
$$\cot 2\alpha = \cot(90 + \phi')$$

$$2\alpha = 90 + \phi'$$

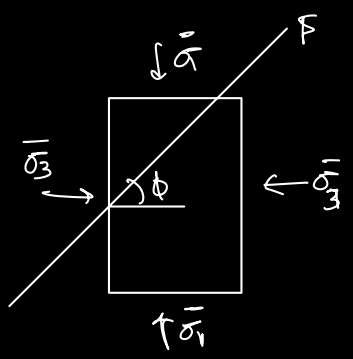
$$\boxed{\alpha = 45 + \frac{\phi}{2}}$$

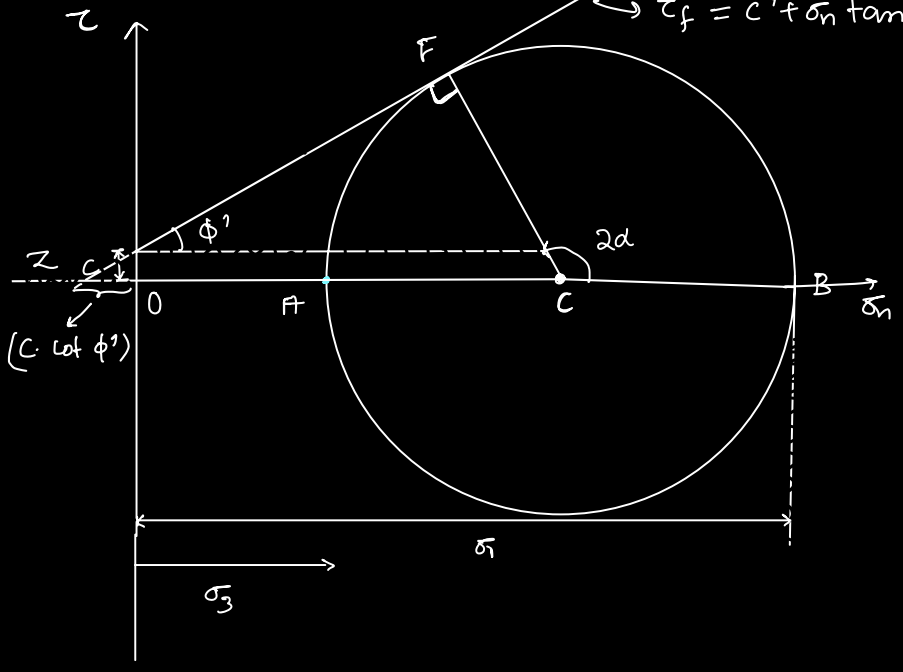
α: Angle of failure plane with respect to major principal plane





→ The mohr coulomb failure criteria can also be expressed in the terms of a relationship between $\bar{\sigma}_1$ and $\bar{\sigma}_3$.





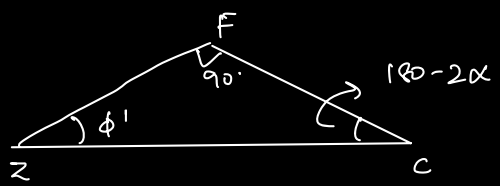
→ $\tau_f = c' + \bar{\sigma}_n \tan\phi$.

$OA = \bar{\sigma_3}$ $\qquad OZ = C' \cot\phi'$

$OB = \bar{\sigma_1}$ $\qquad CZ = OC + OZ$

$OC = \dfrac{\bar{\sigma_1} + \bar{\sigma_3}}{2}$ $\qquad CF = \text{Radius} = \dfrac{\bar{\sigma_1} - \bar{\sigma_3}}{2}$

In the $\triangle$ CZF:



→ Angle of failure plane w.r.t major principle plane measured in degree.

$90 + 180 - 2\alpha + \phi' = 180$

$\alpha = \dfrac{90 + \phi}{2} = 45 + \dfrac{\phi}{2}$

$$\sin\phi' = \frac{CF}{CZ} = \frac{\left(\frac{\bar{\sigma_1} - \bar{\sigma_3}}{2}\right)}{\left(\frac{\bar{\sigma_1} + \bar{\sigma_3}}{2}\right) + C' \cot\phi'}$$

$$\left(\frac{\bar{\sigma_1} + \bar{\sigma_3}}{2}\right)\sin\phi' + C'\cos\phi' = \frac{\bar{\sigma_1} - \bar{\sigma_3}}{2}$$

$$p\sin\phi' + C'\cos\phi' = q$$

↳ Equation of **STRESS PATH**

where,

$$p = \frac{\bar{\sigma_1} + \bar{\sigma_3}}{2} \quad \& \quad q = \frac{\bar{\sigma_1} - \bar{\sigma_3}}{2}$$

$$\left(\frac{\bar{\sigma_1} + \bar{\sigma_3}}{2}\right)\sin\phi' + C'\cos\phi' = \frac{\bar{\sigma_1} - \bar{\sigma_3}}{2}$$

$$(\bar{\sigma_1} + \bar{\sigma_3})\sin\phi' + 2C'\cos\phi' = (\bar{\sigma_1} - \bar{\sigma_3})$$

$$\sigma_3(1 + \sin\phi') + 2C'\cos\phi' = \bar{\sigma_1}(1 - \sin\phi')$$

$$\bar{\sigma_1} = \bar{\sigma_3}\left(\frac{1 + \sin\phi'}{1 - \sin\phi'}\right) + 2C'\left(\frac{\cos\phi'}{1 - \sin\phi}\right)$$

$\because \quad \dfrac{1 + \sin\phi'}{1 - \sin\phi'} = \tan^2\left(45 + \dfrac{\phi}{2}\right) = N_\phi$ **(FLOW VALUE)**

$\because \quad \dfrac{\cos\phi'}{1 - \sin\phi} = \tan\left(45 + \dfrac{\phi}{2}\right) = \sqrt{N_\phi}$

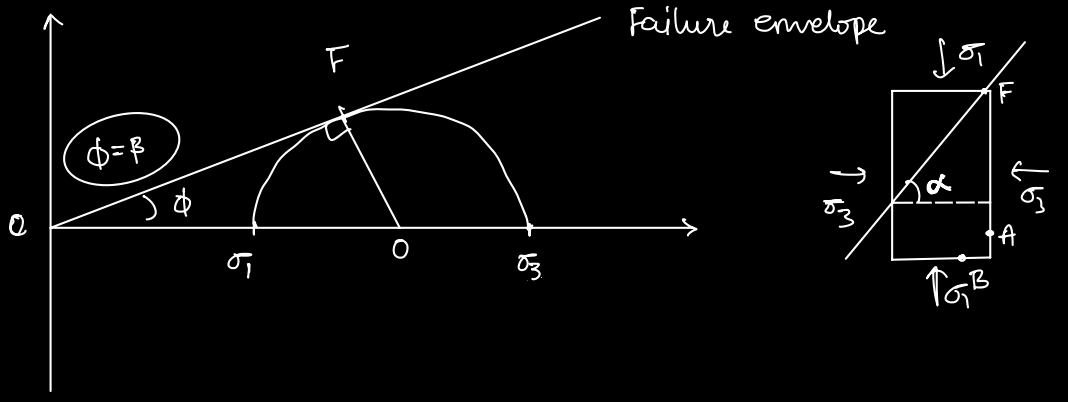
$$\bar{\sigma}_1 = \bar{\sigma}_3 N_\phi + 2c\sqrt{N_\phi}$$

or

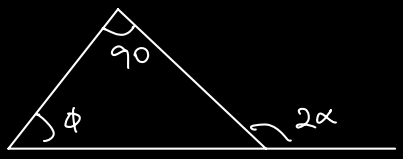
$$\bar{\sigma}_1 = \bar{\sigma}_3 \tan^2\left(45 + \frac{\phi}{2}\right) + 2c \tan\left(45 + \frac{\phi}{2}\right)$$

$$\left\{ \bar{\sigma}_1 = \bar{\sigma}_3 \tan^2(\alpha) + 2c \tan\alpha \right\}$$

(Case ii) : Cohesionless soil [C = 0] (Sand, Gravel) :



In the Δ COF :



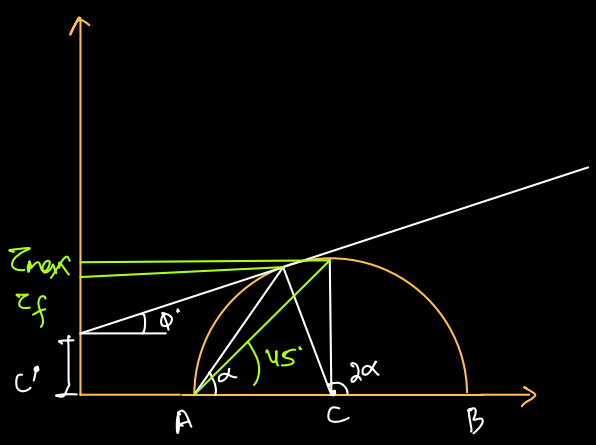
$$\sin\phi' = \frac{CF}{OC} = \frac{(\bar{\sigma}_1 - \bar{\sigma}_3)/2}{(\bar{\sigma}_1 + \bar{\sigma}_3)/2}$$

$$(\bar{\sigma}_1 + \bar{\sigma}_3)\sin\phi = (\bar{\sigma}_1 - \bar{\sigma}_3)$$

$$\bar{\sigma}_1 = \bar{\sigma}_3 \left(\frac{1+\sin\phi}{1-\sin\phi}\right)$$

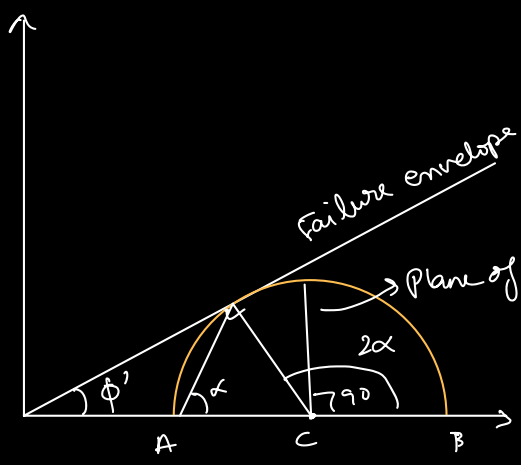
$$\bar{\sigma}_1 = \bar{\sigma}_3 \tan^2\left(45 + \frac{\phi}{2}\right)$$

Silt



$$\bar{\sigma}_1 = \bar{\sigma}_3 \tan^2\left(45 + \frac{\phi}{2}\right) + 2c' \tan\left(45 + \frac{\phi}{2}\right)$$

Sand

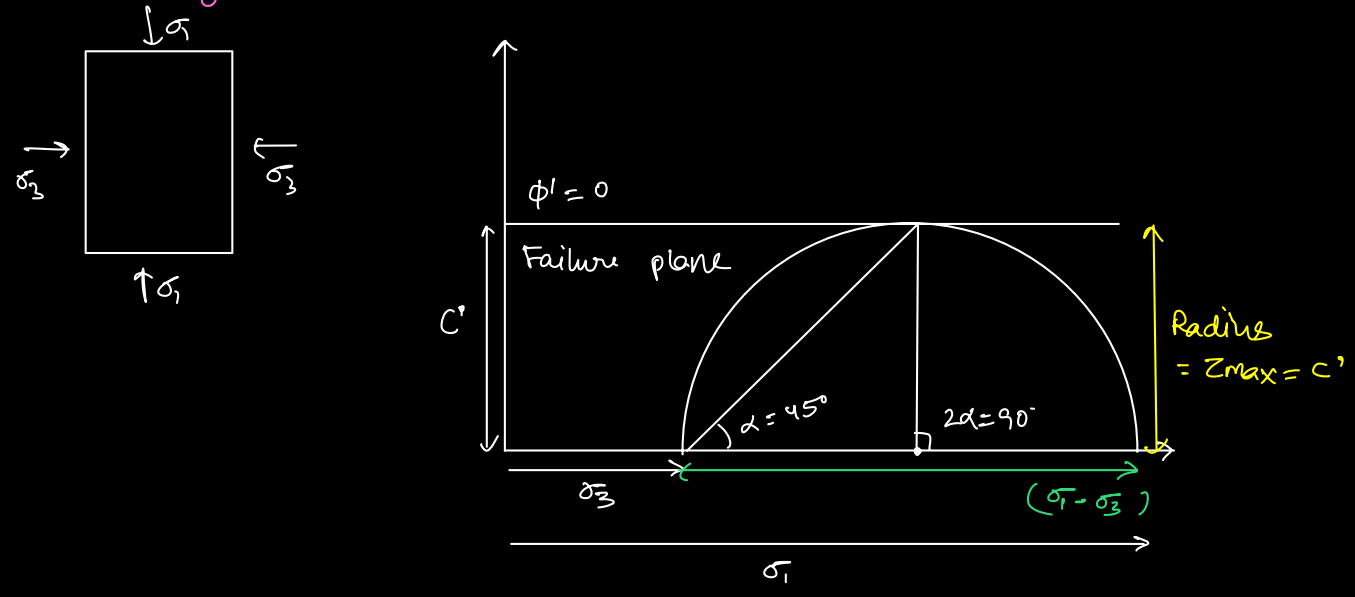


Note: The soil doesnot fail at the plane of maximum shear stress because at the plane of maximum shear stress, shear strength is more than maximum shear stress (τ_{max})

Note: The angle of maximum shear stress is inclined at an angle of 45° with the major principal plane.

Note: The failure or shearing of soil observed at an angle of $\alpha = 45 + \frac{\phi}{2}$ measured with respect to the major principle plane, at this failure plane Normal stress (σ_n) & shear stress (τ) both are not maximum

Case iii : clayey soil ($\phi = 0$)



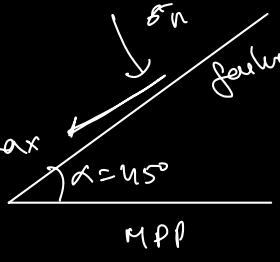
$\sigma_1 = \sigma_3 + 2C'$ } (सीधा दिख रहा है)

$\alpha = 45°$

α: angle of failure plane which is coming out to be equal to 45° just in case of plane of maximum shear stress.

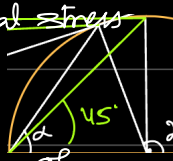
$\sigma_n = \frac{\sigma_1 + \sigma_3}{2}$

$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = C'$



Note: In case of silt & sand - failure doesnot takeplace at the

plane of maximum shear stress. The failure takes place where the angle of plane α becomes $45 + \frac{\phi}{2}$ which is formed as failure plane on which neither normal stress is maximum nor shear stress.



But in case of clay, soil failure takes place of maximum shear stress which is inclined at an angle of $\alpha = 45°$, with the plane of major principal stress. That means in case of clay soil at the failure plane τ_{max} is maximum. but normal stress is not maximum

Types of shear strength that can be determined:

(i) Undrained shear strength:

In case of saturated clay, if rapid construction (fast rate of loading is done) then for short term Analysis undrained shear strength is calculated.

Keypoints: Total Stress Analysis, undrained analysis, drainage is not happening in short duration of clay, fast rate of loading.

Undrained shear strength can be given as

$$\tau_f = c_u + \bar{\sigma_n} \tan \phi_u$$

↳ c_u & ϕ_u are the undrained shear strength parameters obtained in the undrained condition.

(ii) Drain shear strength:

If the rate of loading is slow (slow construction) and major portion of expulsion of pore water completes during the loading then shear strength corresponds to drained condition can be given as

$$\tau_f = c' + \bar{\sigma_n} \tan \phi'$$

or

$$\tau_f = c_d + \bar{\sigma_n} \tan \phi_d$$

c_d: drain cohesion or effective cohesion

ϕ_d: drain angle of shearing resistance or effective angle of shearing resistance

Note: If rate of loading is slow then in short term & long term analysis drained shear strength is determined. For clays in long term analysis drain shear strength is determined.

Note: Rapid construction → Saturated clay → short term Analysis is Undrained Shear strength. ← Total stress Analysis ← No drainage

Note: Slow construction → sands → "free drainage" → drain Analysis is drain shear strength ← both short term long term ← effective stress Analysis is

Note: Slow construction → saturated clay → slow drainage → drain Analysis → effective stress Analysis → long term Analysis → drain shear strength.

SHORT TERM ANALYSIS IN CLAY

" undrained shear strength.

LONG TERM ANALYSIS IN CLAY

" drained shear strength.

SHORT TERM STABILITY ANALYSIS IN SAND

" Drain shear strength"

LONG TERM STABILITY ANALYSIS IN SAND

" Drain shear strength"

Different types of shear strength test:

(i) Direct shear test
(ii) Triaxial test
(iii) Unconfined compression test
(iv) Vane shear test.

Selection of test:

Selection of test depends upon

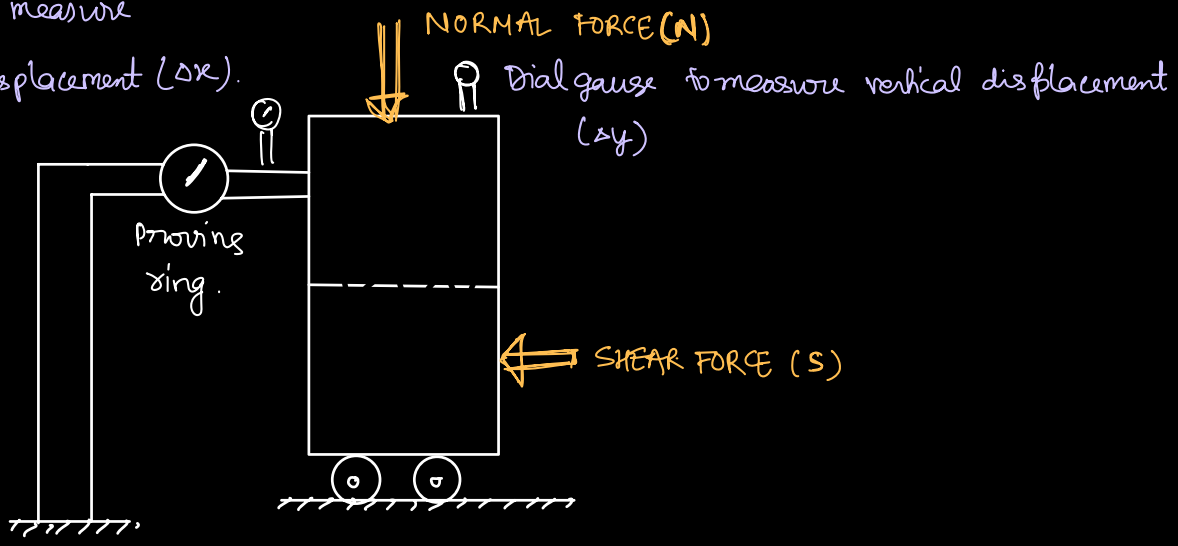
(i) Type of soil

(ii) Purpose of test (short term or long term)

(iii) drainage condition.

1. DIRECT SHEAR TEST (LABORATORY METHOD):

Dial gauge to measure horizontal displacement (Δx).



PROCEDURE:

1. Apply the Normal force & calculated the Normal stress

Applied Normal force : N

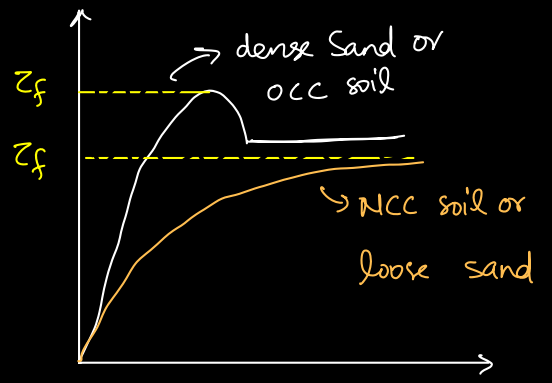
Normal stress : $\frac{N}{A}$

2. Apply the shear force and increase it till shearing & calculate shear stress & corresponding Δx.

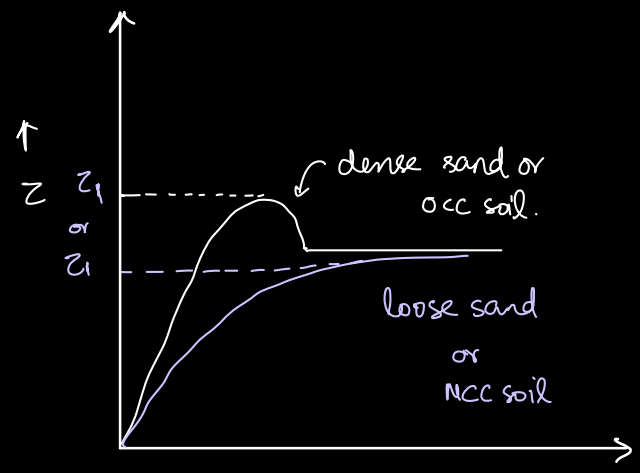
$$\tau = \frac{\text{Shear force}}{\text{Shearing Area}} = \frac{F}{A}$$

Shear force (F)	Shear Stress (τ)	Horizontal displacement (Δx)

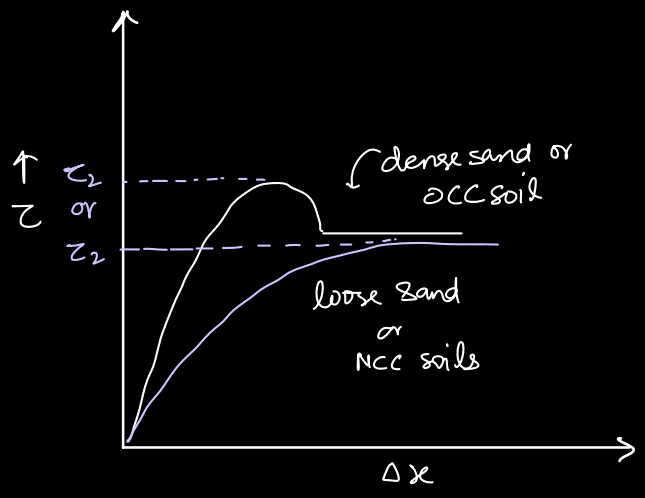
3. Draw the plot between τ & Δx.



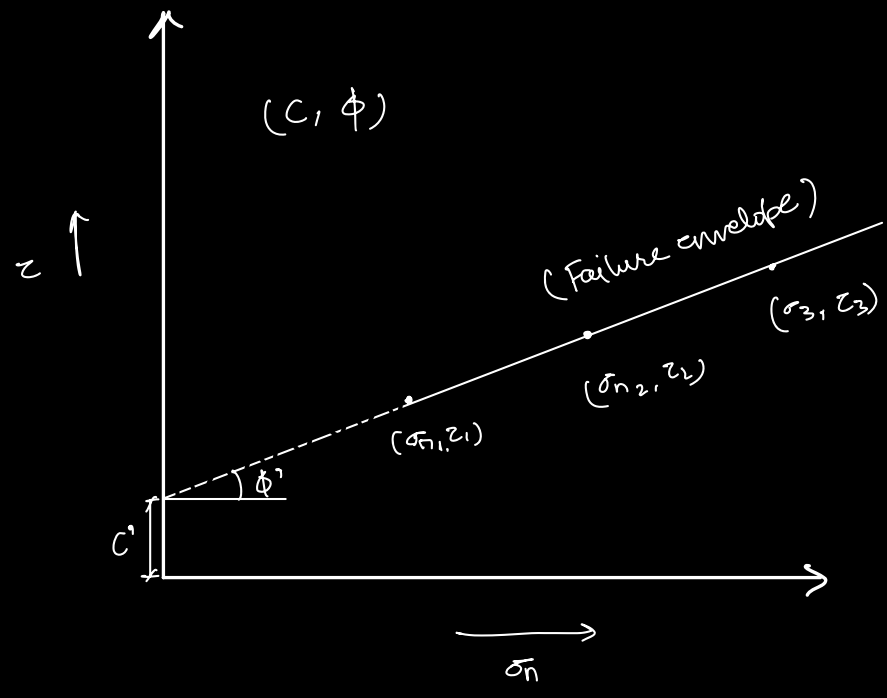
τ_f: Shear stress on the failure plane at the time of failure.



Sample 1 (σ_{n_1}, τ_1)



Sample 2 (σ_{n_2}, τ_2).



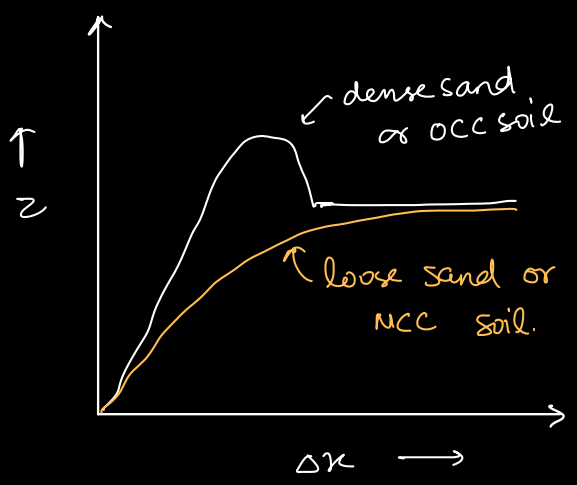
{Direct shear test !!}

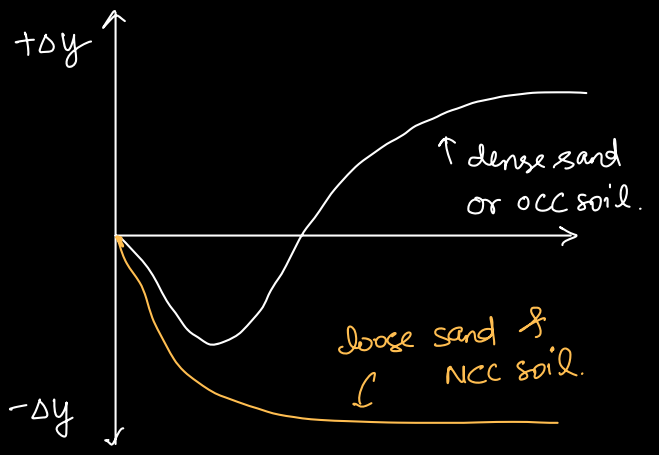
The shear strength of the soil can be computed as

$$\tau_f = c' + \bar{\sigma}_n \tan\phi'$$

↳ equation of failure envelope.

c' & ϕ' are already determined by drawing the failure envelope.

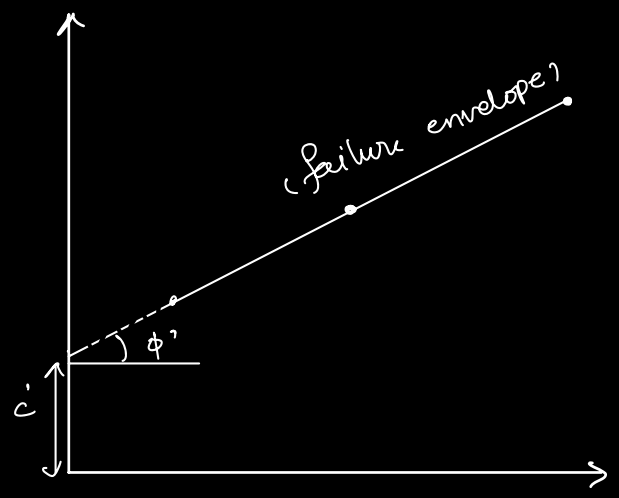




Note: At the time of shearing dense sand & over consolidated soil shows volume increase tendency because particle start rolling over each other.

At the time of shearing loose sand and NCC soils shows volume decrease tendency because it goes towards densification.

1. In the direct shear test (box shear test), the soil sample is kept in metallic box of square & circular cross section which is spited into the two parts along the horizontal plane
2. Solid metallic plates are placed at top and bottom of soil sample if soil is dry
3. If soil mass is partially & fully saturated, porous plates for free drainage can be placed at the top & bottom.
4. The top half portion of the box and fixed & bottom half portion can be moved. Since there is no control on the drainage therefore this test is not very much suitable for fine grained soil
5. This test is very much suitable for free draining soils such as Sand.
6. In the direct shear test principal stresses & principle planes are unknown they can be determined after drawing the failure envelope.



7. As we draw the failure envelope, we can determine the shear strength parameters c' & ϕ'.

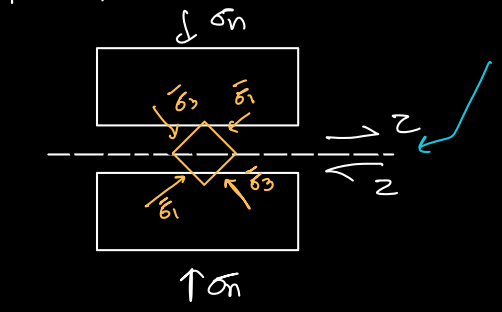
8. In each observation we have the value of σ_n and τ at the time failure.

$$\bar{\sigma_n} = \frac{\bar{\sigma_1} + \bar{\sigma_3}}{2} + \frac{\bar{\sigma_1} - \bar{\sigma_3}}{2} \cos 2\alpha \quad \text{— eqn (1)}$$

$$\tau = \frac{\bar{\sigma_1} - \bar{\sigma_3}}{2} \sin 2\alpha. \quad \text{— eqn (2)}$$

$$\alpha = 45 + \frac{\phi}{2}$$

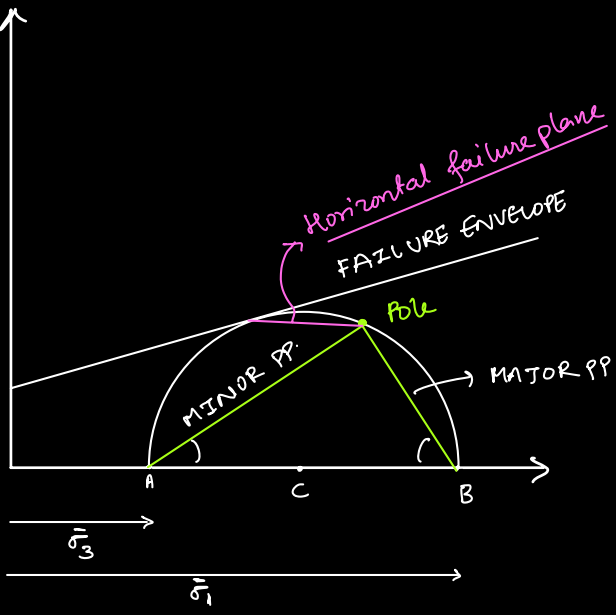
→ Solve equation (1) & equation (2) to determine major and minor principal stress $\bar{\sigma_1}$ & $\bar{\sigma_3}$:-



Failure plane.

FROM SOM!!

$$\tan\theta = \frac{2\tau_{xy}}{(\bar{\sigma_x} - \bar{\sigma_y})}$$

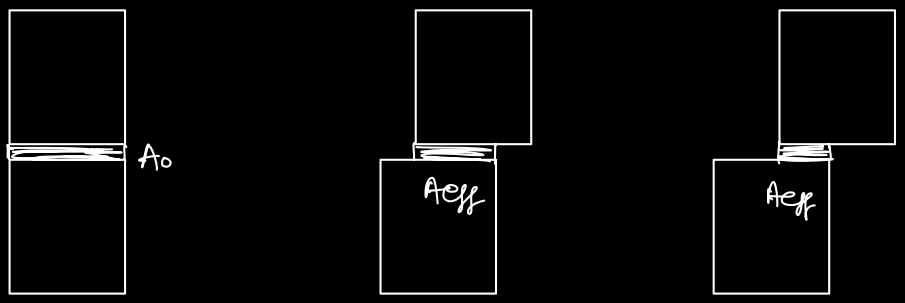


Limitations of Direct Shear test:

(i) There is no control on the drainage, no facility to measure pore water pressure

(ii) The failure plane is predetermined i.e horizontal but actually failure plane occurs at $\alpha = 45 + \frac{\phi}{2}$

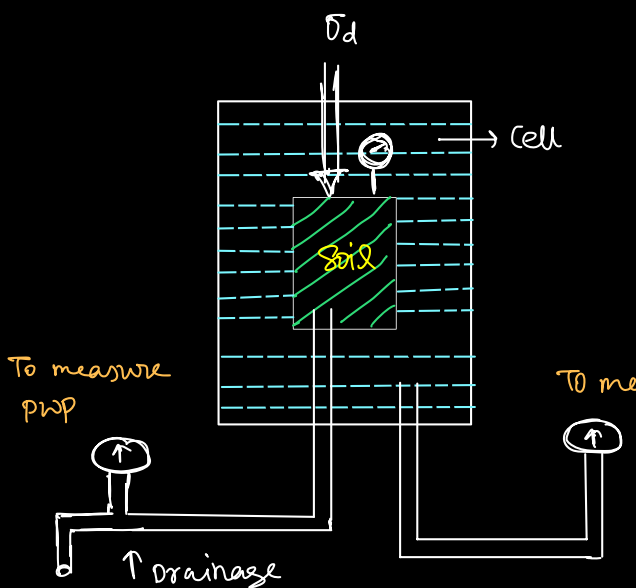
(iii) The results of this test are not reliable because σ_n and τ are calculated for initial value of ϕ's area & shearing area respectively, but during shearing the area is changing, thus effective area should have been taken in calculation of σ_n & τ



Ao: Initial Area

Aeff: Effective Area.

2. TRIAXIAL TEST

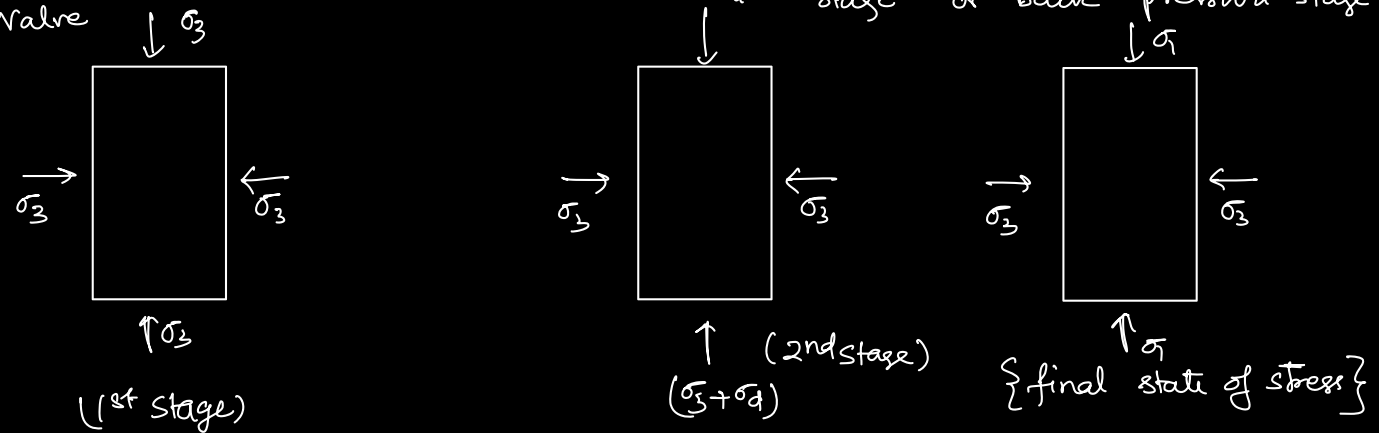


* If valve is kept open it is termed as drained Analysis if wall is kept close undrained Analysis

* There are two stages in this test.

i) Cell pressure stage or confining pressure stage.

ii) Shearing Stage or deviatoric Stage or back pressure stage



$\{\sigma_1 = \sigma_3 + \sigma_d\}$

σ_1: Major principal stress, σ_3: Minor principal stress or Shell pressure.

σ_d : Deviatoric stress.

→ In the triaxial machine following type of tests can be performed

(i) Consolidation - drain test (CD)

(ii) Consolidation - undrain test (CU)

(iii) Unconsolidation - undrain test (UU)

(i) CONSOLIDATION - DRAIN TEST (CD Test / Slow test / S-test:

↳ This test is suitable for saturated clay for long term stability analysis under drain condition with slow rate of loading.

↳ To complete this test few weeks (2 weeks) required.

Stage 1: Cell pressure stage:

a) drainage is allowed

b) apply the confining pressure (σ_3).

c) excess pwp (u_c) will develope, since drainage is allowed, u_c becomes zero at the end of cell pressure stage soil is consolidated.

Stage 2: Shearing stress:

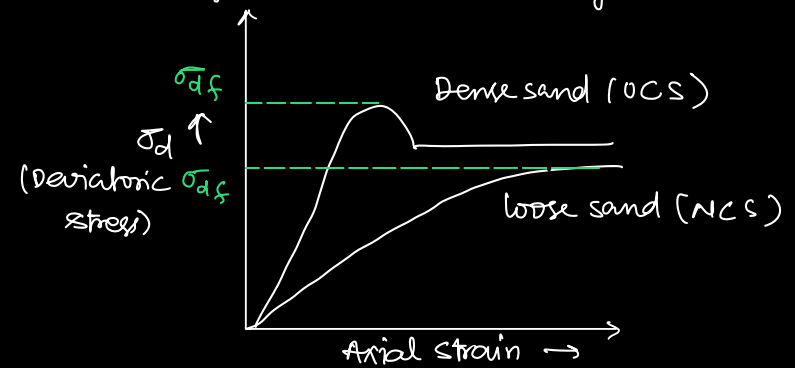
a) Drainage is allowed

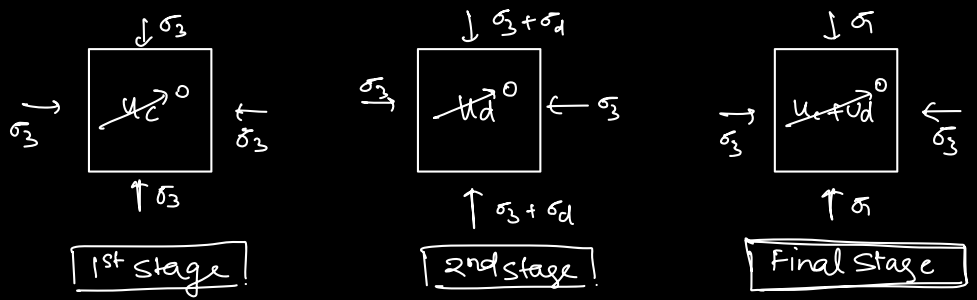
b) Apply and increase the deviatory stress till shearing corresponding Axial strains are also being measured

c) Plot the curve between deviatoric stress and Axial strain and determine the value of deviatoric stress at the time of failure.

d) Due to the deviatoric stress excess pwp (u_d) will develope

e) Since Drainage is allowed therefore u_d becomes zero.





$$\sigma_1 = \sigma_3 + \sigma_d$$

$$\bar{\sigma_1} = \sigma_1 - (u_c + u_d) = \sigma_1$$

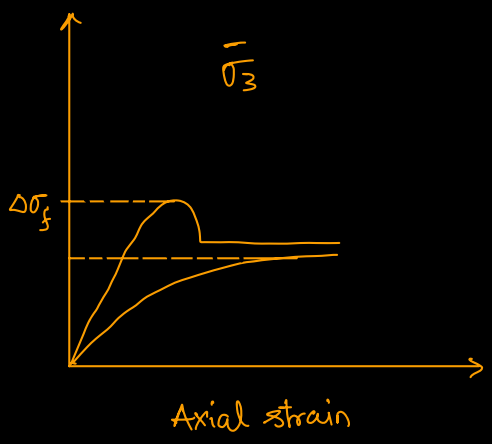
$$\bar{\sigma_3} = \sigma_3 - (u_c + u_d) = \sigma_3$$

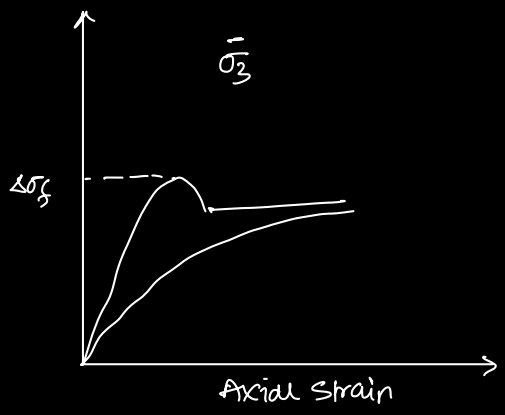
$\bar{\sigma_1}$: Effective major principle stress

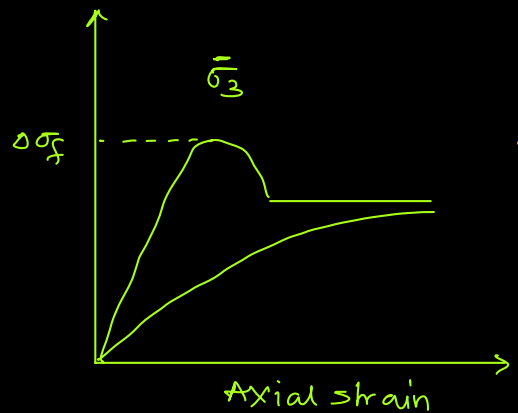
$\bar{\sigma_3}$: Effective minor principle stress

→ The above procedure is repeated for minimum three soil samples with increasing value of confining pressure (σ_3)

- ⊙ Sample (1) ($\bar{\sigma_1}, \bar{\sigma_3}$)
- ⊙ Sample (2) ($\bar{\sigma_1}, \bar{\sigma_3}$)
- ⊙ Sample (3) ($\bar{\sigma_1}, \bar{\sigma_3}$)

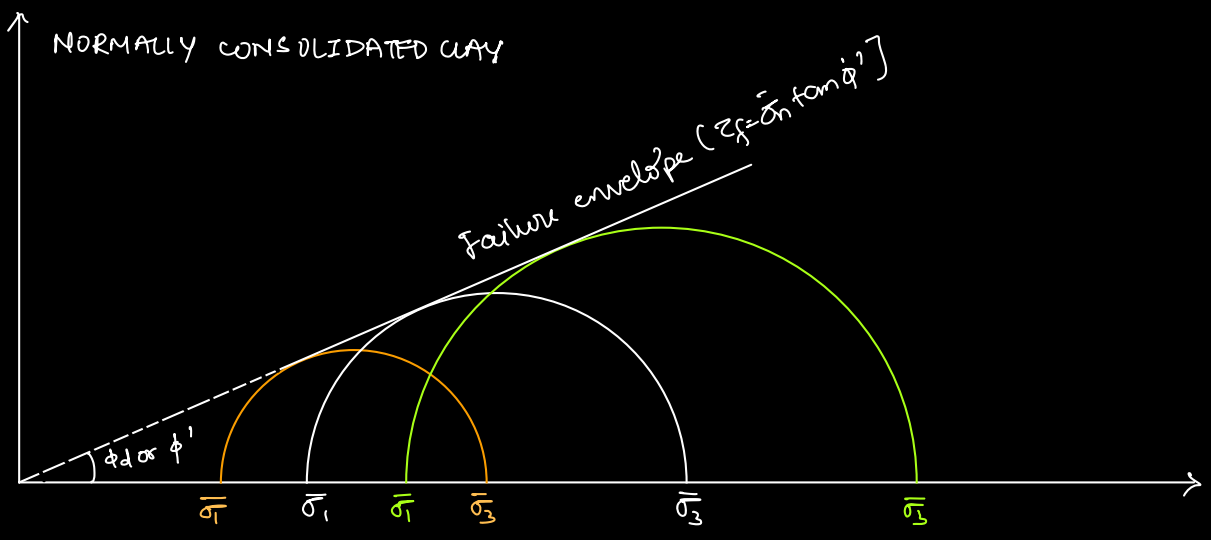






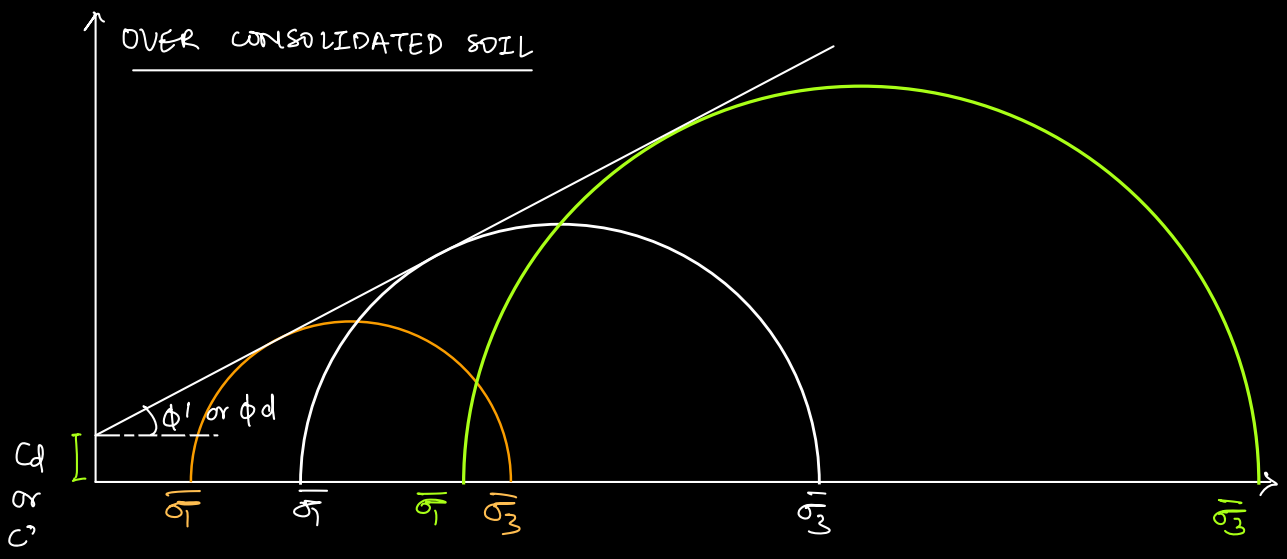
$\bar{\sigma_3}_{(1)} < \bar{\sigma_3}_{(2)} < \bar{\sigma_3}_{(3)}$

$\Delta\sigma_{f(1)} < \Delta\sigma_{f(2)} < \Delta\sigma_{f(3)}$



$\tau_f = \bar{\sigma}_n \tan\phi'$

ϕ' or ϕ_d : drain or effective angle of internal friction.



Note: In this test drained shear strength parameters [C_d and ϕ_d] or effective stress parameter [c' & ϕ'] are obtained. In drained Analysis cohesion is not developed in case of NC soils. but in case of overconsolidated soil it can be developed.

Note: Shear strength parameters can also be calculated without drawing Mohr's circles as shown below,

↳ 1st sample ($\bar{\sigma}_1$, $\bar{\sigma}_3$) :

$$\bar{\sigma}_1 = \bar{\sigma}_3 \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

↳ 2nd sample ($\bar{\sigma}_1$, $\bar{\sigma}_3$) :

$$\bar{\sigma_1} = \bar{\sigma_3} \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

Solve equation (1) & (2) to get c' & ϕ'.

Note: In undrain analysis expulsion of porewater pressure doesnot take place therefore no change in volume of soil is observed. But in case of drain Analysis expulsion of porewater takes place then change in volume of soil sample is observed.

Note: In the direct shear test σ_n and τ_u were calculated corresponding to initial cross-section and shearing area which was its limitations But in case of triaxial test, area at the time of failure is taken in the deviatoric stress

$$\Delta\bar{\sigma}_f = \sigma_{df} = \frac{\text{Applied Compression force}}{\text{Area at the time of failure}}$$

$$= \frac{P}{A_f}$$

$$A_f = \frac{\text{Volume at failure}}{\text{length at failure}} = \frac{V_o \pm \Delta V}{L_o - \Delta L} = \frac{V_o\left(1 \pm \frac{\Delta V}{V}\right)}{L_o\left(1 - \frac{\Delta L}{L}\right)}$$

$$A_f = A_o\left(\frac{1 \pm \varepsilon_v}{1 - \varepsilon_L}\right)$$

ε_v: volumetric strain at failure

ε_L: Axial (or longitudinal strain at failure).

Note: In triaxial machine the test is carried on the cylindrical soil specimen of height to diameter ratio. 2. Generally They are taken as 100mm (H) × 50mm (D) and 76mm (H) × 38mm (D)

Note: In the triaxial test the shear stress distribution is uniform on the failure plane whereas in case of direct shear test shear stress distribution was uniform on failureplane.

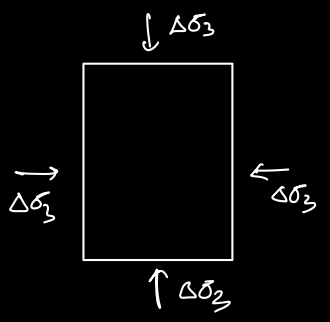
Note: In this test pore water pressure can be measured using "bishop's pressure measuring apparatus".

Note: In this test there is control on drainage

SKEMPTON'S PORE PRESSURE PARAMETERS (A and B):

Skempton have analysed change in pore pressure on changing the stresses in the soil.

1st stage: Cell pressure stage



The Skempton pore pressure parameter "B" is defined as ratio of change in pore pressure (Δu_c) to the change in cell pressure at cell pressure stage. ($\Delta\sigma_3$)

$$\left(B = \frac{\Delta u_c}{\Delta\sigma_3} \right)$$

for partially saturated soil = 0 - 1

for fully saturated soil = 1

for dry soil = 0.

2nd stage: Shearing stage or deviatoric stage

$$\bar{A} = \frac{\text{change in pore water pressure}}{\text{change in deviatoric stress}}$$

$$\bar{A} = \frac{\Delta u_d}{\Delta\sigma_d}$$

Skempton's pore pressure parameter (A) can be given as:

$$\bar{A} = A \cdot B$$

$$A = \frac{\bar{A}}{B}$$

$A = \bar{A}$ [For fully saturated soils]

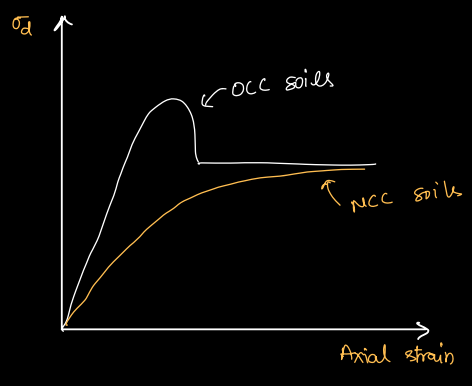
- at the time of failure, A can be represented as

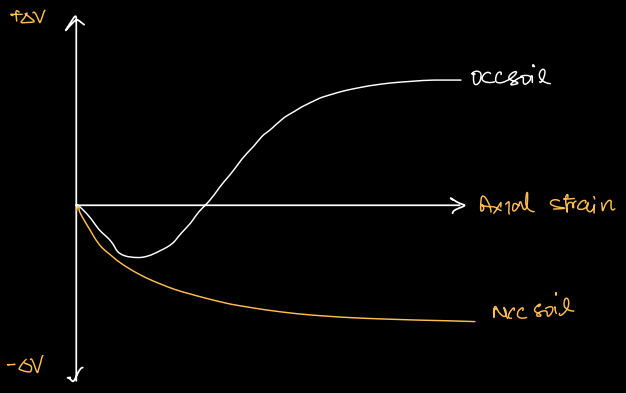
"A_f" (skempton parameter) which depends upon the magnitude of deviatoric stress at the time of failure.

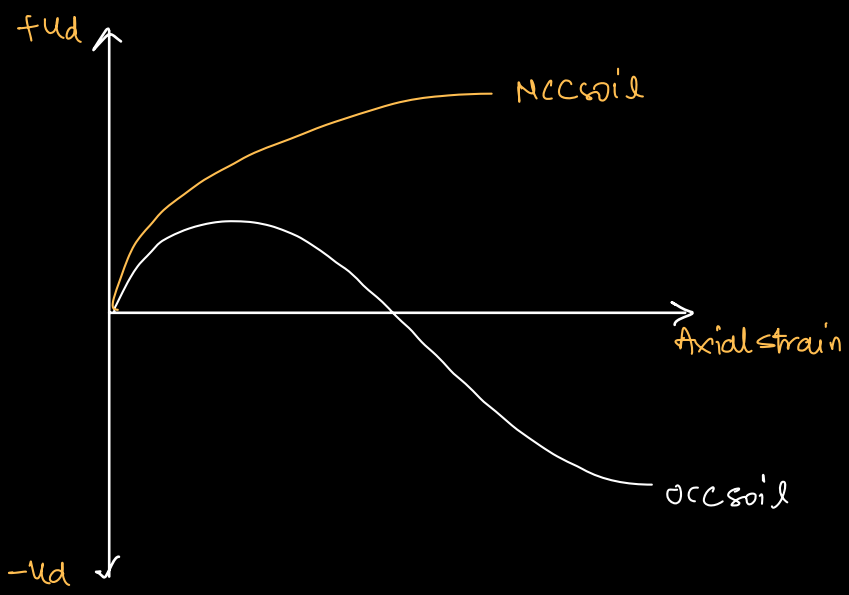
"A_f": it can be taken as positive for Normally Consolidated soils [NCC]

"A_f": it can be taken as negative of overconsolidated soils [OCC]

Note: That means at the time for shearing & failure poorewater pressure is true for Normally consolidated soils and negative for over consolidated soils







Note: In the cell pressure stage the pore water pressure (uc) will be developed equal to the magnitude of the applied cell pressure, whereas in the second stage the developed pore water pressure (ud) will not be equal to the applied additional deviatoric stress (σ_d).

(ii) CONSOLIDATION UNDRAIN TEST / CU TEST / R-TEST :

1st stage : Cell pressure stage.

a. Drainage is allowed

b. Apply confining pressure σ_3.

c. due to confining pressure, excess pore water pressure u_c will develope

d. ∵ drainage is allowed

∴ $u_c \rightarrow 0$

e. change in volume of soil is observed.

f. At the end of cell pressure stage is soil is consolidated.

2nd stage: Shearing stage:

a. Drainage is not allowed.

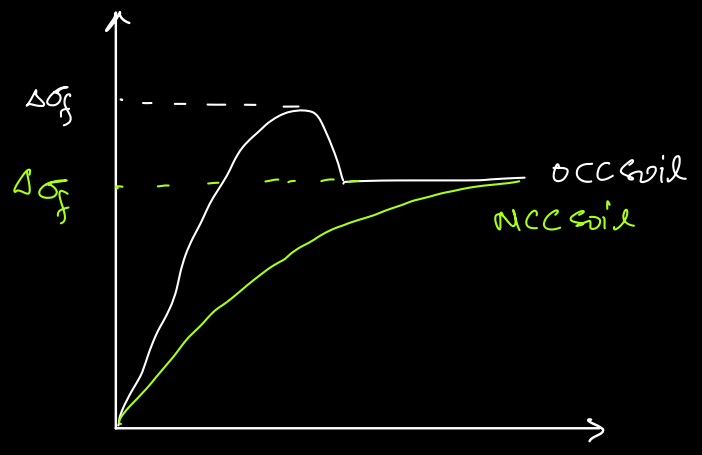
b. apply and increase the deviatoric stress till shearing corresponding Axial strains are measured and graph is plotted between deviatoric stress & axial strain values

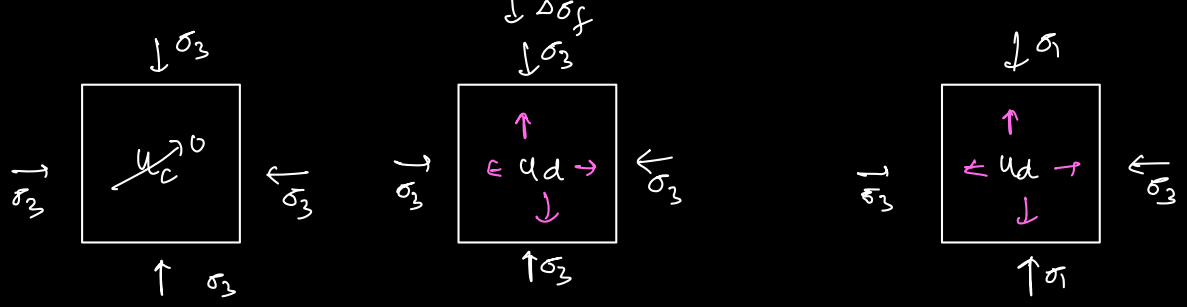
c. From the graph estimate the deviatoric stress at the time of failure

d. Due to deviatoric stress excess pore water pressure Δu_d will develope

e ∵ Drainage is not allowed

∴ $u_d \neq 0$





1st stage | 2nd stage | Final state of stress at the time of failure ($\sigma_1 = \sigma_3 + \Delta\sigma_f$)

$$\bar{\sigma_1} = \sigma_1 - \Delta u_d = \sigma_1 - A_f \Delta\sigma_f$$

$$\bar{\sigma_2} = \sigma_3 - \Delta u_d = \sigma_3 - A_f \Delta\sigma_f$$

$$\because A_f = \frac{\bar{A}}{B} = \frac{\Delta u_d}{\Delta\sigma_f \cdot B}$$

$$\Delta u_d = A_f \Delta\sigma_f \times 1$$ (For fully saturated soil, $B = 1$)

$\because$ A_f is +ve for NCC soils

$$\therefore \bar{\sigma_1} = \sigma_1 - A_f \Delta\sigma_f$$

$$\bar{\sigma_3} = \sigma_2 - A_f \Delta\sigma_f$$

$\because$ A_f is -ve for OCC soils

$$\therefore \bar{\sigma_1} = \sigma_1 - (-A_f \Delta\sigma_f)$$

$$\bar{\sigma_3} = \sigma_3 - (-A_f \Delta\sigma_f)$$

→ Now number of test samples are prepared & tested with increasing value of confining pressure and corresponding deviatoric stress for each sample is determined at the time of failure.

Sample ①	Sample ②	Sample ③
(σ_1, σ_3)	(σ_1, σ_3)	(σ_1, σ_3)
$(\bar{\sigma_1}, \bar{\sigma_3})$	$(\bar{\sigma_1}, \bar{\sigma_3})$	$(\bar{\sigma_1}, \bar{\sigma_3})$

$$\bar{\sigma_{3①}} < \bar{\sigma_{3②}} < \bar{\sigma_{3③}}$$

(** Trend is same for remaining stresses)

→ Shear strength parameters can be determined using Mohrs circle or using Analytical approach

→ Here, total stress or effective stress Analysis can be carried out.

→ In this test also cohesion is not developed for NCC soils but it is there in case of OCC soils.

ANALYTICAL APPROACH:

$$\sigma_{1(1)} = \sigma_{3(1)} \tan^2\left(45 + \frac{\phi_{cu}}{2}\right) + 2c_{cu} \tan\left(45 + \frac{\phi_{cu}}{2}\right) \quad -(1)$$

$$\sigma_{1(2)} = \sigma_{3(2)} \tan^2\left(45 + \frac{\phi_{cu}}{2}\right) + 2c_{cu} \tan\left(45 + \frac{\phi_{cu}}{2}\right) \quad -(11)$$

Solve equation (1) & (2) & determine c_{cu} and ϕ_{cu}

$$\bar{\sigma}_{1(1)} = \bar{\sigma}_{3(1)} \tan^2\left(45 + \frac{\phi'_{cu}}{2}\right) + 2c'_{cu} \tan\left(45 + \frac{\phi'_{cu}}{2}\right) \quad -(1)$$

$$\bar{\sigma}_{1(2)} = \bar{\sigma}_{3(2)} \tan^2\left(45 + \frac{\phi'_{cu}}{2}\right) + 2c'_{cu} \tan\left(45 + \frac{\phi'_{cu}}{2}\right) \quad -(11)$$

c_{cu}: total consolidated undrained cohesion

ϕ_{cu}: total consolidated undrain angle of internal friction.

c_{cu}': effective consolidated undrained cohesion

ϕ_{cu}': effective consolidated undrain angle of internal friction.

OCC

*For total stress Analysis:

$$\tau_f = c_{cu} + \sigma_n \tan\phi_{cu}$$

*For effective stress Analysis:

$$\tau_f = c_{cu}' + \bar{\sigma}_n \tan\phi'_{cu}$$

NCC

* For total stress Analysis:

$$\tau_f = \sigma_n \tan\phi_{cu}$$

* For effective stress Analysis

$$\tau_f = \bar{\sigma}_n \tan\phi'_{cu}$$

Note: In this test also cohesion is not developed in NC soils but it is there in case of OC soils

Note: In CU test:

1st stage takes approximately 24 hours &

2nd stage takes approximately 2 hours

(That means in 1 to 2 days this test can be completed

Note: when stability of earthen dam is checked in the sudden drawdown condition then "consolidation undrain" shear strength is determined by performing "CU-test".

(iii) Unconsolidation undrain test / UU test / Quick test / Q-test

→ This test is suitable only for clay soils only. when they are subjected to fast rate of loading (Rapid construction).

→ For short term stability analysis in the clay soil this test can be used and the undrained unconsolidated shear strength of the soil can be determined.

→ UU test can be over in 5-10 minutes.

1st Stage: Cell pressure stage.

a) Drainage is not allowed

b) Cell pressure (σ_3) is applied

c) Due to cell pressure excess pore pressure u_c will be developed

d) ∵ drainage is not allowed

∴ $u_c \neq 0$

2nd Stage: Shearing stage

a) Drainage is not allowed

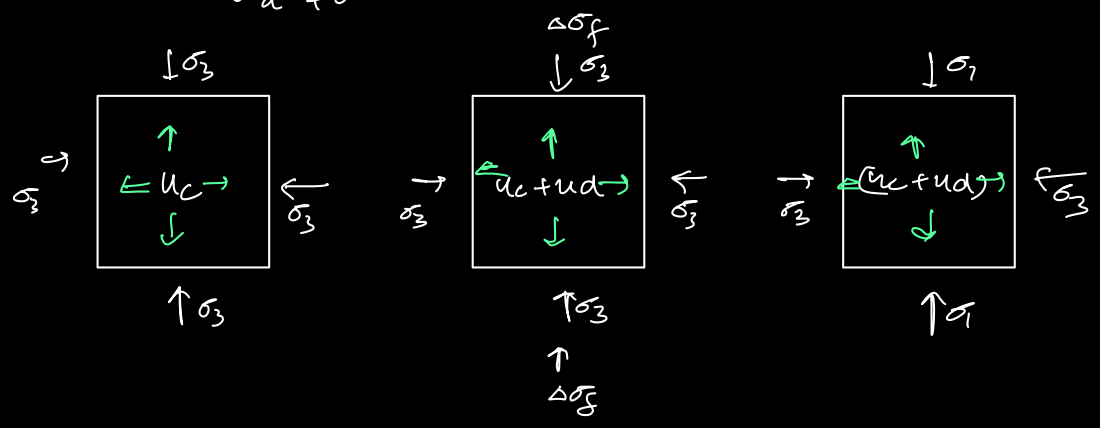
b) Apply and increase the deviatoric stress till shearing

c) Draw the plot between deviatoric stress and Axial strain & determine ($\Delta\sigma_f$)

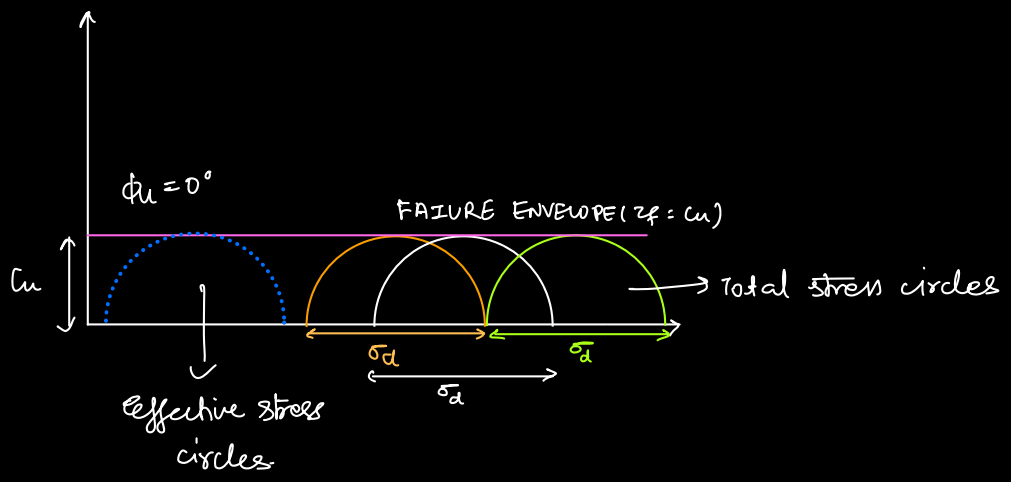
d) Excess pwp u_d is developed

e) ∵ drainage not allowed

∴ $u_d \neq 0$.



f) Now tests can be performed on the number of samples with increasing values of σ_3.



c_u : Undrain cohesion

→ Undrain shear strength can be given as

$\tau_f = c_u + \sigma_n \tan\phi_u$ ($\because \phi_u = 0$ for clay)

$$\boxed{\tau_f = c_u}$$

g) To calculate the deviatoric stress at the time of failure area of soil (A_f) at the time of failure should be used

∴ for drain analysis,

$$\sigma_{df} = \Delta\sigma_f = \frac{P}{A_f}$$

$$A_f = A_0 \left(\frac{1 \mp \varepsilon_v}{1 - \varepsilon_1} \right)$$

$\therefore$ for undrain analysis : $\varepsilon_v = 0$

$$\left\{ A_f = A_o \left(\frac{1}{1-\varepsilon_L} \right) \right\}$$

$\because \Delta V = 0$

(No expulsion of pore water)

$$A_f = \frac{A_o}{1 - \frac{\Delta L}{L_o}}$$

A_o: Initial cross sectional area

ΔL: change in length (settlement of soil)

$L_o \rightarrow$ orginal length or initial length

ε_L: Axial strain

Note: In the triaxial test, UD test is not performed because such condition is not encountered in the field.

(3) UNCONFINED COMPRESSION TEST:

It is the special test of triaxial machine in which sample is tested with zero confining pressure.

Only deviatoric stress (compressive force) is applied directly to shear the given soil sample. Therefore this test is termed as unconfined compression test.

1st Stage: Cell pressure stage:

$$\boxed{\sigma_3 = 0}$$

2nd stage: Shearing stage

a) drainage is not permitted.

b) Apply & increase the deviatoric stress till shearing. Note the corresponding axial strains

c) Since no drainage is allowed therefore undrained shear strength shall be determined.

Note: Test can be performed on the single soil sample.

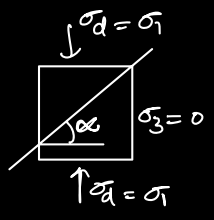
Note: In the place of UU triaxial test this unconfined compression test can be used to find undrained shear strength from simplicity point of view.

$$\because \quad \bar{\sigma_1} = \bar{\sigma_3} \tan^2\left(45 + \frac{\phi}{2}\right) + 2c \tan\left(45 + \frac{\phi}{2}\right)$$

$$\therefore \quad \sigma_1 = 2c \tan\left(45 + \frac{\phi}{2}\right)$$

$$\bar{\sigma_1} = 2c \tan\left(45 + \frac{\phi}{2}\right)$$

$$\because \quad \bar{\sigma_1} = \bar{\sigma_3} + \bar{\sigma_{df}}$$



$$\boxed{\bar{\sigma_{df}} = 2c_u \tan\left(45 + \frac{\phi}{2}\right)}$$

→ unconfined compressive strength of C-ϕ soil.

FOR frictionless soil ($\phi = 0$):

$$\bar{\sigma_{df}} = 2c_u \tan(45)$$

$$\boxed{\bar{\sigma_{df}} = 2c_u} \rightarrow \text{For saturated clay soils.}$$

Note: This test is suitable for frictionless soil like saturated clays and Non fissured clays

Note: Not suitable for sand & fissured clays.

(c_u: undrained cohesion)

$$\boxed{c_u = \frac{\bar{\sigma_{df}}}{2}}$$

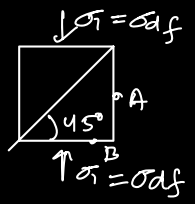
- undrained shear strength of clay soil can be given as:

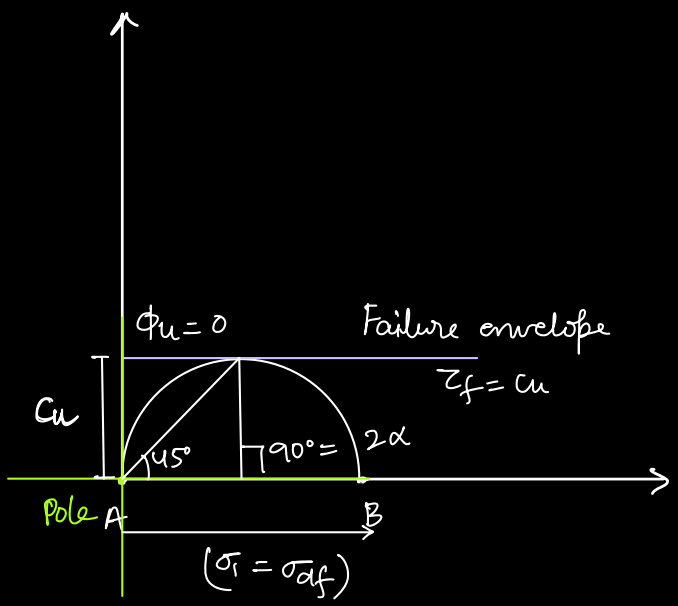
$$\tau_f = c_u + \bar{\sigma_n} \tan \cancel{\phi_u} \rightarrow 0.$$

$$\boxed{\tau_f = c_u}$$

- FOR CLAY SOIL, undrained shear strength or undrained cohesion is half of unconfined compressive strength

⊙ in case of clay soil





→ In unconfined compression test in clay soil, the failure takes place at the plane of max. shear stress

→ unconfined compression strength σ_{df} can be measured as,

$$\sigma_{df} = \frac{\text{Applied load at failure}}{\text{Area at failure}}$$

$$\sigma_{df} = \frac{P}{A_f} = \frac{P(1+\varepsilon_l)}{A_o}$$

$$\left\{ A_f = A_o \left(\frac{1}{1-\varepsilon} \right) \right\}$$

$$\sigma_{df} = \frac{P}{A_o}\left(1 - \frac{\Delta l}{L_o}\right)$$

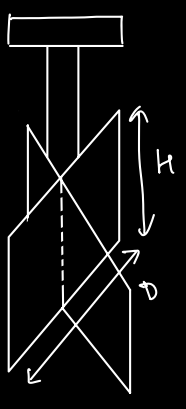
Note: This unconfined compressive strength of soil is used to determine sensitivity of the soil.

4 VANE SHEAR TEST :

a) This test can be used to determine the shear strength of soil in the field & lab

b) Only there is difference in size of the veins

VANES	LAB	FIELD
Height	20mm	10-20cm
Diameter	12mm	5-10cm
Thickness	0.5-1mm	2.5mm



(c) In this test there is no mechanism to control the drainage & porewater pressure cannot be measured

(d) This test is carried out under the undrained condition. therefore undrained shear strength is determined.

e) The equipments of vaneshear test is punched into the ground of soil and vanes are rotated at a rate equal to 6°/minute

f) Applied torque is measured at $T = K\theta$

{ K: Torsional stiffness
θ: Angle of twist. }

$$\theta = \frac{R_f - R_i}{180}$$

R_f: final dial gauge reading

R_i: Initial dial gauge reading.

(g) This test is suitable for soft saturated clay soils.

(h) This test is not suitable for dry soils (sands etc.

(i) Generally height to diameter ratio of the vanes is approximately 2

(j) There are two types of shearing to be observed in this test.

(i) Two way shearing:

when top and bottom of the vanes are below the ground surface. when shearing occurs at top, at side & bottom then it is termed as twoway shearing.

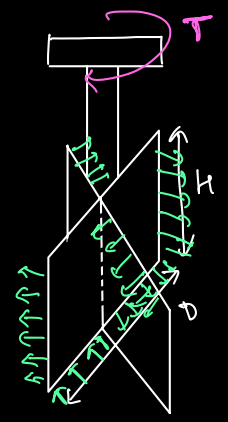
(ii) one way shearing:

when only bottom portion of the value is punched below the ground surface upto any depth H.

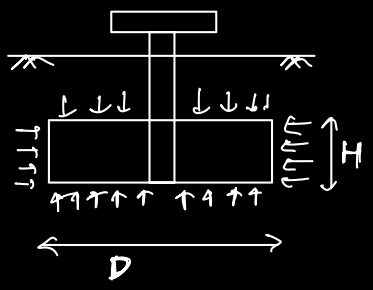
when shearing occurs at bottom and at sides (upto some height) but not at the top then it is termed as oneway shearing.

CONDITION-1: Assuming uniform shear stress distribution on the top, sides & bottom of the veins

$$T = T_{top} + T_{side} + T_{bottom}.$$

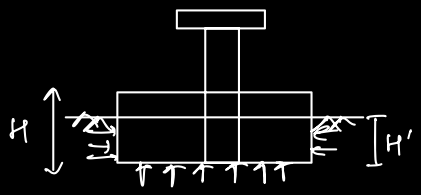


Case I- TWO WAY SHEARING:



$$\tau_f = \frac{T}{\pi D^2\left[\frac{H}{2}+\frac{D}{6}\right]}$$

Case II: ONE WAY SHEARING:



$$\tau_f = \frac{T}{\pi D^2\left[\frac{H'}{2}+\frac{D}{12}\right]}$$

∵ vane sheer test is suitable for soft saturated clay soils

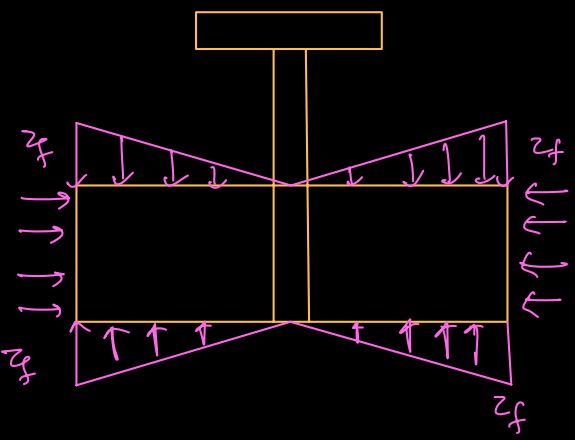
∴ $\tau_f = c_u + \sigma_n \tan\phi_u$ ($\phi_u \to 0$)

$\boxed{\tau_f = c_u}$ @ undrain sheer strength

$$\tau_f = c_u = \frac{T}{\pi D^2\left(\frac{H'}{2}+\frac{D}{12}\right)} \quad \text{(FOR ONE WAY)}$$

$$\tau_f = c_u = \frac{T}{\pi D^2\left(\frac{H}{2}+\frac{D}{6}\right)} \quad \text{(FOR TWO WAY)}$$

CONDITION 2: Linear distribution of Shear stress at top & bottom:



$(T = T_{top} + T_{side} + T_{bottom})$

1. Case (i): TWO WAY SHEAR

$$\tau_f = c_u = \frac{T}{\pi D^2\left[\frac{H}{2}+D/8\right]}$$

Case (ii): ONE WAY SHEAR

$$\tau_f = c_u = \frac{T}{\pi D^2\left[\frac{H'}{2}+\frac{D}{16}\right]}$$

Determination of sensitivity using vane shear test:

$$S_t = \frac{(UCS)\ \text{undisturbed soil}}{(UCS)\ \text{remoulded soil}} = \frac{(\sigma_{0f})\ \text{undisturbed soil}}{(\sigma_{0f})\ \text{remoulded soil}}$$

$$S_t = \frac{(C_u)_{undisturbed\ soil}}{(C_u)_{remoulded\ soil}}$$

Note: Undrained cohesion [Cu] can be determined in the undisturbed soil initially.

After determining Cu for undisturbed soil, the vanes are rotated rapidly to remould the soil, after remoulding we can perform the vane shear test and undrain cohesion Cu can be determined for Remoulded soil.

STRESS PATH:

(i) The locus of stress points is called stress path.

(ii) Stress path represents the state of stress and successive state of stress

(iii) Stress path need not to be a straight line.

It will often be required to represent the successive state of stress in a test specimen or an in situ element of soil during their loading & unloading

(iv) A series of Mohr circles can be drawn to serve this purpose. For simplification it can be represented with the help of stress path.

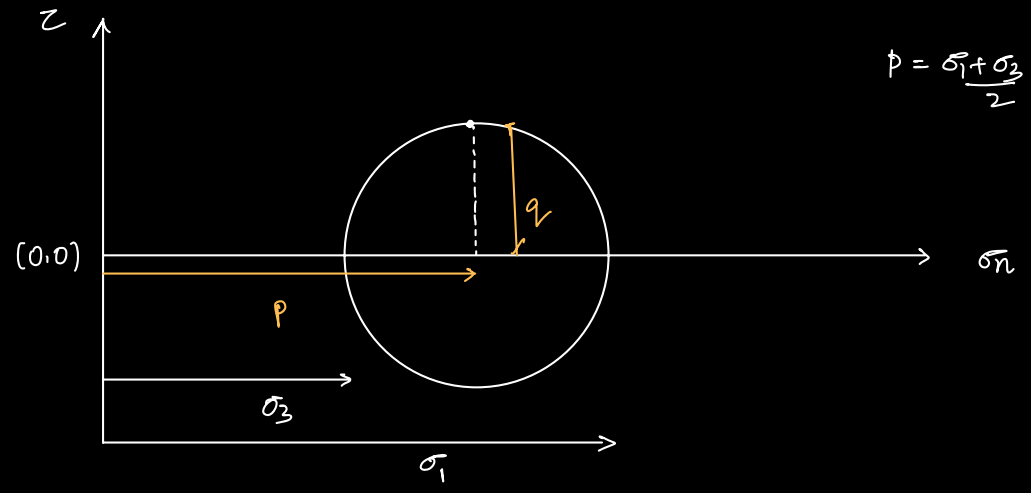
THREE possible stress paths can be drawn:

a) Effective stress path

b) Total stress path

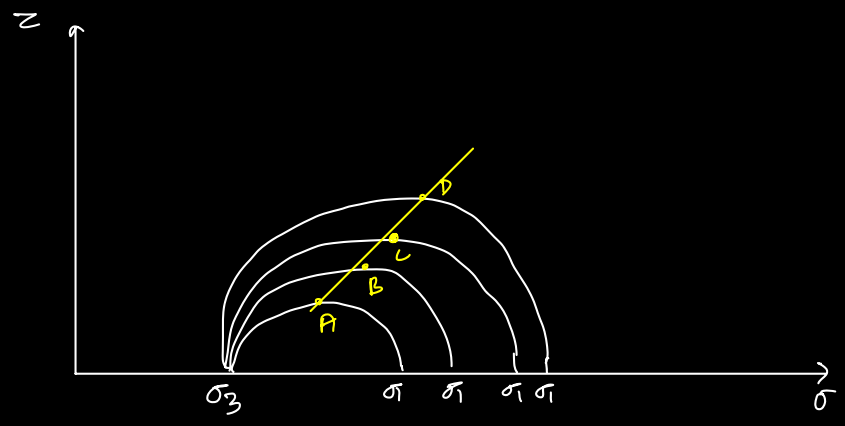
c) Stress path of total stress – static pore water pressure

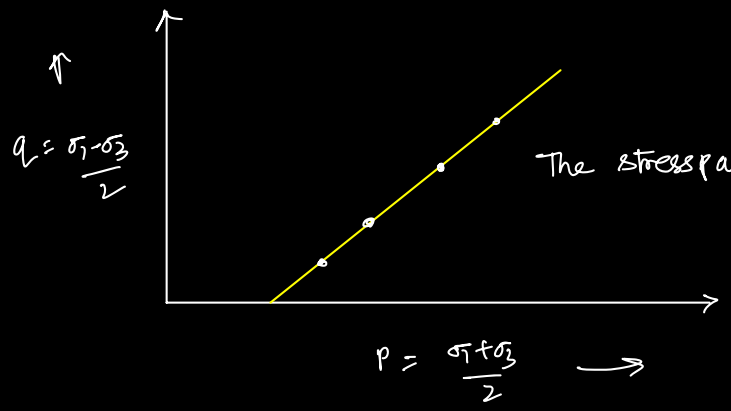
Note: Mohr's circle is drawn on $\sigma - \tau$ plot whereas stress path is drawn on $p-q$ plot.



$p = \frac{\sigma_1 + \sigma_3}{2}$ & $q = \frac{\sigma_1 - \sigma_3}{2}$

- Let's draw the series of Mohr circle for constant value of minor principle stress and successive values of Major principle stress





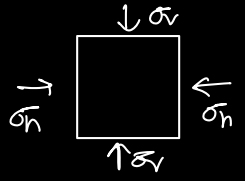
↳ The lines or the line connecting the points A, B, C, D are termed as stress path

↳ All the points A, B, C, D have the stress value in pq plot as that of σ - Z plot.

Let's draw some stress path:

- Let given soil sample is subjected to hydrostatic pressure (i.e)

$\sigma_V = \sigma_h$



σ_v : Major principle stress

σ_h : Minor principle stress

$$p = \frac{\sigma_v + \sigma_h}{2} = \frac{\sigma_v + \sigma_v}{2} = \sigma_v$$

$$q = \frac{\sigma_v - \sigma_h}{2} = 0.$$

- origin of stress path : $(\sigma_v, 0)$
- Now we can draw different stress path for different loading and unloading conditions

Case(1) : $\Delta\sigma_h = \Delta\sigma_v$

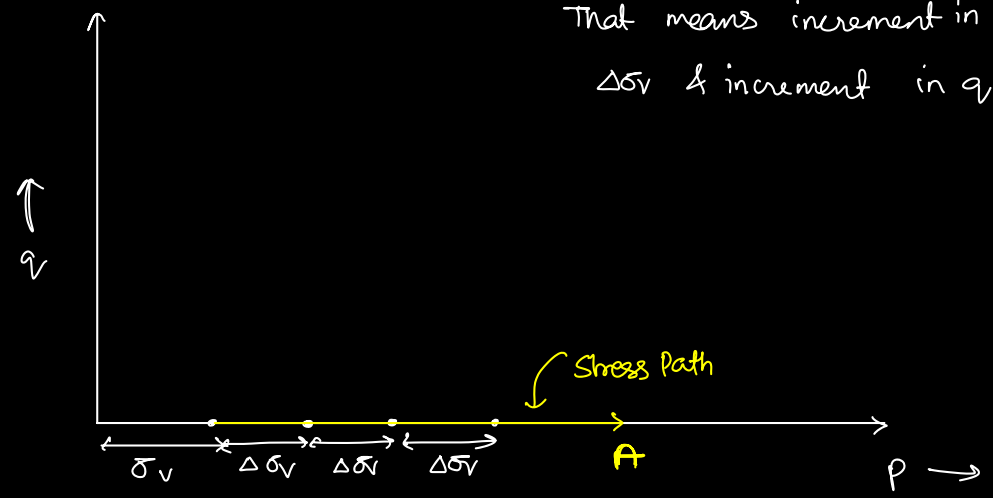
$$p = \frac{\sigma_v + \sigma_h}{2}$$

$$p' = \frac{(\sigma_v + \Delta\sigma_v) + (\sigma_h + \Delta\sigma_h)}{2} \Rightarrow \left(\frac{\sigma_v + \sigma_h}{2}\right) + \Delta\sigma_v$$

$$q' = \frac{\sigma_v + \sigma_h}{2} = \frac{(\sigma_v + \Delta\sigma_v) - (\sigma_h + \Delta\sigma_h)}{2}$$

$$= \frac{\sigma_v - \sigma_h}{2}$$

That means increment in p is $\Delta\sigma_v$ & increment in q is 0.



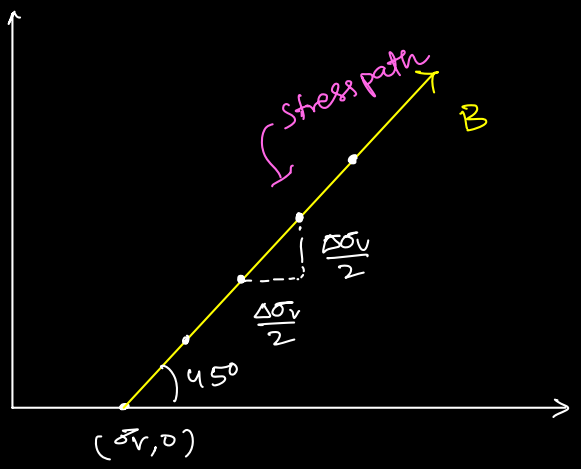
Case ii) : $\Delta\sigma_h = 0$ [σ_h is constant] & σ_v is increased by $\Delta\sigma_v$:

$$p = \frac{\sigma_v + \sigma_h}{2}$$

$$p' = \frac{\sigma_v + \Delta\sigma_h + \sigma_h}{2} = \frac{\sigma_v + \sigma_h}{2} + \frac{\Delta\sigma_v}{2}$$

$$q = \frac{\sigma_v - \sigma_h}{2}$$

$$q' = \frac{\sigma_v + \Delta\sigma_v - \sigma_h}{2} = \frac{\sigma_v - \sigma_h}{2} + \frac{\Delta\sigma_v}{2}$$



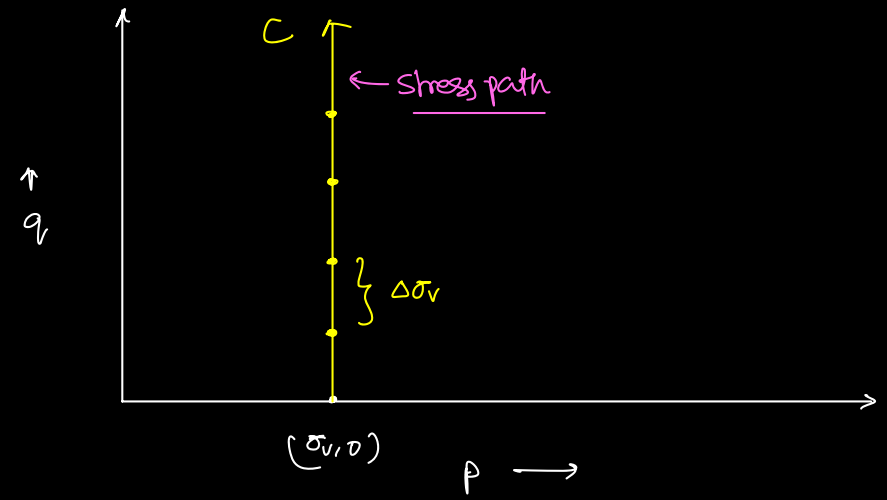
Case iii): $\Delta\sigma_h = -\Delta\sigma_v$

$$p = \frac{\sigma_v + \sigma_h}{2}$$

$$p' = \frac{(\sigma_v + \Delta\sigma_v) + (\sigma_h + \Delta\sigma_h)}{2} = \frac{\sigma_v + \sigma_h}{2}$$

$$q = \frac{\sigma_v - \sigma_h}{2}$$

$$q' = \frac{(\sigma_v + \Delta\sigma_v) - (\sigma_h + \Delta\sigma_h)}{2} = \frac{\sigma_v - \sigma_h}{2} + \Delta\sigma_v$$



Case iv): σ_h decreases & σ_v is constant:

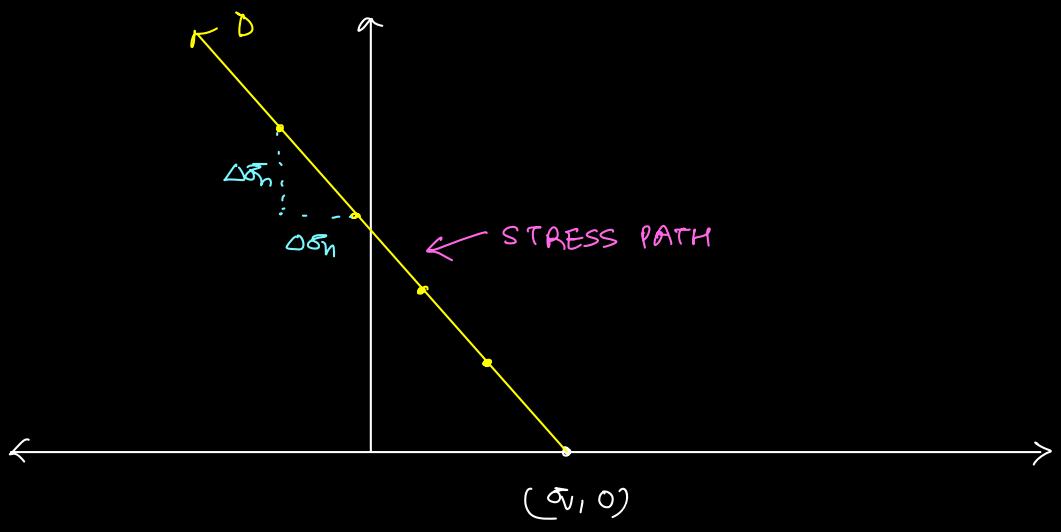
$(\Delta\sigma_h < 0)$, $(\Delta\sigma_v = 0)$

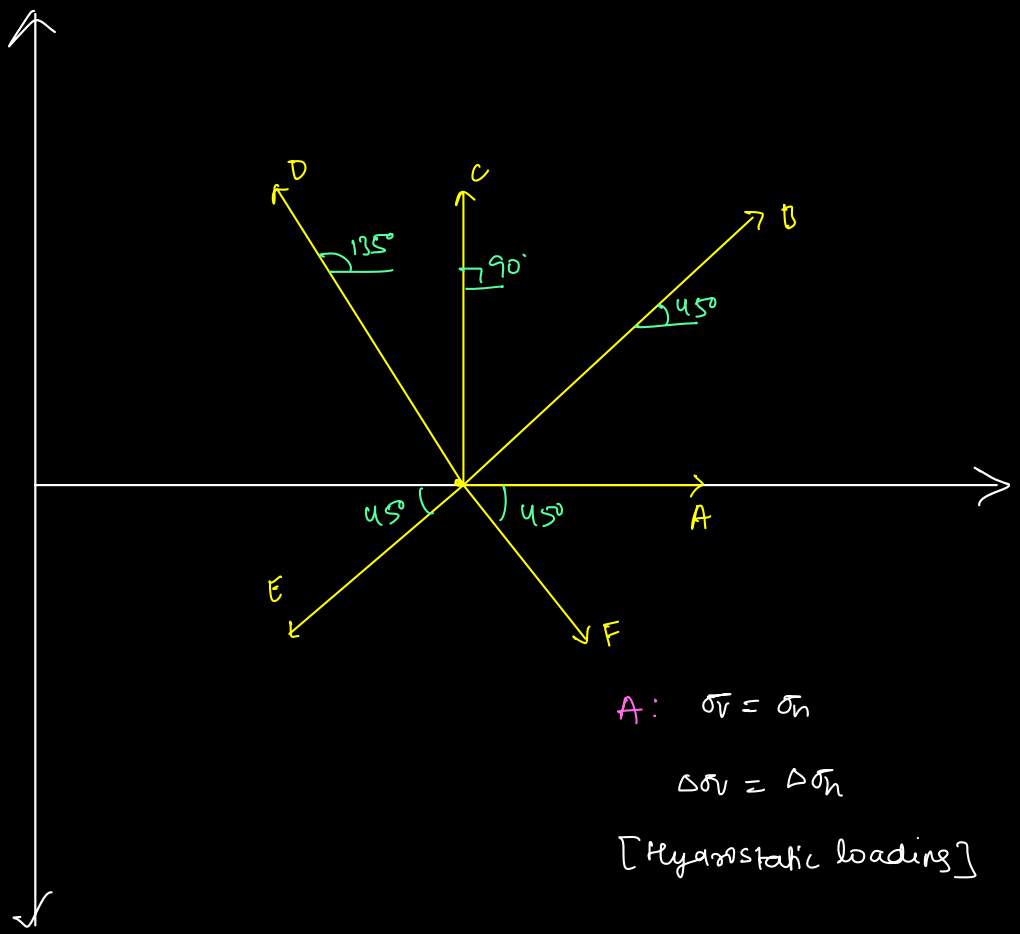
$$p = \frac{\sigma_v + \sigma_h}{2}$$

$$p' = \frac{(\sigma_v + 0) + [\sigma_h + (-\Delta\sigma_h)]}{2} = \frac{\sigma_v + \sigma_h}{2} - \frac{\Delta\sigma_h}{2}$$

$$q = \frac{\sigma_v - \sigma_h}{2}$$

$$q' = \frac{(\sigma_v + 0) - [\sigma_h - \Delta\sigma_h]}{2} = \frac{\sigma_v - \sigma_h}{2} + \Delta\sigma_h$$





A: $\sigma_v = \sigma_h$

$\Delta\sigma_v = \Delta\sigma_h$

[Hydrostatic loading]

B: σ_h is constant and σ_v increases
[Compression loading]

D: σ_h decreases & σ_v constant
[Compression unloading]

F: σ_h increases and σ_v constant
[Extension loading]

E: σ_h : constant
σ_v : decrease
extension unloading.

Note: The Mohr coulomb failure criteria can also be expressed in the forms of stress path equations

$$p \sin\phi + c \cdot \cos\phi = q$$

$$p = \frac{\sigma_1 + \sigma_3}{2}$$

$$q = \frac{\sigma_1 - \sigma_3}{2}$$

VERTICAL STRESS DISTRIBUTION

→ The final or total value of stress at any point below the ground surface can be given as:

Final stress = Stress due to self weight of soil + Stress due to surface loading

→ In this chapter stress due to surface loading shall be discussed.

The vertical stress distribution due to point load:

It can be Analyzed using boussinesque's theory or westergaard theory

Boussinesq's theory:

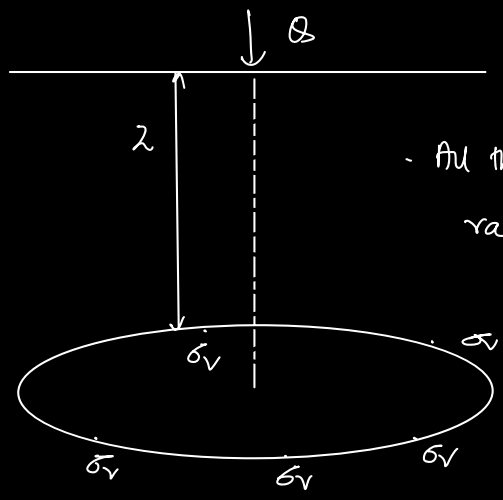
Assumptions:
1. Surface load is point load
2. Soil mass is homogeneous, isotropic and seminfinite
3. Soil mass is linearly elastic medium
[E & μ are constant]
4. Hooke's law is valid
5. Self wt of the soil is ignored

6. No change in volume of soil takes place due to application of loads.

7. Soil is initially unstressed

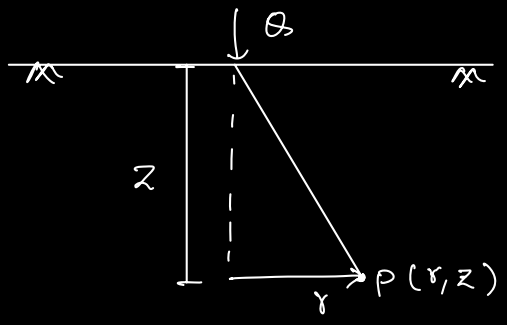
8. The surface of the soil is free from shear stress & subjected only to point load.

9. Distribution of the vertical stress is symmetrical about the vertical axis of the load line.



- All the points at same radial distance.

10. The vertical stress at any point 'P' due to the applied point load "Q" can be given as



$$\sigma_z = K_B \cdot \frac{Q}{z^2}$$

Q: Point load

z: depth

K_B: Boussinesque's influence factor

$$K_B = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

$$\therefore \quad \sigma_z = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

r: Radial distance of given point P from the given load Q.

↳ The vertical stress under the point load ($r = 0$) is given as

$$\left\{ \sigma_z = \frac{3}{2\pi} \cdot \frac{Q}{z^2} \right\}$$

$$\boxed{\sigma_z = 0.477 \cdot \frac{Q}{z^2}}$$

Note: Since vertical stress at any point P is given as

$$\sigma_z = \frac{3}{2\pi} \cdot \frac{Q}{z^2} \cdot \left[\frac{1}{1+\left(\frac{r}{z}\right)^2} \right]^{5/2}$$

↳ The whole equation is independent of elastic constant μ & E, which justify the linearly elastic medium of the soil.

↳ That means Boussinesque's theory is applicable for Non-Stratified soil deposits

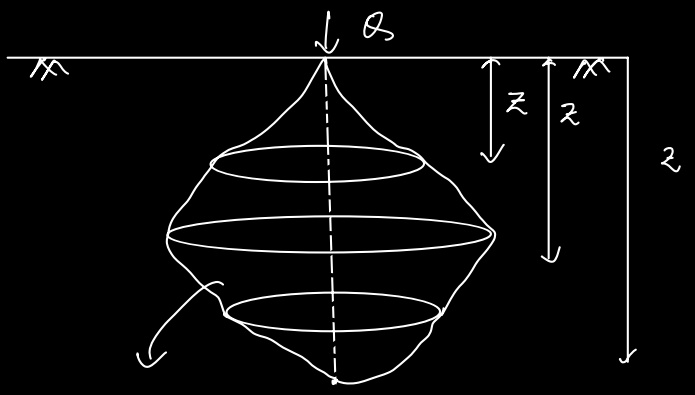
Pressure Distribution Diagram:

Using Boussinesque's theory, following type of pressure distribution diagrams can be drawn

(i) Pressure bulb / stress Isobar
(ii) vertical pressure distribution on horizontal plane
(iii) vertical pressure distribution on the vertical plane

(i) Pressure Bulb / Stress Isobar:

- An isobar is a curve or contour connecting all the points below the ground surface of equal vertical pressure
- An isobar is a spatial, curved surface of the shape of a bulb, because the vertical stress on a given horizontal plane is same in all directions of the points located at equal radial distances around the axis of loading



Pressure bulb
or stress isobar.

$$\sigma_z = K_B \cdot \frac{Q}{z^2}$$

$$K_B = \frac{\sigma_z \cdot z^2}{Q}$$

Let's draw the pressure bulb for $\sigma_z = 0.25Q$ per unit area

$$K_B = \frac{(0.25)Q \cdot z^2}{Q} = 0.25 \cdot z^2$$

↳ for different values of z, K_B can be determined.

↳ using following equation $\frac{r}{z}$ can be determined

$$K_B = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

z	$K_B = 0.25z^2$	r/z	r
0.2	✓ .01	1.92	0.384
0.4	✓ .04	1.30	0.52
0.6	✓	⋮	⋮
⋮	✓	1	⋮

The depth at which the isobar $\sigma_z = 0.25Q$ the axis of loading is calculated first by finding K_B with $r=0$ value.

$$0.25Q = 0.477 \cdot \frac{Q}{z^2}$$

$$0.477 = z^2 (0.25)$$

$$\{z = 1.38 \text{ unit}\}$$

Note: larger size pressure bulb connects the points of lower vertical stress

Smaller sized pressure bulbs connects points of higher vertical stress.

Question

At what depth stress isobar of $\sigma_z = 100 \text{ kN/m}^2$ will cross line of action of load?

$$Q = 1000 \text{ kN}.$$

$\sigma_z = 0.1 \cdot Q$ per unit area

$$0.1 \cdot Q = 0.477 \cdot \frac{Q}{z^2}$$

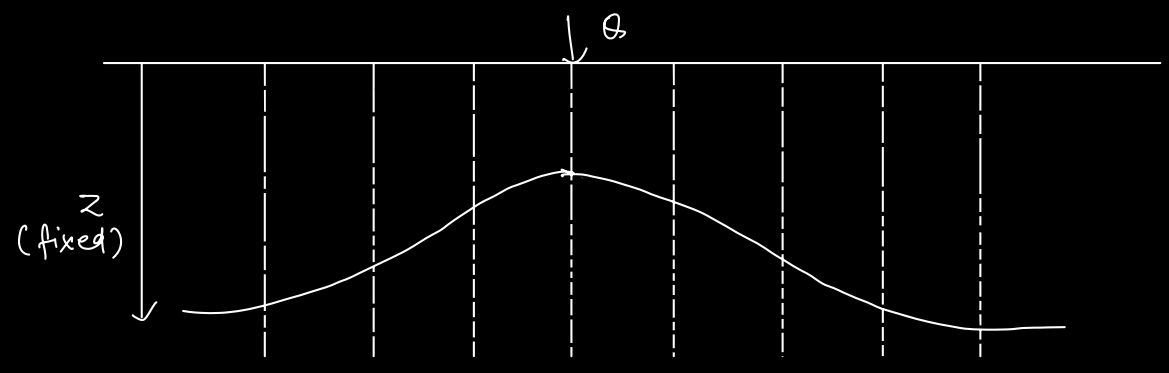
$$z^2 = 4.77$$

$$\boxed{z = \sqrt{4.77} = 2.18}$$

(ii) Vertical pressure distribution on the Horizontal plane at a constant depth z:

$$\sigma_z = K_B \cdot \frac{Q}{z^2} \quad (z \text{ is fixed})$$

r/z	$K_B = \frac{3}{2\pi}\left[\frac{1}{1+(\frac{r}{z})^2}\right]^{5/2}$	$Q = K_B \cdot \frac{Q}{z^2}$	
0	0.477	$0.477 \frac{Q}{z^2}$	$\sigma_{z\max}$
0.5	0.2733	$0.2783\ Q/z^2$	58% of $\sigma_{z\max}$
1.0	0.0844	"	17.7%
2.0	0.0085	"	1.8%



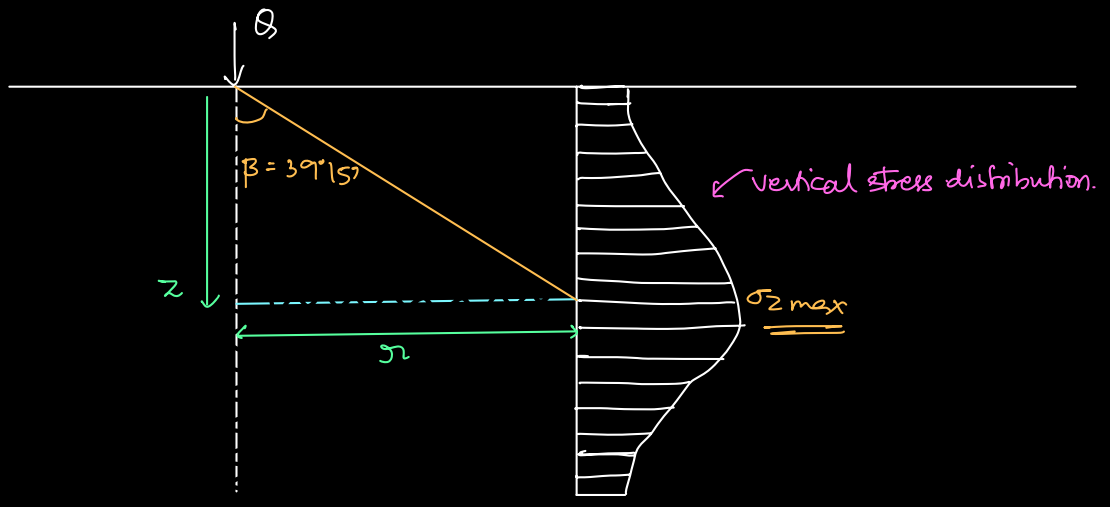
→ from the above table and drawing, it can be concluded that at a given depth when horizontal radial distance is equal to twice the depth, the vertical pressure due to concentrated load can be considered negligible

→ The above diagram of vertical stress distribution shows that as we move away from the load in radial direction the influence of load decreases

(iii) Vertical stress distribution on the vertical plane at a constant radial distance r.

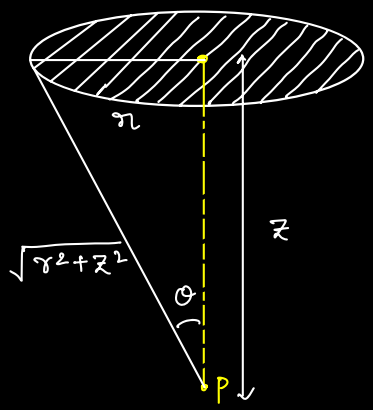
- In this case the vertical stress distribution is found on the vertical plane taken at a constant radial distance "r" with different values of z.
- It is observed that the vertical stress first increases and attains a maximum value and then decreases.

z	r/z	$K_B = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$	$\sigma_z = K_B \cdot \frac{Q}{z^2}$
–	–		
–	–		
–	–		



Vertical Pressure / Stress distribution due to uniformly loaded circular Area :

- This is the extension of boussinesques theory for uniform pressure acting on the circular area
- Assumptions: · Soil mass is homogeneous, isotropic, semi-infinite, linearly elastic and Hookes law is valid.
  - Suitable for Non-stratified soil deposits



r: radius of circular area

z: depth of the point of interest exactly below the centre of the circle

The vertical stress can be given as

$$\sigma_z = K_B \cdot q \quad kN/m^2$$

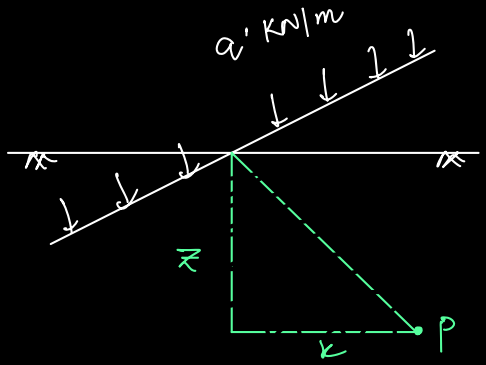
$$K_B = [1 - \cos^2\theta]$$

$$\cos\theta = \frac{z}{\sqrt{r^2+z^2}}$$

Boussinesques influence factor. ← $$K_B = \left[1 - \frac{z^2}{(r^2+z^2)^{3/2}}\right]$$

$$\sigma_z = \left[1 - \frac{z^2}{(r^2+z^2)^{3/2}}\right] \cdot q$$

Vertical stress distribution due to line load:



The vertical stress due to infinitely long line load at a point which is present at a horizontal distance x and vertical distance z can be given as,

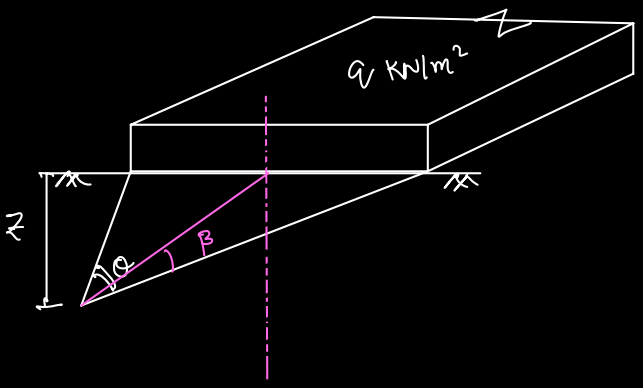
$$\sigma_z = \frac{2q'}{\pi z}\left[\frac{1}{1+\left(\frac{x}{z}\right)^2}\right]^2$$

when the point load is present below the line load depth = z &

$x = 0$

$$\left\{ \sigma_z = \frac{2q'}{\pi z} \ KN/m^2 \right\} = 0.637 \frac{q'}{\pi z}$$

vertical stress distribution due to strip loading:



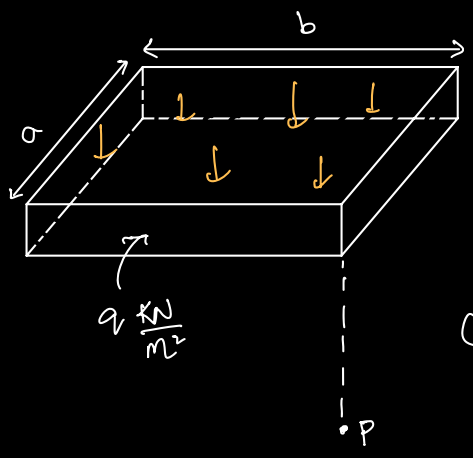
The vertical stress due to ∞ strip of width B loaded with uniformly distributed load intensity q per unit are:

$$(\sigma_z)_q = \frac{q}{\pi} \left(\theta + \sin\theta \cdot \cos 2\beta \right)$$

- if the point P is located at a depth z with a line passing through centre of the strip then $\beta = 0$

$$\sigma_{z_p} = \frac{q}{\pi} (\theta + \sin\theta) \ KN/m^2$$

Vertical stress below the corner of the uniformly loaded rectangular area.



$$(\sigma_z)_p = K \cdot q$$

K: influence factor

$$(\sigma_z)_p = \frac{q}{4\pi} \left[\frac{2mn\,(m^2+n^2+1)^{1/2}}{m^2+n^2+m^2n^2+1} \cdot \frac{(m^2+n^2+2)}{m^2+n^2+1} + \tan^{-1} \left(\frac{2mn\,(m^2+n^2+1)^{1/2}}{m^2+n^2-m^2n^2+1} \right) \right]$$

$$m = \frac{a}{z} \ , \ n = \frac{b}{z}$$

Note: 'm' and 'n' are interchangeable

Note: In the above expression the 2nd term of $\tan^{-1}$ shall be

Calculated in radians.

Question:

Calculate intensity of the vertical stress induced at a point 2m vertically below the corner of a square footing of 2m side trans. a bearing pressure of $12\ t/m^2$ to the surface of the soil deposites?

Solution.

$a = b = 2m$

$z = 2m$

$m = n = 1$

$m = \frac{a}{z} = 1$

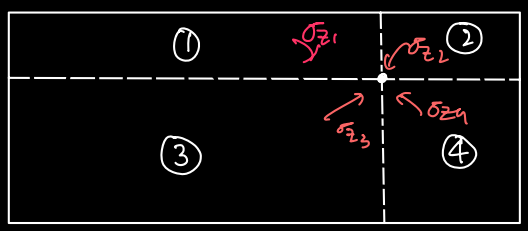
$n = \frac{b}{z} = 1$

$$\sigma_z = \frac{q}{4\pi}\left[\frac{2mn\,(m^2+n^2+1)^{1/2}}{m^2+n^2+m^2n^2+1}\cdot\frac{m^2+n^2+2}{m^2+n^2+1} + \tan^{-1}\left(\frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2-m^2n^2+1}\right)\right]$$

$$\sigma_z = \frac{q}{4\pi}\left[\frac{2\sqrt{3}}{4}\cdot\frac{4}{3} + \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right)\right]$$

$$\sigma_z = 2.1027\ t/m^2$$

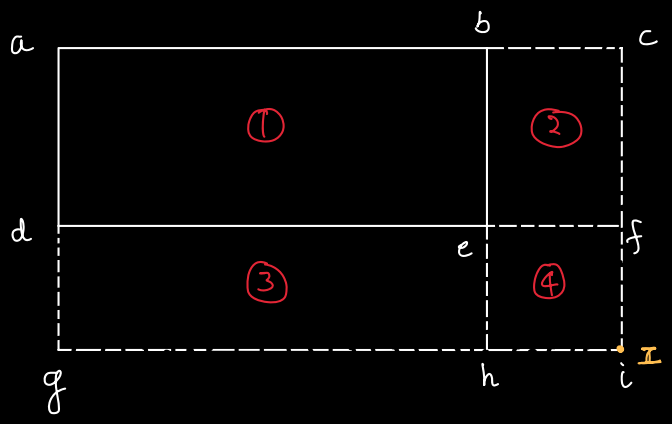
Note①: To determine the vertical stress inside the rectangular loaded area but not below the corner, the given area is divided into the four rectangles so that the point 'P' is made the common corner of each rectangle. The vertical stress due to each rectangle is determined at the point P and sum of all four values can be expressed as the final vertical stress at given point P.



$$(\sigma_z)_p = \sigma_{z1} + \sigma_{z2} + \sigma_{z3} + \sigma_{z4}$$

Note②: When the point is at some depth z but not below the corner of the rectangular loaded area it is outside the uniformly loaded rectangular area then area of rectangle can be extended

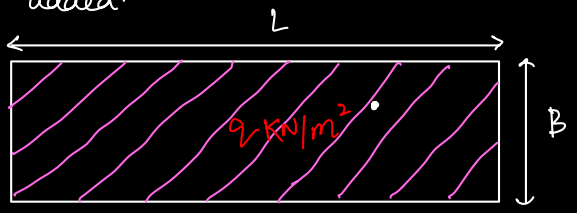
upto the given points and stress can be computed by adding & subtracting stresses

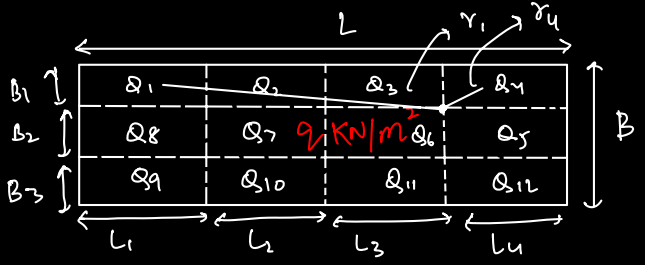


$$(\sigma_z)_I = (\sigma_z)_{acig} + (\sigma_z)_{efhi} - (\sigma_z)_{bcih} - (\sigma_z)_{dgfi}$$

Approximate method for vertical stress distribution:

(i) Equivalent point load method: In this method the loaded area is divided into a number of small area units and the total pressure present over the small area units are replaced by an equivalent point load acting at CG of the area units. The contribution of the point load is calculated and all are added.





Generally we can calculate

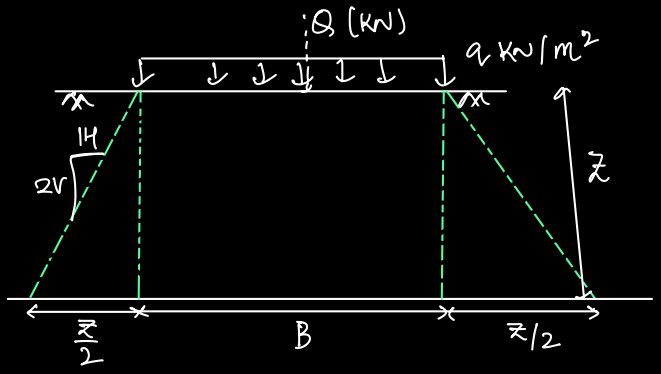
$$(\sigma_z)_p = \sigma_{z_1} + \sigma_{z_2} + \cdots + \sigma_{z_{12}}$$

$$(\sigma_z)_p = K_{B_1}\frac{Q_1}{z^2} + K_{B_2}\frac{Q_2}{z^2} + K_{B_3}\frac{Q_3}{z^2} + \cdots K_{B_{12}}\frac{Q_{12}}{z^2}$$

$$\left\{\begin{array}{l} Q_1 = q \cdot B_1 L_1 \\ Q_2 = q \cdot B_1 L_2 \\ \vdots \qquad\quad \vdots \end{array}\right\}$$

(ii) Two to one method:

- The surface loading can be distributed over an area of same shape as the loaded area on the surface
- The load is distributed with the slope 2V : 1H method
- First pressure (q) is converted into the point load.



$$\sigma_z = \frac{Q}{(L+z)(B+z)} = \frac{\text{Point load}}{\text{Area of dispersion.}}$$

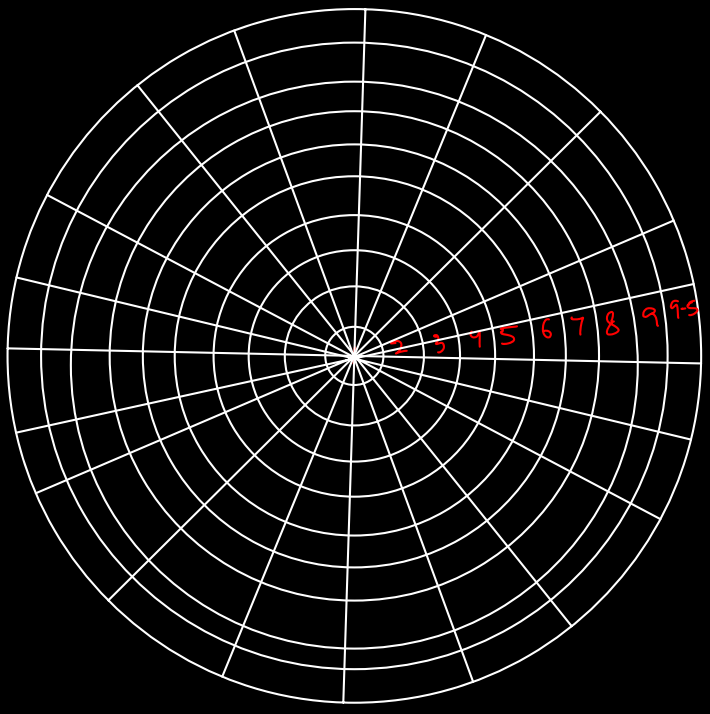
$$= \frac{qBL}{(L+z)(B+z)}$$

Newmark's Influence chart method:

- Newmark's influence chart method can be used to estimate vertical stress at any point inside or outside the uniformly loaded area of any shape
- In this method soil is assumed to be homogeneous, isotropic, semi-infinite and linearly elastic that means this method can't be used for stratified soil deposits
- In this method 'n' number of concentric circles and "m" number of radial lines are drawn. Therefore total number of segments or small area units will be mxn.
- The vertical stress of the centre of the circles (or centre of the chart) by each segment will be same or equal
- That means each segment has equal at the point which is located exactly below the centre of the circle which can be termed as Influence factor (If).

$$\left\{ I_f = \frac{1}{m \times n} \right\}$$

- In the Newmark's chart method the influence factor has be considered as 0.005.



$I_f = 0.05$

- The Newmark's chart has been drawn with the scale Z = 5cm (distance AB) mentioned on the chart
  The length AB represent the depth Z.
- Now draw the plot of the loaded area with scale Z = 5cm or whatever is given on the tracing paper
- Keep the tracing paper on the Newmark's chart such as the point of interest (where σ_z is required to be calculated at any depth z coincides with centre of the influence chart
- Count the number of segments [including the fractions also] on the influence chart covered by loaded area as "N".
- The vertical stress at any point of interest can be given as

$$\boxed{(\sigma_z)_p = I_f \times N \times q}$$

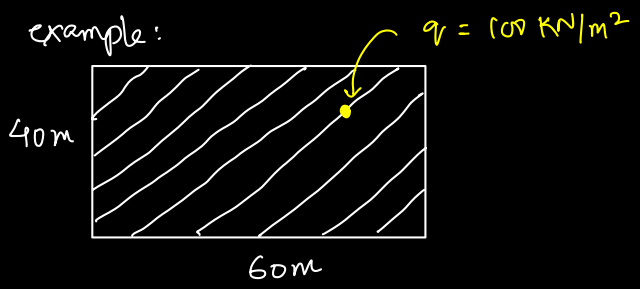
I_f: Influence factor ($I_f = \frac{1}{m \times n}$)

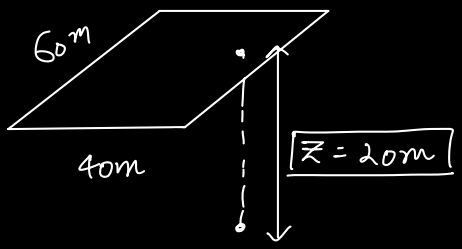
N: Number of segments covered by loaded area on influence chart

q: Applied pressure on the loaded area

$$\sigma_z = 0.0059 q \cdot N$$

For example:



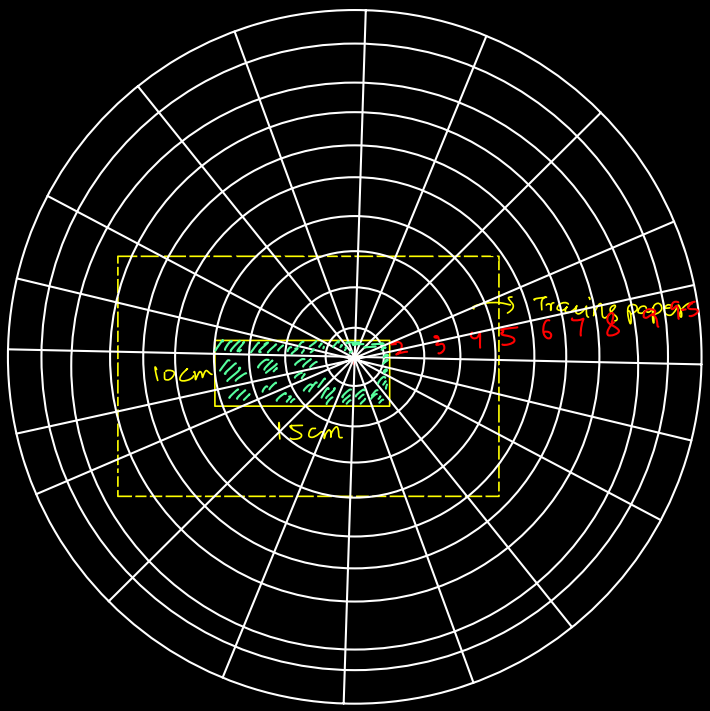


∵ In the given newmark's chart

distance AB represents the depth Z

20m = 5cm

4m = 1cm : scale



Total segments covered

: 35.5 segments

$\sigma_z = 0.005 \times q \times N$

$\sigma_z = 0.005 \times 100 \times 35.5$

$\sigma_z = 17.75 \ KN/m^2$

Note: The 10th Number circle cannot be drawn because it has ∞ radius but 9½ circle can be drawn.

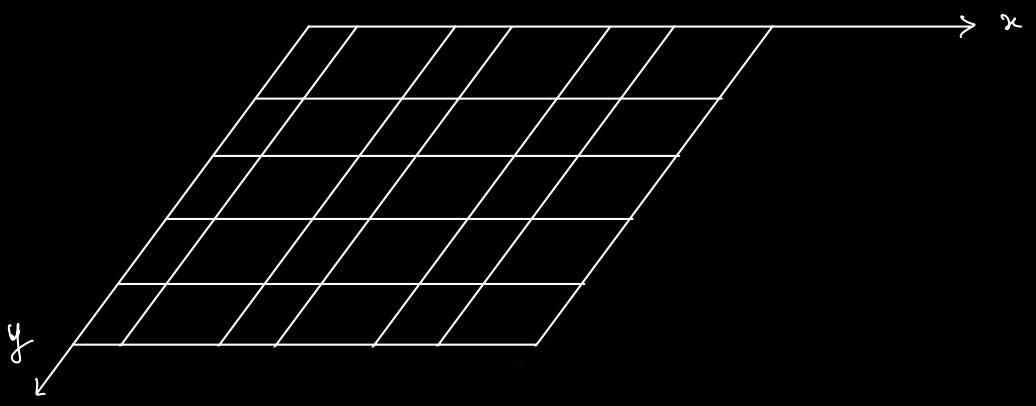
WESTERGAARD'S THEORY

- The vertical stress Theories discussed earlier are all based on boussinesque's vertical stress equation which assumes soil to be isotropic But in actual soils are non isotropic & found in stratified condition. For such soils or conditions westergaard Analysis is best suitable.

Assumptions: 1. Soil is homogeneous, semi-infinite & Non-isotropic (Anisotropic)

(That means these theory can be used for Non-stratified soil deposits)

2. Soil is assumed to be an elastic medium of semi-∞ extent but containing numerous closely spaced horizontal sheets of negligible thickness of infinitely rigid material which permits only "downward deformation" on the mass as a whole without allowing it to undergo any lateral strain.

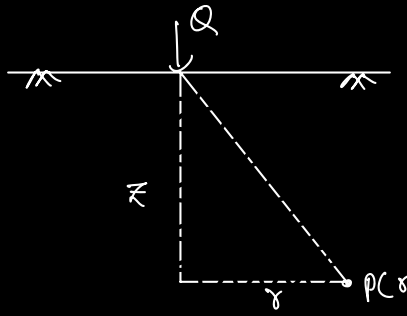


$$\underbrace{\varepsilon_x = 0 \;,\; \varepsilon_y = 0,}_{\text{(lateral strain)}} \quad \underbrace{\varepsilon_z \neq 0}_{\text{(longitudinal strain)}}$$

Note: In general poisson's ratio of elastic material varies from 0 to 0.5. But in this case lateral restraints are assumed to be acting in the soil.

$$\therefore \mu = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\varepsilon_x \text{ or } \varepsilon_y}{\varepsilon_z} = 0$$

for zero value of poisson's ratio ($\mu = 0$) the vertical stress at any point below depth 'z' due to point load 'Q' can be given as:



$$\sigma_z = K_w \cdot \frac{Q}{z^2} \quad KN/m^2$$

K_w: Westergaard's influence factor.

$$K_W : \quad \frac{1}{\pi}\left[\frac{1}{1+2\left(\frac{r}{z}\right)^2}\right]^{3/2}$$

$$\sigma_z = \frac{1}{\pi}\cdot\frac{Q}{2}\left[\frac{1}{1+2\left(\frac{r}{z}\right)^2}\right]^{3/2}$$

if $r=0$, (under the load)

$$\sigma_z = 0.318\,\frac{Q}{z^2}$$

for $r=0$, $\left[(\sigma_z)_B = 0.477\,\frac{Q}{z^2}\right] > \left[(\sigma_z)_W = 0.318\,\frac{Q}{z^2}\right]$

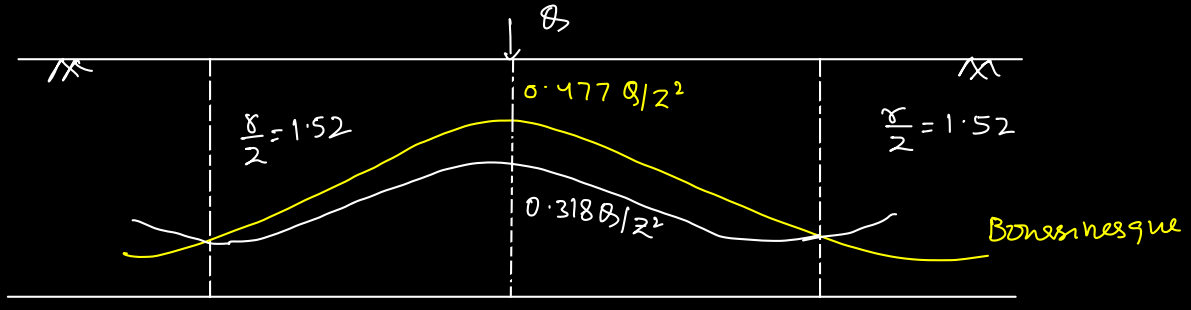
— Determining the value of $\frac{r}{z}$ at which $(\sigma_z)_B = (\sigma_z)_W$

$$(\sigma_z)_B = (\sigma_z)_W$$

$$K_B = K_W$$

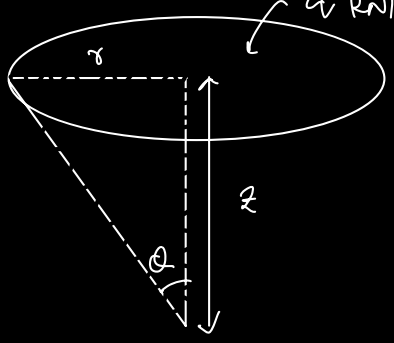
$$\frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} = \frac{1}{\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2\cdot 2}\right]^{3/2}$$

$$\frac{r}{z} : 1.52$$



Note: The soil is actually found in Anisotropic condition in the field hence the results obtained by westergaard theory are more accurate & closer to the actual results But then also we use boussinesques theory because it gives more conservative results which makes the structure more safe.

Vertical stress distribution for uniformly loaded circular area using Westergaard's Analysis :



$(\sigma_z)_p = K_w \cdot q$

r: radius of the circular area

z: depth of point of interest (exactly below centre of the circle)

$$\sigma_z = q\left[1 - \left[\frac{1}{1+2\left(\frac{r}{z}\right)^2}\right]^{\frac{1}{2}}\right]$$

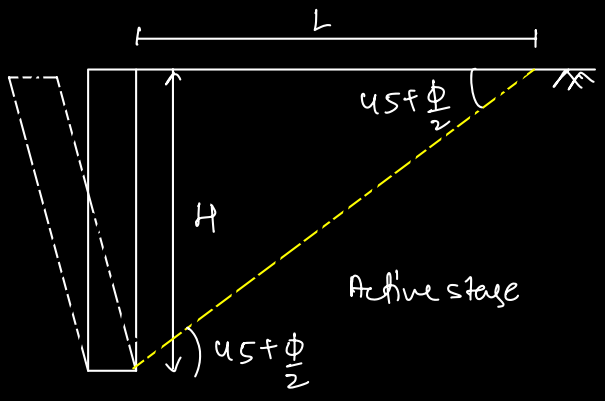
This equation used for Non-isotropic soils

[suitable for stratified soil deposites]

#LATERAL EARTH PRESSURE ANALYSIS

Notes pdf given by sir!!

downloaded in laptop!!

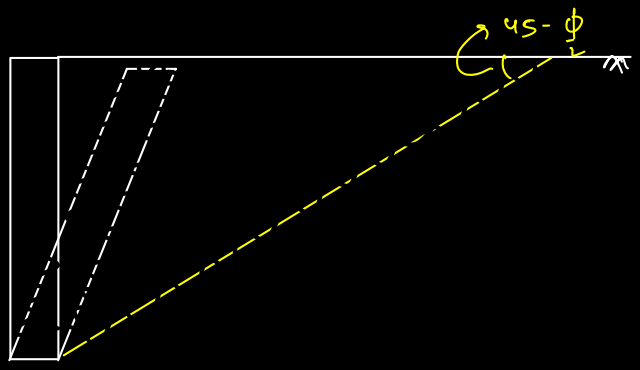


$\left[\begin{array}{l} \varepsilon = 0.5\% \\ \theta = 0.25\% \end{array}\right\}$ Loose sand, $\Delta = 0.0025H$.

$\tan\left(45+\frac{\Phi}{2}\right) = \frac{H}{L}$

$L = H \cot\left(45+\frac{\Phi}{2}\right)$

$L \simeq \frac{H}{2}$



Rankine: cohesionless soil:

FOUNDATION ENGINEERING

Difference between the foundation and footing:

The foundation is the substructure which transmits the load of the superstructure to the earth (soil), whereas footing is the part of the foundation which is in contact with the soil and that transmits the load / pressure over the large area

Note: All the footings can be foundations but all the foundations can not be footings. They are provided for shallow depth.

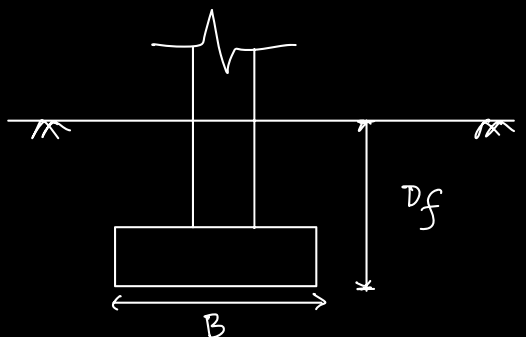
Note: A foundation is a substructure which transmits the structural load to the earth in such a way that the supporting soil is not overstressed & not undergo deformation that would cause excessive settlement of the structure.

The various types of structural foundation can be broadly classified into two groups:

1) SHALLOW FOUNDATION
2) DEEP FOUNDATION

1. Shallow foundation: As per terzaghi definition, the given foundation is termed to be shallow foundation if

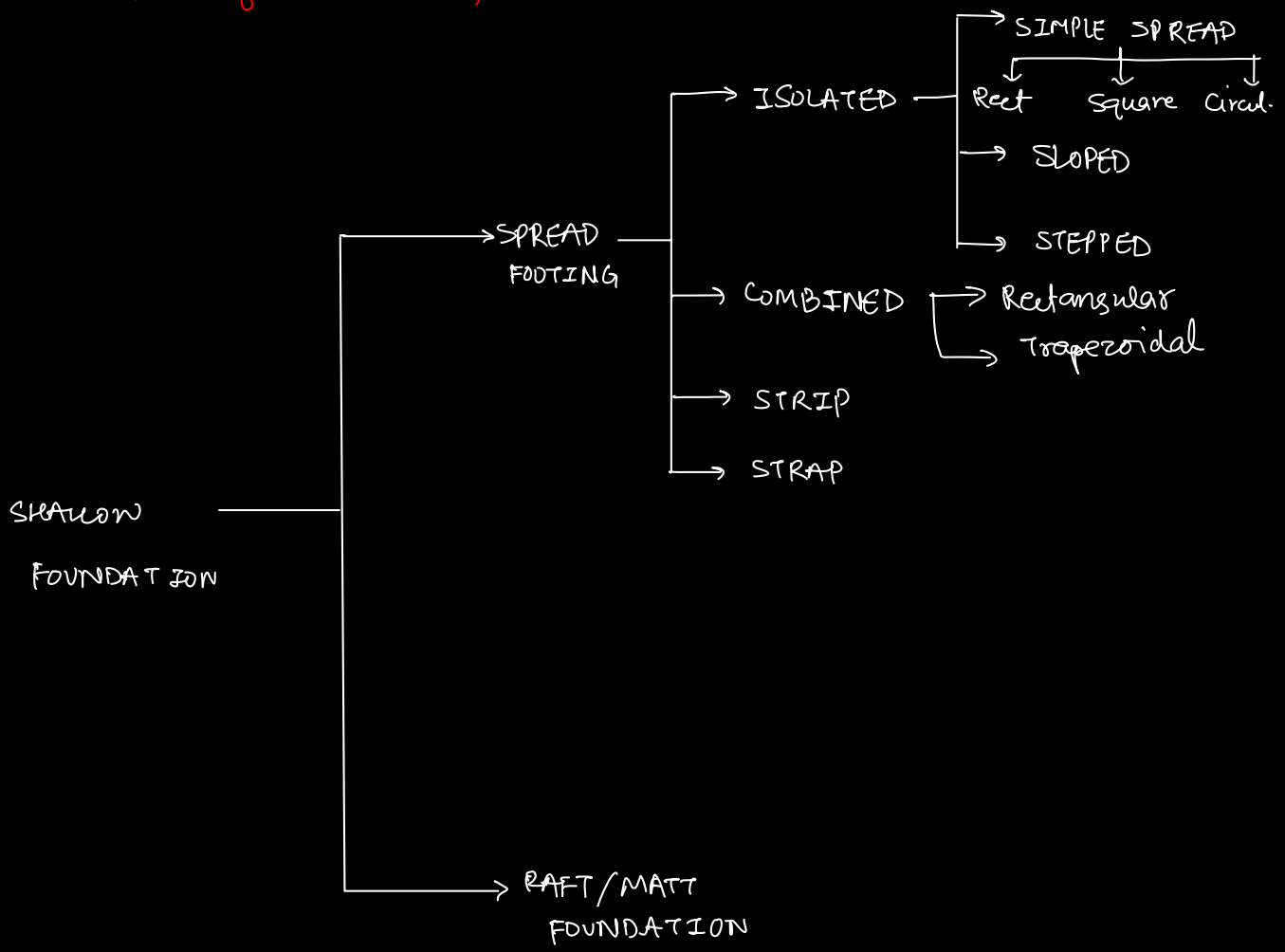
$\frac{D_f}{B} < 1.$



→ as per terzaghi if $\frac{D_f}{B} \leq 1.0$ @ shallow foundation

→ as per Skempton if $\frac{D_f}{B} \leq 2.5$ @ shallow foundation

Types of shallow foundation :



2. **Deep foundation** : The foundation for which depth of foundation is greater than width/ size of foundation is termed as deep foundation.

If $1 < \frac{Df}{B} < 15$: Moderately deep foundation

If $\frac{Df}{B} > 15$: Deep foundation (such as pile)

Types of deep foundation :
- Pile foundation
- Caisson / well foundation.

Note: Shallow foundations are constructed in open excavations & disturbance of soil is minimum.

whereas in case of deep foundation it is impossible to visually inspect the construction.

The disturbance of soil also extends for larger zone all along the depth of deep foundation.

For reasons of economy, shallow foundations are the first choice.

General requirements of foundation:

For satisfactory performance of foundation, following key criteria must be satisfied

1. Location & depth criteria
2. Shear feilure criteria
3. Settlement based criteria

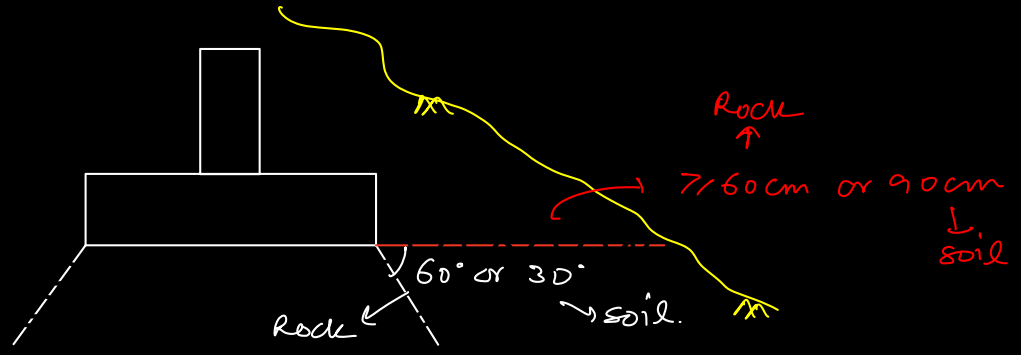
1. Location & depth criteria:

a.) Any foundation should be placed at a depth where soil stratum is adequate from the point of bearing capacity (shear feilure criteria) & settlement based criterion.

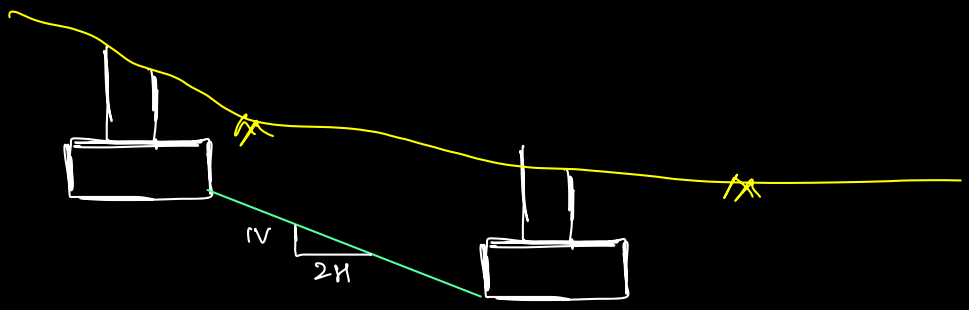
b) As per IS 1904-1976 the minimum depth of foundation should not be less than 50cm below the natural ground level.

c) The foundation must be placed below the zone of black cotton soil, volume change, swelling & shrinkage & frost heaving.

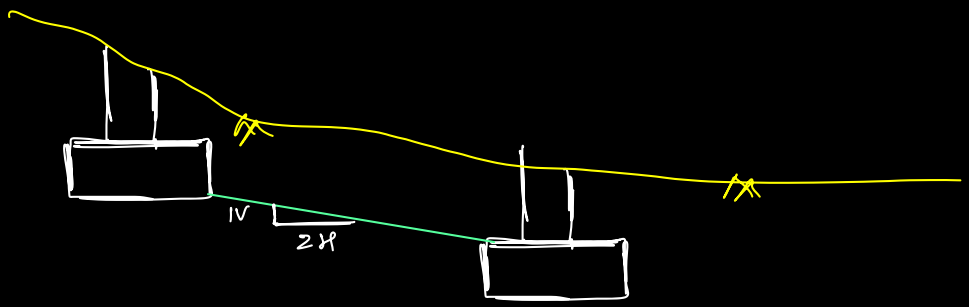
d) when the ground surface slopes downward adjacent to a footing, sloping should not encroach on the frustum of bearing material under the footing having sides which makes and an angle of 60° with horizontal for rock & 30° for soil & the horizontal distance from the lower edge of footing to the sloping surface should be atleast 60cm for rock & 90cm for footing.



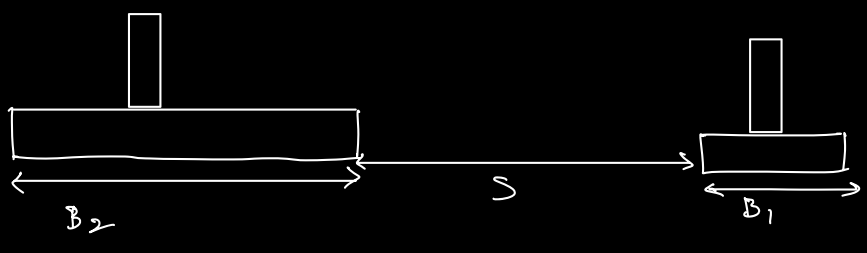
e) For footings in regular soil, line joining the lower adjacent edges of adjacent footing should not have a slope steeper than two horizontal & one vertical.



f) In clayey soils, the slope of lines joining the lower adjacent edge of the lower adjacent edge of the upper footing and upper adjacent edge of the lower should not be steeper than two horizontal to one vertical



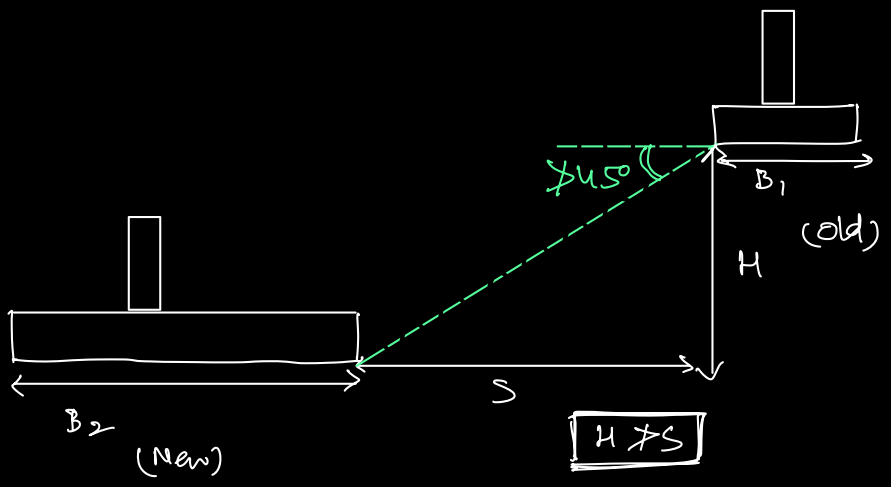
g) To avoid the damages to an existing structure, the foundation for a new structure at an adjacent site should be located atleast at a distance "S" from the edge of the existing footing whereas "S" is the width of larger footing.



$S = \max(B_1, B_2)$

↳ (minimum spacing.)

h) The line from the edge of the new footing to the edge of the existing footing should make an angle of 45° or less with the horizontal plane, that is the difference between elevation of footings must not exceed the distance "S."



i) Excavation must not, be too close to the existing footing because the existing structure may be endangered due to lateral flow of soil from beneath the existing footing.

Note: The maximum pressure carrying capacity of the soil can be determined based on following two criterion:

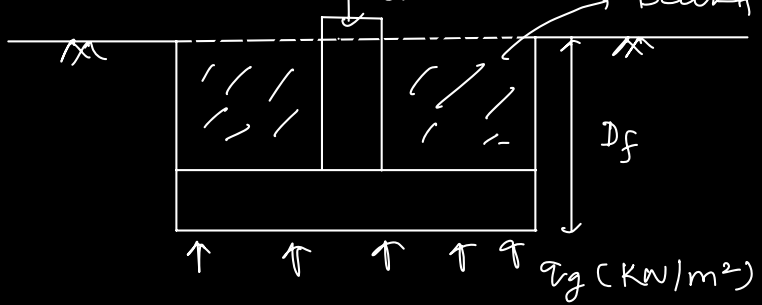
a. Shear failure criterion

b. Settlement failure criterion

Different types of bearing capacity based on shear failure criteria:

1. GROSS BEARING CAPACITY (GROSS PRESSURE): It is the total pressure at the base of the footing including the pressures due to self weight of the footing, overburden pressure applied by backfill & the pressure applied by column.

↓ Q → Backfilling



2. NET BEARING CAPACITY (NET PRESSURE): The difference between the grose bearing capacity & overburden pressure is termed as net bearing capacity (q_n).

$$q_n = q_g - \bar{\sigma}$$

$\bar{\sigma}$: Effective overburden pressure

$\bar{\sigma} = \gamma D_f$ (Bulk condition)

$\bar{\sigma} = \gamma_{sub} \cdot D_f$ (fully submerged condition)

3. ULTIMATE BEARING CAPACITY: The maximum grose bearing capacity or the pressure that the soil can support without failing in shear is called as ultimate bearing capacity.

$$q_u = \max(q_g)$$

4. NET ULTIMATE BEARING CAPACITY: It is the maximum net intensity of loading at the base of the footing that the soil can support before failing in shear.

$$q_{nu} = q_u - \bar{\sigma}$$

[$\bar{\sigma}$: overburden pressure due to backfilling.]

5. NET SAFE BEARING CAPACITY: The maximum net intensity of loading that the soil can safely support without the risk of shear failure. It is calculated by dividing by Factor of Safety (FOS), usually FOS is taken as 2.5 to 3.

$$q_{ns} = \frac{q_{nu}}{FOS} = \frac{\text{col + Footing}}{FOS}$$

6. GROSS SAFE BEARING CAPACITY or SAFE BEARING CAPACITY:

It is the maximum gross intensity of loading that the soil can carry safely without failing in shear.

$$q_s = q_{ns} + \bar{\sigma}$$

$$q_s = \frac{q_{nu}}{FOS} + \bar{\sigma}$$

$$q_s = \frac{q_u - \bar{\sigma}}{FOS} + \bar{\sigma} \quad KN/m^2$$

Note: It seems logical not to use the FOS to the overburden pressure because it is acting from the past so many years

Note: Bearing capacity term, we are using for shear failure criteria and bearing pressure term we are using in settlement failure criteria.

Note: while designing a foundation it is also necessary to ensure that the settlement of the foundation under the applied load intensity is within the permissible limit.

Based on settlement failure criteria the net load intensity / pressure can be expressed as:

Net safe bearing pressure (q_{ns}): It is the maximum net intensity of loading that can be imposed on the soil without the settlement exceeding the permissible value. No factor of safety is used when dealing with settlement criteria.

Note: Now we have "net safe bearing capacity" from the shear failure criteria and "net safe bearing pressure" from the settlement based criteria

"Allowable bearing capacity or Allowable bearing capacity":

($q_{a\,net}$): It is the maximum net intensity of the loading that can be imposed on the soil with no possibility of shear failure or the possibility of excessive settlement. Hence it is smaller of net safe bearing capacity (shear failure criteria)" and net safe bearing pressure (settlement criteria)"

$$q_{a\text{-}net} = \text{minimum} \begin{cases} \text{Net safe bearing capacity} \\ \text{and} \\ \text{Net safe bearing pressure.} \end{cases}$$

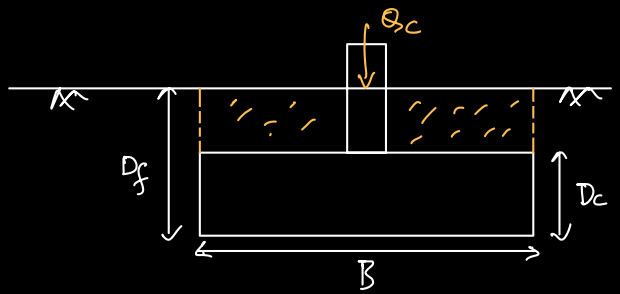
Choice of using net allowable bearing pressure:

→ There is often some confusion whether net allowable bearing pressure or net allowable bearing pressure should be used in determining net allowable bearing pressure.

→ It is conventional & convenient to use the net allowable bearing pressure in proportioning the footing.

→ Therefore the net load intensity at the base of foundation must be smaller than net allowable bearing pressure.

(i) Use of gross allowable bearing pressure:



$$Q_g = Q_c + W_s + W_f$$

$$Q_g = Q_c + B^2 \times (D_f - D_c) \times \gamma + D_c \times B^2 \times \gamma_c$$

$$\frac{Q_g}{B^2} = \frac{Q_c}{B^2} + (D_f - D_c) \cdot \gamma + D_c \cdot \gamma_c$$

$$q_{a\text{-}g} = \frac{Q_c}{B^2} + (D_f - D_c) \cdot \gamma + D_c \cdot \gamma_c$$

After rearranging:

$$B = \sqrt{\frac{Q_c + B^2 (D_f - D_c)\gamma + B^2 D_c \gamma_c}{q_{a-g}}}$$

→ very complex.

We have to solve this using trial & error ←

Note: From the above equation it is clear that only trial and error solution is possible which is not convenient.

(ii) Use of net allowable bearing pressure:

$$\because \quad Q_g = Q_c + B^2 (D_f - D_c)\gamma + B^2 D_c \gamma_c$$

$$q_g = \frac{Q_c}{B^2} + \gamma D_f - \gamma D_c + D_c \gamma_c$$

$$q_g - \gamma D_f = \frac{Q_c}{B^2} + D_c (\gamma_c - \gamma)$$

$$q_n = \frac{Q_c}{B^2} + D_c (\gamma_c - \gamma)$$

$\because$ $\gamma_c = 24\ KN/m^3$ (unit wt of concrete) and $\gamma = 20\ KN/m^3$ (unit wt of soil). Therefore considering $\gamma_c - \gamma \simeq 0$

$$q_n = \frac{Q_c}{B^2}$$

This net load intensity should be smaller than the net allowable bearing pressure ($q_{a\text{-net}}$)

$$q_n \leq q_{a\text{-net}}$$

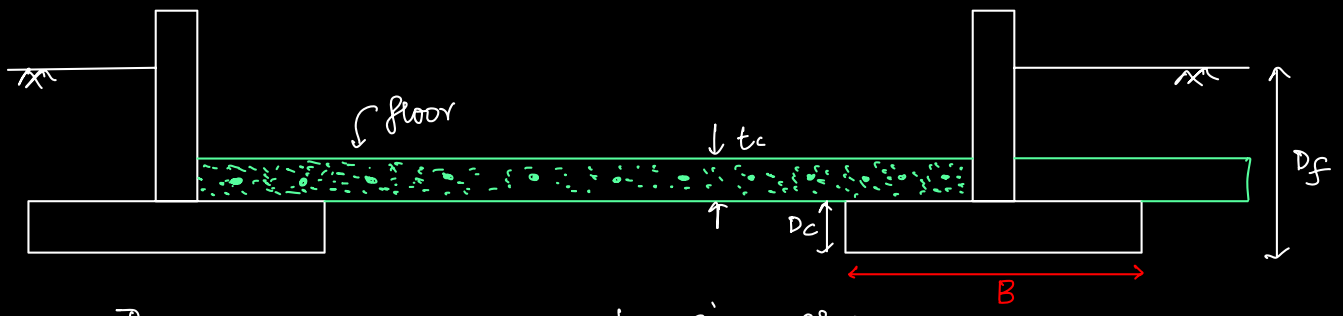
$$\frac{Q_c}{B^2} \leq q_{a\text{-net}}$$

$$B \geq \sqrt{\frac{Q_c}{q_{a\text{-net}}}}$$

→ Net allowable bearing pressure

From the above equation it is clear that if we use the net allowable bearing pressure then the calculation becomes easy & convenient.

(iii) SPECIAL CASE: when the foundation is not backfilled (in case of basements)



The gross pressure can be given as:

$$q_g = \frac{Q_c}{B^2} + \frac{B^2 D_c \gamma_c}{B^2} + t_c \cdot \gamma_c$$

$$q_g = \frac{Q_c}{B^2} + D_c \gamma_c + t_c \gamma_c$$

in the above equation the value of 2nd & 3rd term are smaller than the first term therefore ignoring the 2nd & 3rd term from the above equation.

$$q_g - \gamma D_f = \frac{Q_c}{B^2} - \gamma D_f$$

$$q_n = \frac{Q_c}{B^2} - \gamma D_f \quad \text{— (A)}$$

↳ net load intensity

The above net load intensity should be smaller than or equal to net allowable bearing pressure ($q_{a\text{-net}}$)

$$q_n \leq q_{a\text{-net}}$$

$$\frac{Q_c}{B^2} \leq q_{a\text{-net}} + \gamma D_f$$

→ from the above equation it is clear that load carrying capacity is enhanced if foundation is not backfilled

→ from the eqn (A) it is clear that the net load intensity becomes zero if $\frac{Q_c}{B^2} = \gamma D_f$

Mode of Shear failure:

when a horizontal strip footing is placed on the surface of the homogeneous soil & it is subjected to the gradually increasing the load and its characteristics are determined.

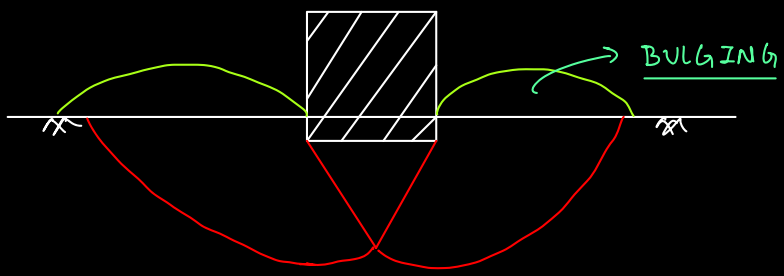
Once soil reaches to the q_u then three different types of failure mechanisms based on the failure pattern of shearing zone are observed. They are:

(i) GENERAL SHEAR FAILURE

(ii) LOCAL SHEAR FAILURE

(iii) PUNCHING SHEAR FAILURE

(i) GENERAL SHEAR FAILURE: $I_D > 70$



a. General shear failure is observed in medium to dense sand and OCC type of soils

b. A sudden failure occurs due to tilting of foundation.

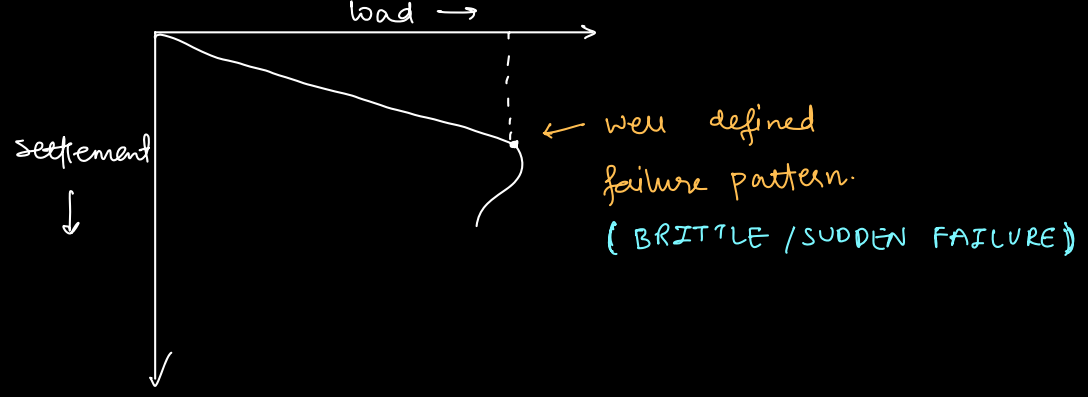
c. The final slip movement would occur only on one side, accompanied by tilting of foundation.

d. Before failure, very less settlement is observed

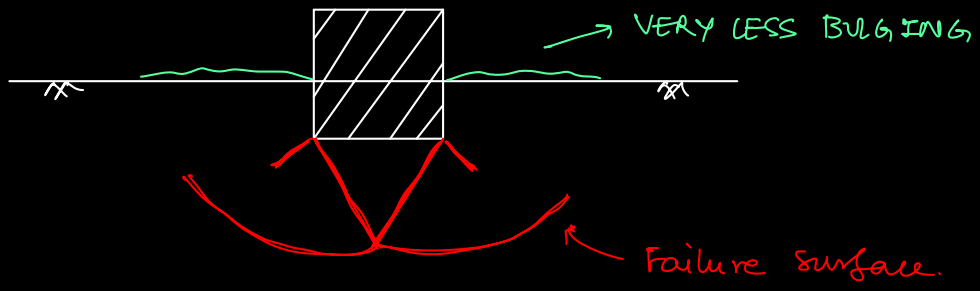
e. In General shear failure due to tilting of footing large bulging / heaving is observed adjacent to the foundation.

f. In this failure, failure surface reaches to the ground surface.

g. The ultimate load is well defined & recognised in the load settlement curve.

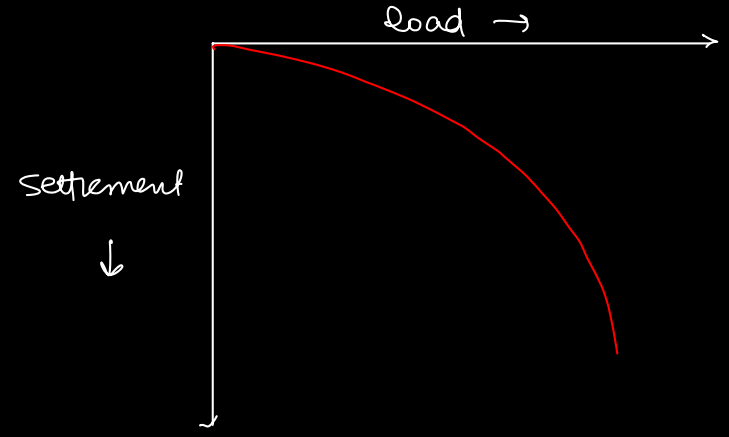


(ii) LOCAL SHEAR FAILURE: ($35 < I_D < 70$)

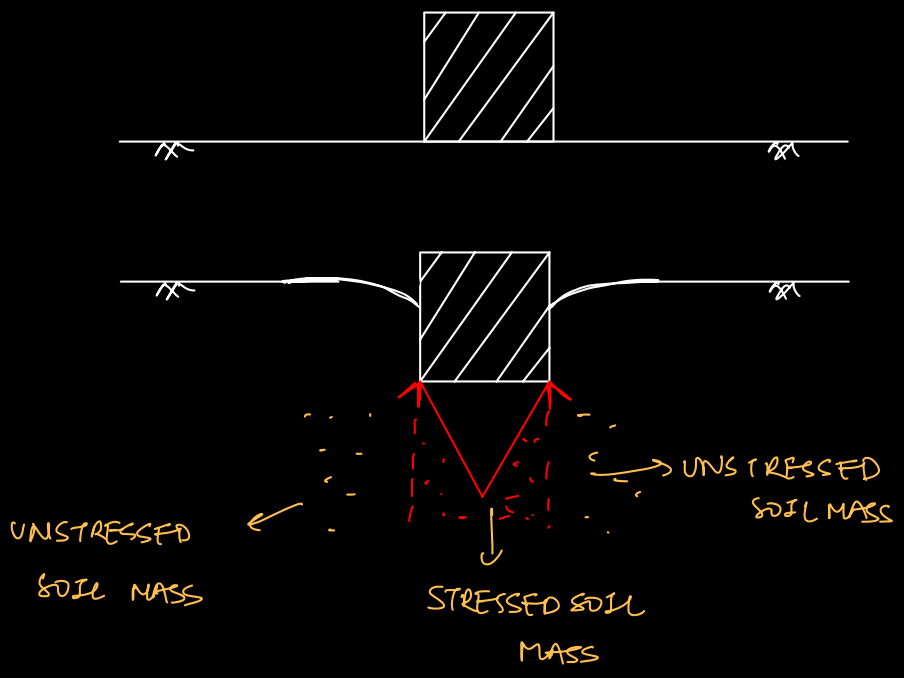


- Local shear failure is observed in loose to medium dense sand & NCC soils
- The failure is defined by the large settlement, very less tilting or no tilting is observed.
- ∴ Very less bulging or no bulging is observed
- ∵ No sudden failure.
- ∴ Failure surface doesnot reach upto ground surface in general conditions.
- ∵ well defined failure surface is observed only beneath the foundation, but it is not clearly defined beyond the edge of the footing
- ∵ It is generally observed when Relative density varies from 35% to 70%.
- ∵ Let c' & ϕ' are the shear strength parameters in general shear failure then the parameter in local shear failure will be,

$$c_l' = \frac{2}{3} c' \quad \text{and} \quad \tan \phi_l' = \frac{2}{3} \tan \phi'$$



(iii) PUNCHING SHEAR FAILURE: $I_D < 35$



→ Punching shear failure occurs in relatively loose sand in case of shallow foundations

→ It is generally observed when relative density (I_D) is less than 35%.

→ Punching shear failure occurs where there is high compression of the soil under the footing accompanied by shearing of vertical direction all around the edges of footing.

→ Punching shear failure is also observed in case of deep foundation

→ In case of punching shear failure, the soil present beneath the foundation is highly effected and stressed. But the soil present

beyond the edge of the footing is little or not effected.

→ The failure pattern is not well defined. Therefore ultimate load cannot be well defined or determined.

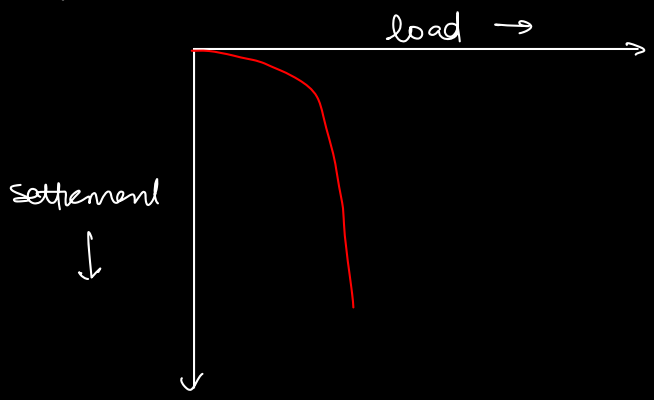
→ No sudden failure

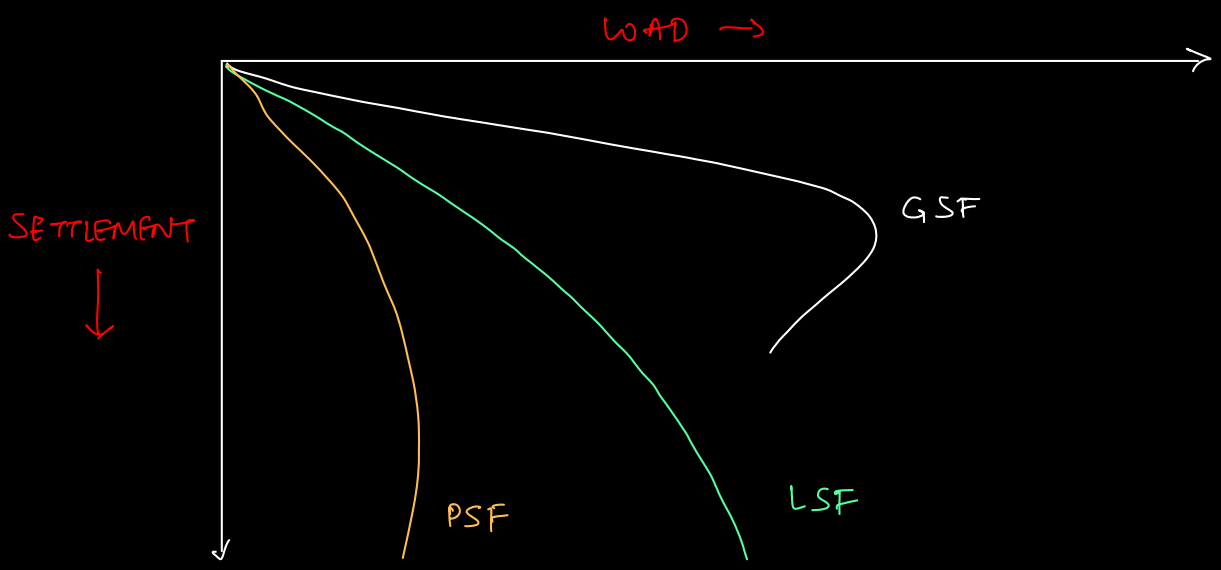
→ No bulging of soil adjacent to the foundation.

→ large continuous / progressive settlement is observed

→ Therefore failure is characterized in terms of very large settlement

→ No tilting of foundation.





General shear failure		local shear failure	
Sands	Clay	Sands	Clay
$I_D > 70\%$	UCS $> 100 KN/m^2$	$35\% < I_D < 70\%$	UCS $< 80 KN/m^2$
$\phi > 36°$		$\phi > 28°$	
$e < 0.55$		$e > 0.75$	
SPT No > 30		SPT No: $< 5-10$	

METHODS TO DETERMINE BEARING CAPACITY

(i) Analytical Method / Static Method:

a) Rankine's Theory
b) Prandtle's Analysis
c) Terzaghi's Theory
d) Skempton's Analysis
e) Meyerhoff's Analysis
f) Fellenius Method
g) Vesic Method
h) IS Code method.

(ii) Codal Provisions:

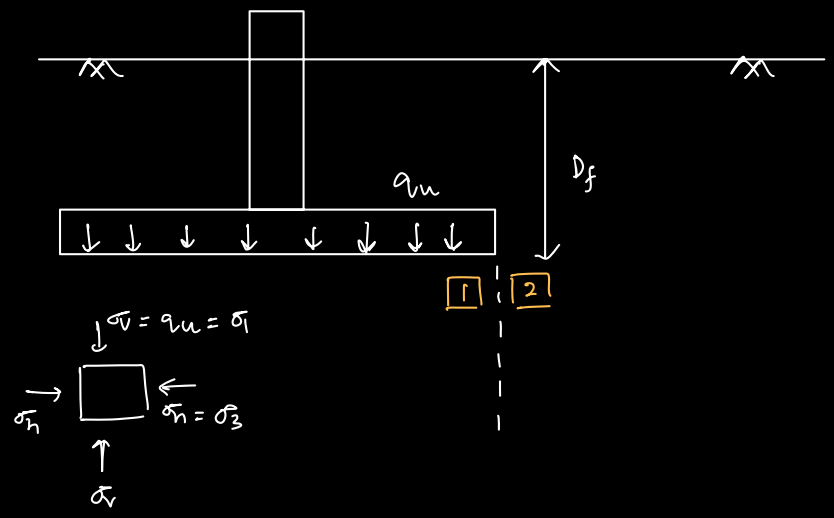
Different centre / State level agencies provide bearing capacity values for the different region (zones for ex- BIS, IRC, NBC, CPWD etc.

(iii) Field Methods:

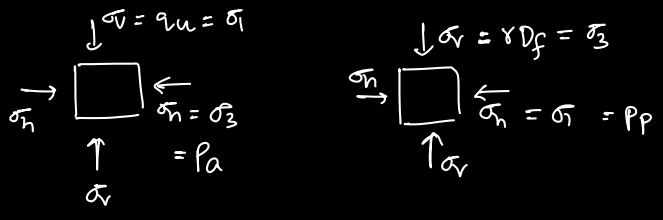
a) Plate load test
b) Standard penetration test
c) Cone penetration test

1. RANKINE'S ANALYSIS:

a) Rankine's Analysis of Bearing capacity is the extension of (or based on) lateral earth pressure analysis
b) This theory is applicable for cohesionless soils
c) In this theory, two soil elements are considered in equilibrium condition where 1st element is considered below the base of the foundation. and second element is considered just beyond the edge of the foundation adjacent to the 1st soil element
d) As the load is increased on the foundation and when soil beneath the foundation reaches to the ultimate bearing capacity (q_u) the shear feilure of 1st element occurs that time shear feilure of 2nd element must occur due to lateral thrust from the 1st element.



at plastic stage of equilibrium :



(Active Stage) (Passive stage)

for equilibrium condition, $(\sigma_h)_1 = (\sigma_h)_2$

$$(P_a)_1 = (P_p)_2$$

$$(K_a \cdot \sigma_v)_1 = [K_p (\sigma_v)]_2$$

$$\frac{1-\sin\phi}{1+\sin\phi} \cdot q_u = \frac{1+\sin\phi}{1-\sin\phi} \cdot \gamma D_f$$

ultimate bearing capacity $\leftarrow$

$$q_u = \left(\frac{1+\sin\phi}{1-\sin\phi}\right)^2 \cdot \gamma D_f$$

$$q_u = (K_p)^2 \cdot \gamma D_f \quad KN/m^2$$

If soil is fully submersed

$$q_u = \gamma_{sub} D_f \left(\frac{1+\sin\phi}{1-\sin\phi}\right)^2 \quad KN/m^2$$

The ultimate bearing capacity can be expressed as

$$\therefore \quad q_u = \gamma D_f \tan^4\left(45 + \frac{\phi}{2}\right)$$

using this theory minimum depth of foundation can be determined as,

$$D_{f\,min} = \frac{q_u}{\gamma \cdot \tan^4\left(45+\frac{\phi}{2}\right)} = \frac{q_u}{\gamma} \cdot \tan^2\left(45-\frac{\phi}{2}\right)$$

limitations of Rankine's Analysis :

1) This theory is applicable is useful only for cohesionless soil

2) In actual ultimate bearing capacity depends upon the size & shape of footing also. But in rankine's analysis the effect of the size and shape of footing has not being considered.

3) For the surface footing, D_f is zero

or $q_u = 0$, which is not possible according to Rankine's analysis at groundlevel which is practically not possible.

$$\because \quad q_u = \gamma D_f \left(\frac{1+\sin\phi}{1-\sin\phi}\right)^2$$

if $D_f = 0$; $q_u = 0$

Ls which is practically not possible.

2. TERZAGHI'S BEARING CAPACITY THEORY :

For complete bearing capacity failure the soil can be assumed to be assumed to be ideal plastic material. This concept was first introduced by Prandtle and later it was used by Terzaghi, meyerhoff and others.

Terzaghi's theory of bearing capacity is modified form of prandtle's theory. Prandtle has considered smooth base of the footing whereas terzaghi's has considered rough base of footing.

Assumptions :

(i) This theory is suitable for all the types of soil (cohesive and cohesionless both)

(ii) Terzaghi's theory is given for strip footing for which $L >> B$ ($L > 5B$)

(iii) General shear failure has been considered.

(iv) Footing and Ground surface both should be horizontal

(v) The feilure surface is not extended above the base of the footing that means the resistance offered by the soil present above the base of foundation is neglected in estimation of ultimate bearing capacity

vi) Therefore, terzaghi's bearing capacity theory is not suitable for deep foundations

vii) This soil present above the base of the footing can be replaced by an equivalent surcharge 'q' = γD_f.

viii) Soil is homogeneous and isotropic

ix) The load on the foundation should be vertical & concentric

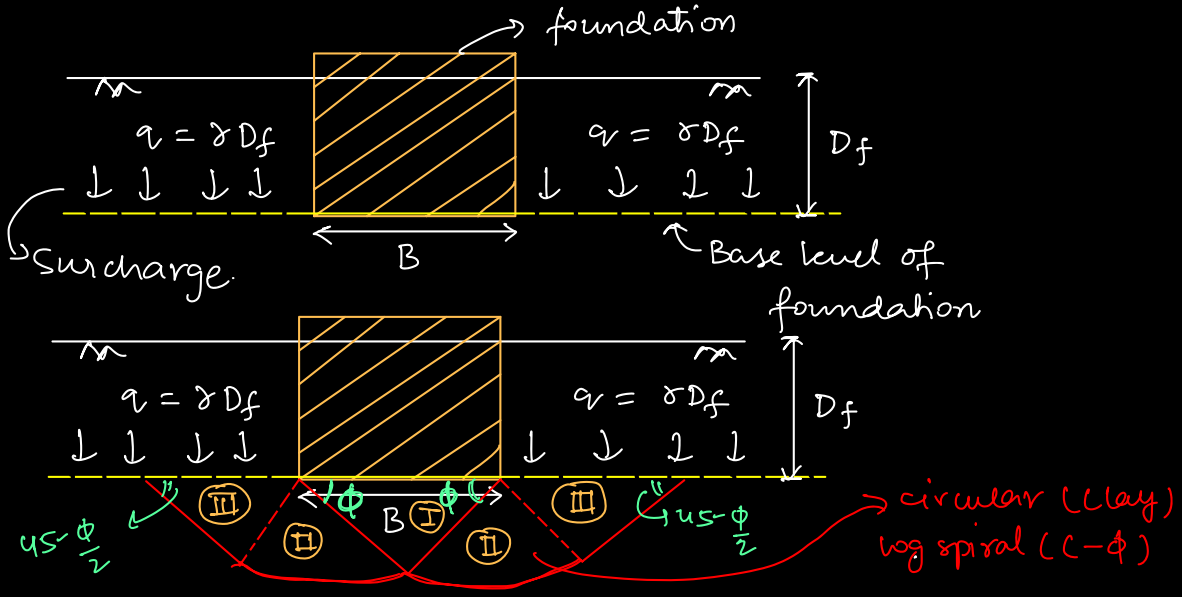
x) The footing has large L/B ratio hence two-dimensional or plane strain analysis shall be carried out.

xi) The base of the footing is laid at a shallow depth that is $D_f \leq B$

xii) General shear feilure is assumed to take place & soil volume is unchanged prior to failure

xiii) The shear strength of soil is governed by Mohr coulomb equation.

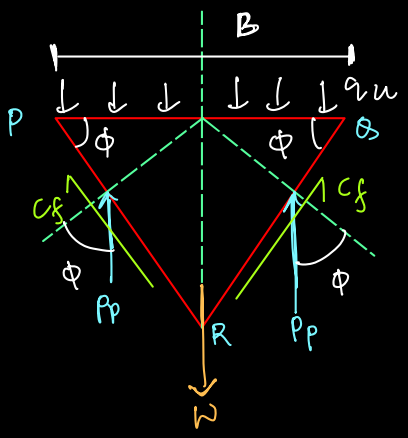
xiv) Water table is far below the zone of shear stress resistance that means effect of water table is not being considered



I: Zone of elastic Equilibrium

II: Zone of radial shear } Zone of plastic equilibrium

III: Zone of linear shear }



For equilibrium condition:

$$\Sigma F_y = 0$$

$q_u \cdot B$ + wt of soil wedge PBR $- 2P_p - 2C_f \sin\phi = 0$

PR: $\frac{B}{2\cos\phi}$

$$q_u B + \frac{1}{2} \times B \times \frac{B}{2} \tan\phi \times 1 \times \gamma - 2P_p - 2C_f \sin\phi = 0$$

$$q_u B = 2P_p + 2C_f \sin\phi - \frac{B^2}{4} \cdot \gamma \cdot \tan\phi$$

The total passive resistance P_p on the surface PR & QR is made up of or consists of three components

P_{pr}: produced by the weight of the soil in shear zone assuming the soil to be cohesionless [C = 0] and surcharge to be zero. [q = 0]

P_{pc}: produced by the soil cohesion ignoring the surcharge & weight of the soil in shear zone [q = 0, γ = 0]

P_{pq}: produced by the surcharge ignoring cohesion & weight of the soil in shear zone. [C = 0, γ = 0]

$$q_u B = 2P_p + 2C_f \sin\phi - \frac{B^2}{4} \cdot \gamma \cdot \tan\phi$$

C_f: cohesive force

C_f: $c \times PR \times 1$

: $c \times \frac{B}{2\cos\phi} \times 1$

$$q_{uB} = 2(P_{p\gamma} + P_{pc} + P_{pq}) + 2cf\sin\phi - \frac{B^2}{4} \cdot \gamma \cdot \tan\phi.$$

$$q_{uB} = 2(P_{p\gamma} + P_{pc} + P_{pq}) + 2 \cdot c \cdot \frac{B}{2\cos\phi} \cdot \sin\phi - \frac{1}{4} \cdot \gamma \cdot B^2 \cdot \tan\phi.$$

$$q_{uB} = 2(P_{p\gamma} + P_{pc} + P_{pq}) + cB\tan\phi - \frac{1}{4} \cdot \gamma \cdot B^2 \cdot \tan\phi$$

Assuming:

$$2P_{pc} + BC\tan\phi = B \cdot [cN_c]$$

$$2P_{pq} = B \cdot [qN_q]$$

$$2P_{p\gamma} - \frac{1}{4} \cdot B^2\gamma \cdot \tan\phi = B \cdot \left[\frac{1}{2} \cdot B \cdot \gamma \cdot N_\gamma\right]$$

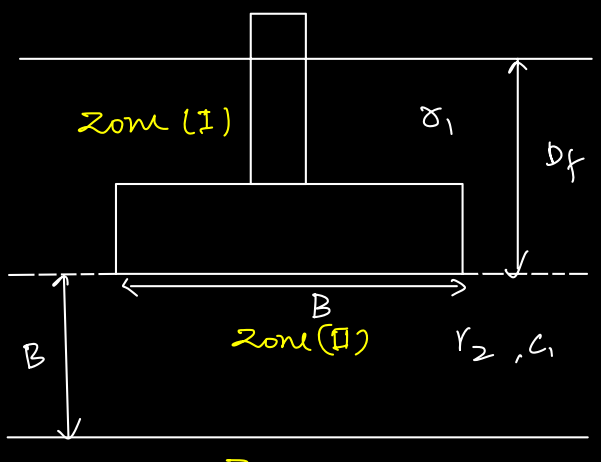
$$q_{uB} = B \cdot [cN_c + qN_q + \frac{1}{2}B\gamma N_\gamma]$$

$$q_u = cN_c + qN_q + \frac{1}{2} \cdot B\gamma N_\gamma$$

$$q_u = cN_c + \bar{\sigma} N_q + \frac{1}{2}B\gamma N_\gamma$$

q_u → ultimate bearing capacity equation proposed by terzaghi for strip footing.

where N_c, N_q & N_γ are the bearing capacity factors which depend upon the angle of internal friction of the soil which is below the base of the foundation.



Terzaghi's ultimate bearing capacity equation for strip footing is given as

$$q_u = cN_c + \bar{\sigma}N_q + 0.5B\gamma_2 N_\gamma$$

$$\{q_u = cN_c + \gamma_1 D_f N_q + 0.5B\gamma_2 N_\gamma\}$$

γ_2: it is the unit wt of soil which is present in the zone II below the base of foundation.

γ_1: it is the unit wt of soil present above the base of the foundation which shall be used in estimation of

shall be used in mid term of the equation.

overburden pressure in 2nd term of the equation

As per terzaghi's bearing capacity equation

$$q_u = CN_c + \bar{\sigma} N_q + \frac{1}{2} B\gamma \cdot N_\gamma$$

Note: Terzaghi and Prandtle's equation are same but only the bearing capacity factors are different

- As per terzaghi, for clay ($\phi = 0$)

  N_c : 5.7

  N_q : 1

  N_γ : 0

As per prandtle (for clay)

N_c : 5.14

N_q : 1

N_γ : 0

Note: As ϕ value increases N_c, N_q, N_γ values also increases

Modifications for shape:

: for different shape of footing some semi-empirical eqn were proposed.

Case I. Strip footing

$$q_u = CN_c + \bar{\sigma} N_q + 0.5 B\gamma N_\gamma$$

Case II. For rectangular footing ($B < L$)

$$q_u = \left(1 + 0.3 \frac{B}{L}\right) CN_c + \bar{\sigma} N_q + \frac{1}{2}\left(1 - 0.2 \frac{B}{L}\right) B\gamma N_\gamma$$

Case III. For square footing:

$$q_u = 1.3 CN_c + \bar{\sigma} N_q + 0.4 B\gamma N_\gamma$$

↳ Shape factor = 0.8 here

Case IV. For circular footing: ($B = D$)

$$q_u = 1.3 CN_c + \bar{\sigma} N_q + 0.3 B\gamma N_\gamma$$

Shape factor	Strip footing	rectangular	Square	Circular
S_c	1	$1+0.3\,B/L$	1.3	1.3
S_q	1	1	1	1
S_γ	1	$1-0.2\,B/L$	0.8	0.6.

Modification for LOCAL SHEAR FAILURE:

→ Terzaghi has considered GSF in his bearing capacity Analysis, punching failure is very uncommon and rarely to be seen because footing is not provided in case of very loose sand.

→ In case of loose sand & soft clay, local shear failure is reported which is common

→ Shear resistance is not mobilized on the full length of failure surface as in case of GSF.

Therefore the shear strength parameters are reduced in case of local shear failure.

$$\left\{ \begin{array}{l} c_l' = \frac{2}{3} c' \\ \phi_l' = \tan^{-1}\left(\frac{2}{3}\tan\phi'\right) \end{array} \right\}$$

In GSF,

$$q_u = c' N_c + \bar{\sigma} N_q + \frac{1}{2} B \gamma N_\gamma$$

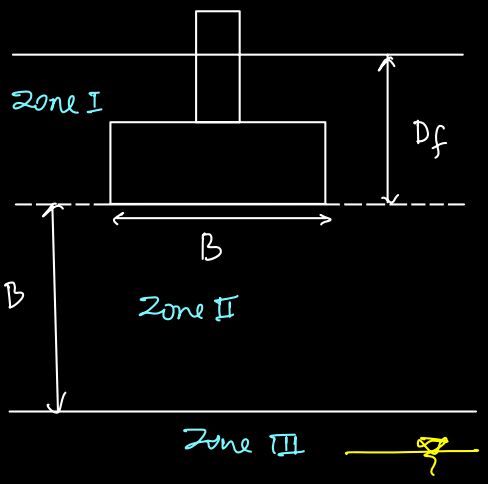
In LSF,

$$q_u = \frac{2}{3} c' N_c' + \bar{\sigma} N_q' + \frac{1}{2} B \gamma N_\gamma'$$

N_c, N_q, N_γ: corresponding to value of ϕ'

N_c', N_q', N_γ': corresponding to value of ϕ_l'

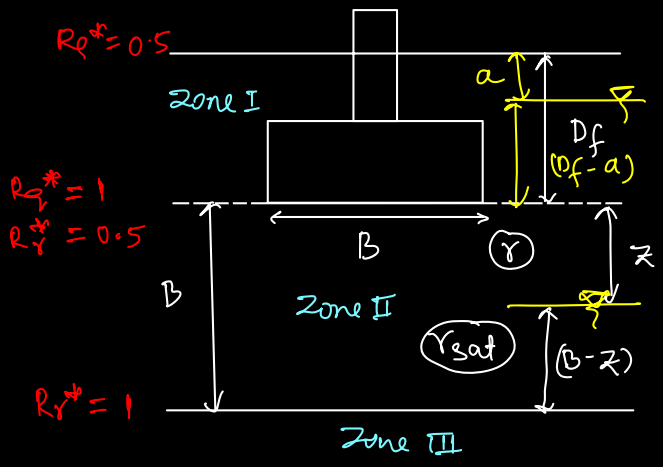
Modifications for water table:



CASE I: When water table is in Zone III

- Water table is assumed to be far below the base of the formal.
- The stress zone is unaffected from the water table
- Therefore the ultimate bearing capacity is unaffected

$$q_u = CN_c + \bar{\sigma} N_q + 0.5 B \gamma N_\gamma$$



CASE 2: When water table is in Zone II.

→ Due to the presence of water table effective shear strength parameters c' & ϕ' shall be considered.

(Note: In the absence of data take $c' = c$ & $\phi' = \phi$).

→ If water table is present & we are using effective angle of friction (ϕ') then bearing capacity factors shall be calculated for ϕ' only. They can be given as N_c', N_q' & N_γ'.

1st Method: Water table correction factor:

- Third term shall be modified
- $q_u = cN_c + \gamma D_f N_q + \frac{1}{2} B \cdot \gamma_{avg} N_\gamma (R_\gamma^*)$

$$\gamma_{avg} = \frac{\gamma \cdot Z + \gamma_{sat} (B - Z)}{B}$$

$$R_\gamma^* = 0.5 \left(1 + \frac{Z}{B}\right)$$

- If $Z = B$; $\gamma_{avg} \to \gamma$; $R_\gamma^* = 1$

$$q_u = CN_c + \gamma D_f N_q + \frac{1}{2} B \gamma N_\gamma$$

↳ Bearing capacity is unaffected.

- If $Z = 0$; $\gamma_{avg} \rightarrow \gamma_{sat}$; $R_\gamma^* = 0.5$

$$q_u = CN_c + (\gamma D_f) N_q + \frac{1}{2} B \gamma N_\gamma (0.5)$$

2nd Method : Effective Parameters method

$$q_u = C \cdot N_c + \bar{\sigma} N_q + \frac{1}{2} \cdot B \gamma_{avg} N_\gamma$$

↳ Third term shall be modified,

$$\gamma_{avg} = \frac{\gamma Z + \gamma_{sub} (B - Z)}{Z + (B - Z)}$$

if $Z = B$; $\gamma_{avg} \rightarrow \gamma$; $q_u = CN_c + (\gamma D_f) N_q + \frac{1}{2} B \gamma N_\gamma$

if $Z = 0$; $\gamma_{avg} \rightarrow \gamma_{sub}$, $q_u = CN_c + (\gamma D_f) N_q + \frac{1}{2} B \gamma_{sub} N_\gamma$

CASE 3: WATER TABLE IN ZONE I:

↳ 2nd & 3rd term shall be modified

$$q_u = CN_c + \gamma D_f N_q + \frac{1}{2} B \gamma N_\gamma$$

1st : Water table correction factor method.

$$q_u = CN_c + \gamma D_f N_q + \frac{1}{2} B \gamma_{sat} N_\gamma (0.5)$$

$$q_u = CN_c + \gamma_{avg} D_f N_q R_q^* + \frac{1}{2} B \gamma_{sat} N_\gamma (0.5)$$

$$\gamma_{avg} = \frac{\gamma \cdot a + \gamma_{sat} (D_f - a)}{a + (D_f - a)}$$

$$R_q^* = 0.5 \left(1 + \frac{a}{D_f} \right)$$

if $a = D_f$; $\gamma_{avg} : \gamma$; $R_q^* = 1$

$$q_u = CN_c + \gamma D_f N_q + \frac{1}{2} B \gamma_{sat} N_\gamma (0.5)$$

if $a = 0$; $\gamma_{avg} : \gamma_{sat}$; $R_q^* = 0.5$

$$q_u = CN_c + \gamma D_f N_q (0.5) + \frac{1}{2} B\gamma N_\gamma (0.5)$$

2nd Method: Effective Parameter method:

$$q_u = CN_c + \gamma_{avg} D_f N_q + \frac{1}{2} B\gamma_{sub} N_\gamma$$

$$\gamma_{avg} = \frac{\gamma_w \cdot a + \gamma_{sub}(D_f - a)}{a + (D_f - a)}$$

when $a = 0$, water table at ground

$$\gamma_{avg} = \gamma_{sub}$$

$$\left\{ CN_c + \gamma_{sub} D_f N_q + 0.5 \times \frac{1}{2} B\gamma N_\gamma \right\}$$

$a = D_f$: base of foundation

$$\gamma_{avg} = \gamma_w \left\{ CN_c + \gamma D_f N_q + 0.5 \times \frac{1}{2} B\gamma N_\gamma \right\}$$

Special case (1): ultimate bearing capacity in cohesionless soil when water table rises to the ground surface

$$q_u = \cancel{CN_c}^{\,0} + \gamma_{sat} D_f N_q (0.5) + \frac{1}{2} B\gamma_{sat} N_\gamma (0.5)$$

$$q_u = \frac{1}{2} \left[\gamma_{sat} D_f N_q + \frac{1}{2} B\gamma_{sat} N_\gamma \right]$$

Note: That means when GW reaches to ground surface in cohesionless soil, the ultimate bearing capacity gets reduced by 50%.

→ In cohesionless soil, ultimate bearing capacity depends upon the size of footing (B) and depth of footing D_f, shape of footing and location of water table.

Special case 2: Bearing capacity in clay soil

$$q_u = CN_c + \bar{\sigma} N_q + \frac{1}{2} B\gamma N_\gamma$$

$$q_u = 5.7\,C + 1 \cdot \bar{\sigma} + 0 \quad [N_c = 5.7,\ N_q = 1,\ N_\gamma = 0]$$

(According to terzaghi)

$$(q_u = 5.7C + \gamma \cdot D_f)$$

- Does not depend on size of footing
- Depth of footing में बदलता
- Depends on shape
- Dependens on location of water table

$$q_{nu} = CN_c + \gamma D_f - \gamma D_f$$

$$q_{nu} = 5.7 \cdot [C]$$

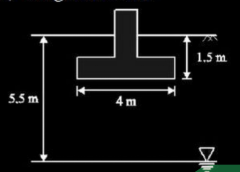
⇒ from the above equation it is clear that the net ultimate bearing capacity of clay soil is independent of size & depth of the footing.

When water table rises to the ground in clay soil, $q_{nu} = 5.7c'$ - In absence of data, $c = c'$ thus the bearing capacity is not very much effected by water table but it depends upon shape of footing.

Question

Q. A 4 m wide strip footing is founded at a depth of 1.5 m below the ground surface in a $c-\phi$ soil as shown in the figure. The water table is at a depth of 5.5 m below ground surface. The soil properties are $c' = 35$ kN/m^2, $\phi' = 28.63^0$, $\gamma_{sat} = 19$ kN/m^3, $\gamma_{bulk} = 17$ kN/m^3 and $\gamma_w = 9.81$ kN/m^3. the values of bearing capacity factors for different ϕ' are given below.

ϕ'	N_c	N_q	N_γ
15^0	12.9	4.4	2.5
20^0	17.7	7.4	5.0
25^0	25.1	12.7	9.7
30^0	37.2	22.5	19.7



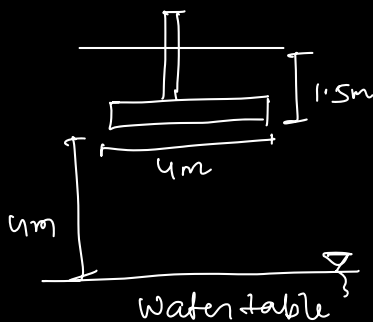
Using Terzaghi's bearing capacity equation and a factor of safety, $F_s = 2.5$, the net safe bearing capacity (expressed in kN/m^2) for local shear failure of the soil is _____.

[GATE 2016 : IISc Bangalore]

Solution

$B = 4m$

$D_f = 1.5m$



$C' = 35 KN/m^2$

$\phi' = 28.63°$

$\gamma_b = 17 KN/m^3$

$\gamma_{sat} = 19 KN/m^3$

$\gamma_w = 9.81 KN/m^3$

$$C_l' = \frac{2}{3} C' = \frac{2}{3} \times 35 = 23.33$$

$$\tan \phi_l' = \frac{2}{3} \tan \phi'$$

$$\phi_l' = \tan^{-1}\left(\frac{2}{3} \tan \phi'\right) = 19.99° \approx 20°$$

$N_c = 17.7$

$N_q = 4.4$

$N_r = 5$

$$q_u = 17.7 \times 23.33 + 17 \times 1.5 \times 4.4 + 0.5 \times 17 \times 4 \times 5$$

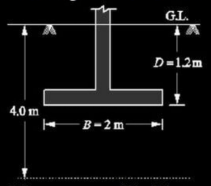
$$\therefore q_u = 771.641 \ KN/m^2$$

$$q_n = 771.641 - 17 \times 1.5 = 746.141$$

$$q_{ns} = \frac{746.141}{2.5} = 298.46 \text{ KN/m}^2$$

Question

Q. A strip footing 2 m wide carries a load intensity of 400 kN/m² at a depth of 1.2 m in sand. The saturated unit weight of sand is 19.5 kN/m³ and unit weight above water table is 16.8 kN/m³. The shear strength parameters are $c = 0$ and $\phi = 35^0$. Determine the factor of safety with respect to shear failure for the following cases of location of water table:



(A) Water table is 4 m below G.L.
(B) Water table is 1.2 m below G.L.
(C) Water table is 2.5 m below G.L.
(D) Water table is 0.5 m below G.L.
(E) Water table is at G.L. itself.

Use Terzaghi's equations.

Assume general shear failure, $N_q = 41.4$ and $N_r = 42.4$

$q = 400 \text{ KN/m}^2$

$D_f = 1.2$ $\quad \gamma_b = 16.8 \text{ KN/m}^3$

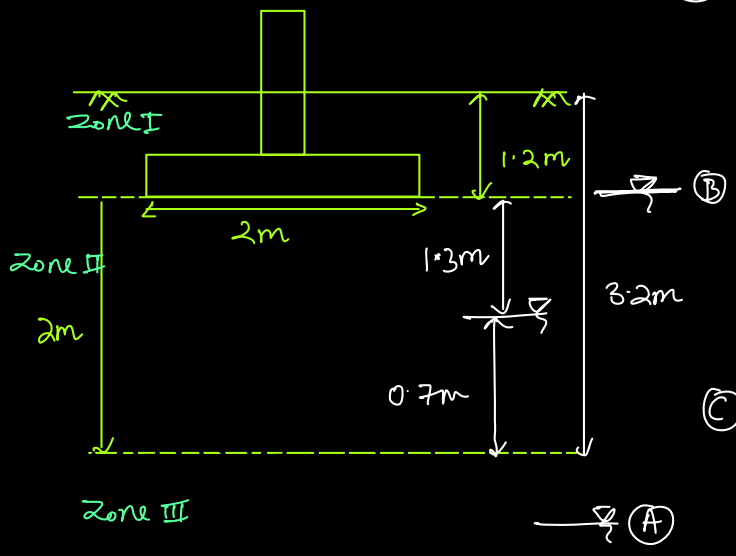
$B = 2\text{m}$ $\quad \gamma_{sat} = 19.5 \text{ KN/m}^3$

$C = 0$, $\phi = 35^\circ$

$N_q = 41.4$

$N_\gamma = 42.4$

load intensity of 400 KN/m² at the 1.2 m depth



(A) $q_u = \bar{\sigma} N_q + \frac{1}{2} B \gamma N_\gamma$

$= \gamma D_f N_q + 0.5 B \gamma N_\gamma$

$= 16.8 \times 1.2 \times 41.4 + 0.5 \times 16.8 \times 2 \times 42.4$

$= 1546.944 \text{ KN/m}^2$

$FOS = \frac{1546.944}{400} = 3.86$

(C) Water table is 2.5m below the Ground level.

$$\left[\begin{array}{l} R^*_\gamma = 0.5 \left(1 - \frac{z}{B}\right) \\ = 0.5 \left(1 + \frac{1.3}{2}\right) \\ R^*_\gamma = 0.825 \end{array} \right]$$

$$\gamma_{avg} = \frac{\gamma \times 1.3 + \gamma_{sat} \times 0.7}{2} = \frac{16.8 \times 1.3 + 19.5 \times 0.7}{2}$$

$$= 17.745 \text{ KN/m}^2$$

$$q_u = \gamma D_f N_q + \frac{1}{2} B \gamma_{avg} N_\gamma \cdot (0.825)$$

$$= 16.8 \times 1.2 \times 41.4 + 0.5 \times 2 \times 17.745 \times 42.4 \times 0.825$$

$$= 1455.3441 \text{ KN/m}^3$$

$$FOS = \frac{1455.3441}{400} = 3.64 \quad \text{Ans}$$

SKEPTON'S ANALYSIS:

- Skepton's Analysis is Applicable only for pure cohesive soils like clay for which $c_u = 0$.
- In Skempton's Analysis, base and side resistance both are considered
- Skepton's Analysis is based on the laboratory field and theoretical observations.
- According to Skepton's analysis, net ultimate bearing capacity of the clay soil can be given as:

$$\boxed{q_{nu} = CN_c}$$

* can be used for shallow as well as deep foundation

C: unit cohesion

N_c: Skempton's bearing capacity factor which depends upon the $\frac{D_f}{B}$ ratio.

Case I: When $\frac{D_f}{B} = 0$ (Surface footing):

$N_c = 5$ (for strip footing)

$N_c = 6$ (for other shape of footing)

Case 2: When $0 \leq \frac{D_f}{B} \leq 2.5$:

$$N_c = 5\left(1 + 0.2\frac{D_f}{B}\right) \quad : \text{STRIP}$$

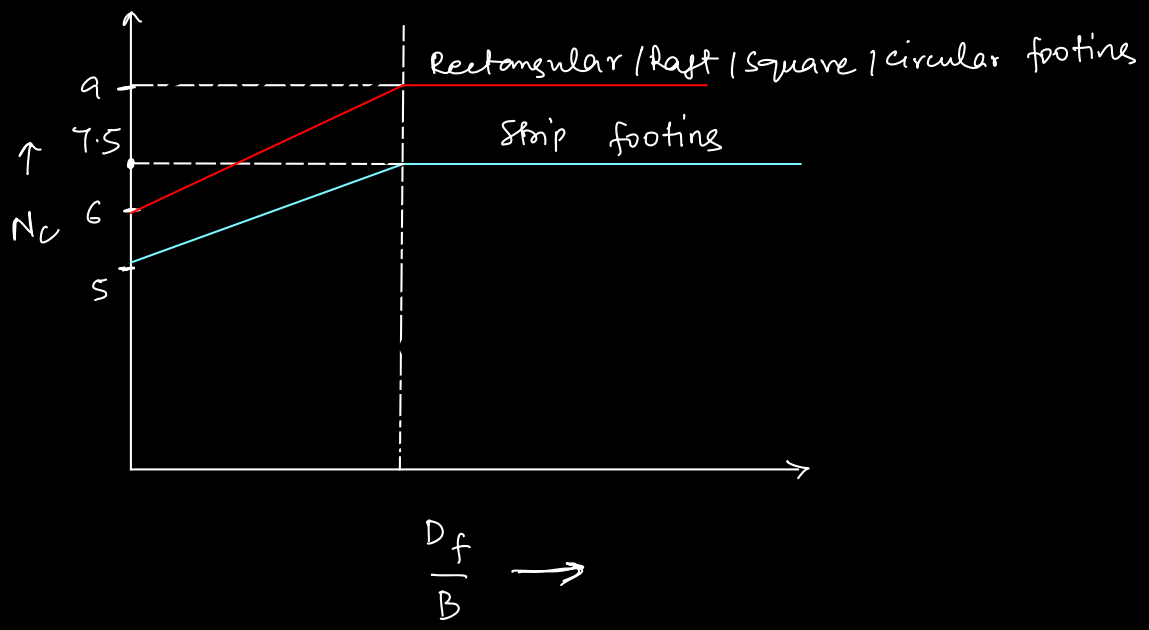
$$N_c = 6\left(1 + 0.2\frac{D_f}{B}\right) \quad : \text{FOR SQUARE CIRCULAR}$$

$$N_c = 5.0\left(1 + 0.2\frac{B}{L}\right)\left(1 + 0.2\frac{D_f}{B}\right) \quad : \text{For rectangular (Raft also)}$$

case3: $D_f > 2.5$

$N_c = 7.5$ [for strip footing]

$N_c = 9.0$ [for other shape]



MEYERHOFF'S ANALYSIS:

↳ Meyerhoff has given comprehensive analysis of bearing capacity of a soil of a strip footing.

↳ In Meyerhoff analysis the failure surface is extended upto the ground level above the base of the foundation. whereas terzaghi has stopped the feilure surface at the base of the foundation only.

→ Hence the meyerhoff has considered base resistance & side resistance both in the ultimate bearing capacity. therefore Meyerhoff's analysis is considered for both shallow & deep foundation

The ultimate bearing capacity as per meyerhoff can be given as:

$$q_u = CN_c S_c d_c i_c + \bar{\sigma} N_q S_q d_q i_q + \frac{1}{2} B \gamma N_\gamma S_\gamma d_\gamma i_\gamma$$

S_c, S_q, S_γ → Shape factor

d_c, d_q, d_γ → depth factor

i_c, i_q, i_γ → Inclination factor

Note: $S_q = S_c = S_r = 1$ for strip footing

Note: Shape factors are introduced for different shape of footing. Depth factors are introduced based on side resistance above base of foundation. Inclination factors are introduced for incline loads.

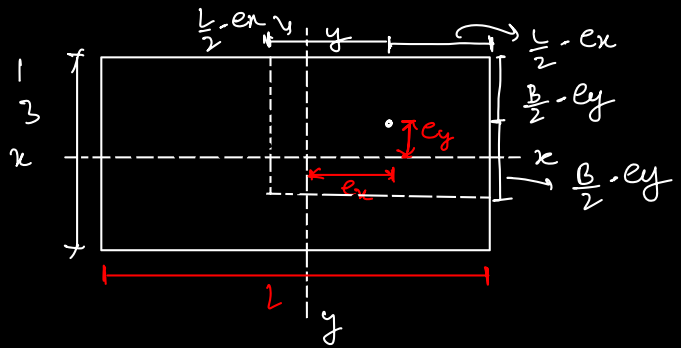
Note: If load is vertical (not inclined) then i_c, i_q, i_γ.

where N_c, N_q, N_γ are the Meyerhoff's bearing capacity factors, they are same as proposed by prandtl and Reisnner

$$\left.\begin{aligned} N_c &= (N_q - 1)\cot\phi \\ N_q &= e^{\pi \tan\phi} \cdot \tan^2\left(45 - \frac{\phi}{2}\right) \\ N_\gamma &= (N_q - 1)\tan(1.4\phi) \end{aligned}\right\}.$$

Note: Meyerhoff has also introduced the concept of use of reduced area or dimension of the footing in the estimation of ultimate bearing capacity in case of eccentric loading.

CaseI. TWO WAY ECCENTRICITY.



Reduced area

$\Rightarrow (L - 2e_x)(L - 2e_y)$

$\Rightarrow L' \cdot B'$

B, L : original width & length of footing

B', L' : reduced length of footing

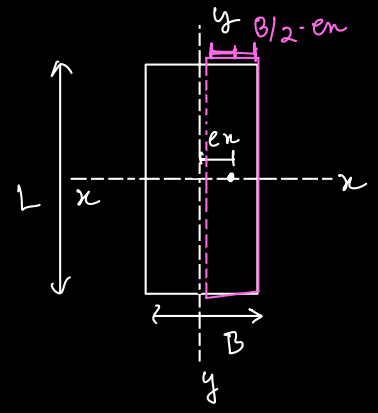
A : original area

A' : Reduced area

$A = B \times L$

$A' = B' \times L'$

Case 2: One way Eccentricity:



$$B' = B - 2e_x$$

$$L' = L$$

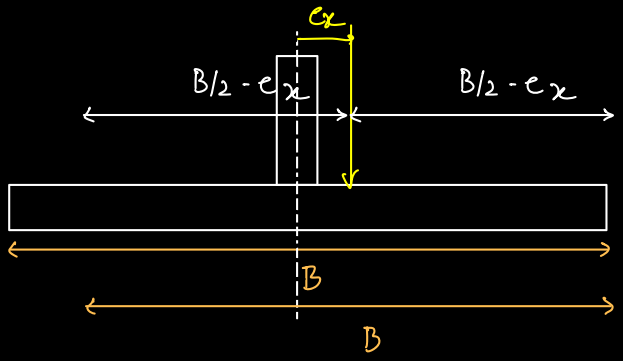
$$A = BL$$

$$A' = (B - 2e_x) \cdot L$$

Case III: Eccentricity in strip footing along the width:

↳ for strip footing $L >> B$

↳ Eccentricity is considered along the width



B' : reduced width of strip footing

$$B' = B - 2e_x$$

Note: In case of eccentric loading reduced width B' shall be used in estimation of load bearing capacity.

$$q_u = cN_c s_c i_c d_c + \bar{\sigma} N_q s_q i_q d_q + \frac{1}{2} B' \gamma N_\gamma s_\gamma i_\gamma d_\gamma$$

(B': Reduced width of footing.)

Calculation of Net safe load in eccentric loading condition

$$Q_{NS} = q_{ns} \times \text{reduced area of footing.}$$

$$Q_{NS} = \frac{q_u - \bar{\sigma}}{FOS} \times \text{reduced area of footing}$$

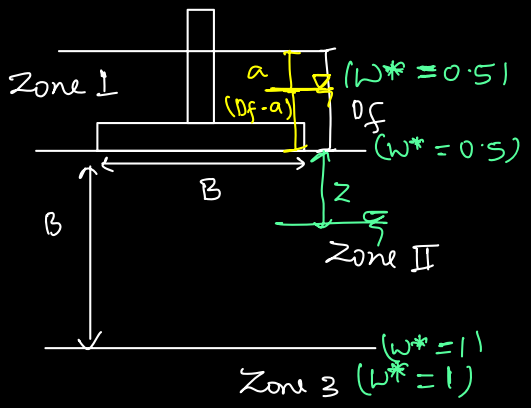
IS Code Method:

- IS code 6403 - 1981 recommends the net ultimate bearing capacity of a shallow foundation in GSF, given as.

$$q_{nu} = cN_c s_c d_c i_c + q(N_q - 1) s_q d_q i_q + \frac{1}{2} B \gamma N_\gamma (s_\gamma d_\gamma i_\gamma) W^*$$

- The third term of the above equation W^* is the water table correction factor.

$$W^* = 0.5 \left(1 + \frac{z}{B}\right)$$



CASE1 : Water table in zone III

In third term

$\gamma \rightarrow \gamma$, $W^* = 1$

In 2nd term

$q = \gamma D_f$

CASE2 : Water table in Zone II

In third term

$\gamma \rightarrow \gamma_{avg}$, $W^* = 0.5 \left(1 + \frac{z}{B}\right)$

$$\gamma_{avg} = \frac{\gamma z + \gamma_{sat}(B - z)}{z + (B - z)}$$

In 2nd term,

$q = \gamma D_f$

CASE3 : Water table in Zone I.

In third term,

$\gamma \rightarrow \gamma_{sat}$, $W^* = 0.5$

In 2nd term,

Is No correction factor required in 2nd term. directly put value as effective surcharge considering the effect of water table.

$$q = \gamma \cdot a + \gamma_{sat}(D_f - a) - \gamma_w (D_f - a)$$

$$q = \gamma \cdot a + \gamma_{sub}(D_f - a)$$

→ For local shear failure, the recommendations of IS 6403-1981 are similar to terzaghi Analysis

$$c_l' = \frac{2}{3} c'$$

$$\phi_l' = \tan^{-1}\left(\frac{2}{3}\tan\phi'\right)$$

In local shear failure bearing capacity factors should be calculated for ϕ_l' value.

→ The bearing capacity factors are as proposed by vesics.

$$\left\{\begin{array}{l} N_c = (N_q - 1)\cot\phi \\ N_q = e^{\pi\tan\phi} \cdot \tan^2\left(45 + \frac{\phi}{2}\right) \\ N_\gamma = 2(N_q + 1)\tan\phi \end{array}\right\}$$

→ As per prandtle, reissner, meyerhoff, Hansen, vesic & IS code:

$$N_c = (N_q - 1)\cot\phi$$

$$N_q = e^{\pi\tan\phi} \cdot \tan^2\left(45 + \frac{\phi}{2}\right)$$

that means as per prandtle, reissner, meyerhoff, Hansen vesic & IS code, for clay soil.

$$N_c = 5.14 \quad , N_q = 1$$

→ N_γ is different in different theories:

$N_\gamma = (N_q - 1)\tan(1.4\phi)$: Prandtl
Reissner
Meyerhoff

$N_\gamma = 1.5(N_q - 1)\tan\phi$: Hansen

$N_\gamma = 2(N_q + 1)\tan\phi$: Vesic
IS code.

Question

Q.9 A square footing (2m×2m) is subjected to an inclined point load, P as shown in the figure below. The water table is located well below the base of the footing. Considering one-way eccentricity, the net safe load carrying capacity of the footing for a factor of safety of 3.0 is _____kN.

The following factors may be used:

Bearing capacity factors:

$N_q = 33.3,\ N_\gamma = 37.16;$

Shape factors :

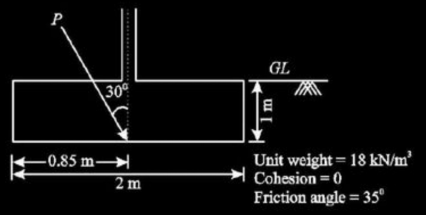
$F_{qs} = F_{\gamma s} = 1.314;$

Depth factors :

$F_{qd} = F_{\gamma d} = 1.113;$

Inclination factors :

$F_{qi} = 0.444,\ F_{\gamma i} = 0.02$



[GATE 2015 : IIT Kanpur]

(net safe load), FOS = 3

square footing

$\phi = 35°$

Reduced, $B' = 0.85 \times 2$

$= 1.7\ m$

$q_u = q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} B' \gamma N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$

$= 18 \times 1 \times 33.3 \times 1.314 \times 1.113 \times 0.444$

$+ 0.5 \times 1.7 \times 18 \times 37.16 \times 1.314 \times 1.113 \times 0.02$

$\{ q_u = 405.84\ KN/m^2 \}$

$q_n = q_u - \gamma D_f = 405.84 - 18 \times 1$

$= 387.84\ KN/m^2$

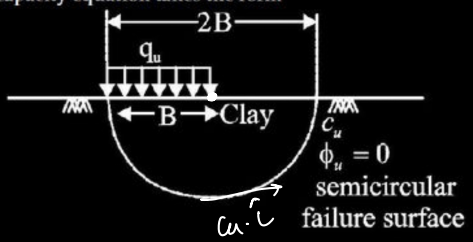
$q_{ns} = \frac{387.84}{3} = 129.28\ KN/m^2$

$Q = 129.28 \times 1.7 \times 2$ (oneway ecc.)

$= 439.552\ KN$

Question

Q.10 For the strip footing on a saturated clay, for the given failure surface, the bearing capacity equation takes the form



Where,

C_u = undrained shear strength

ϕ_u = angle of internal friction

B = width of strip footing

q_c = ultimate bearing capacity of soil

(A) $5.7C_u$ (B) $5.14C_u$

(C) $4\pi C_u$ (D) $2\pi C_u$

[GATE 1996 : IISc Bangalore]

Solution

strip footing, clay

$\phi_u = 0$ (Don't assume)

$C_u = ?$

semicircular failure surface?

overturning moment about point "O":

$M_O = q_u \times B \times \frac{B}{2}$

$M_O = \frac{q_u \cdot B^2}{2}$

Restoring moment $= C_u \cdot \pi B \cdot B$

$= C_u \cdot \pi B \cdot B = C_u \pi B^2$

Restoring moment = overturning moment

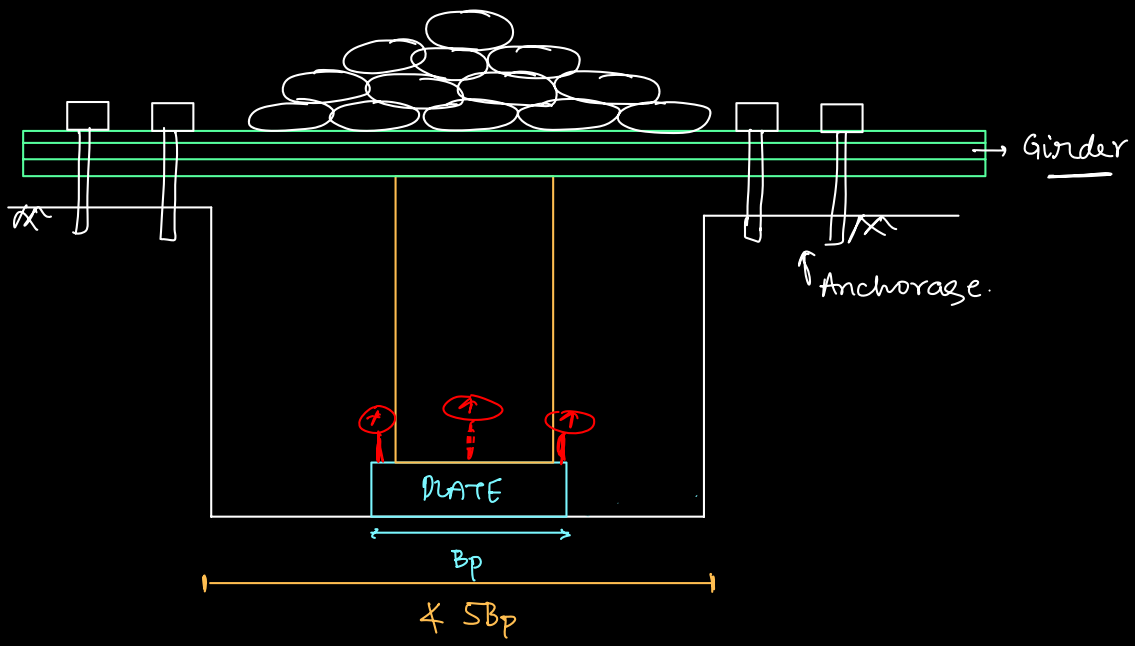
$$\frac{q_u B^2}{2} = c_u \pi B^2$$

$$\boxed{q_u = 2\pi c_u}$$

FIELD METHODS

1. Plate load test
2. Standard penetration test
3. Cone penetration test.

1. PLATE LOAD TEST:



1. Plate load test is a field test performed for the determination of bearing capacity and settlement of the shallow foundation
2. The plate load test is also used to determine the modulus of subgrade reaction used in the design of rigid pavement
3. Test procedure of this plate load test is explained in IS1888:1982 code
4. A rigid plate of mild steel is used in this test.

5. The shape of the plate can be square or circular.

6. The size of the plate varies from 30cm to 75cm. (in general 30 cm / 45cm / 60 cm / 75cm)

7. The minimum thickness of the plate is 25 mm

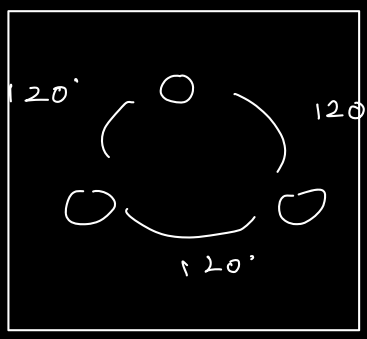
8. The plate is placed in the pit / trench at the proposed depth of the foundation

9. The size of the pit or trench shall not be less than 5 times of the size of the plate

10. The plate is seated at the centre in the pit or trench over a fine sand of maximum thickness 5mm.

11. The smaller sized plates are used in dense or stiff soils where-as larger sized plates are used in loose or soft soils

12. Minimum two dial gauges shall be placed over the plate. In general we place three dial gauge at a angular distance of 120°.



13. Load on the plate can be applied by (i) gravity loading or (ii) Reaction loading.

(i) GRAVITY LOADING: For gravity loading, a platform is created over a vertical column resting on the plate. Loading on the plate is done by placing the sand bags on the constructed platform

(ii) REACTION LOADING: In hydraulic loading, load is provided through a proving ring and a hydraulic jack

14) At each load settlement is recorded at the time interval of 1, 2.25, 4, 6.25, 9, 16 and 25 minutes and thereafter at intervals of one hour.

15) For clay soils, the load is increased when the time settlement-end curve indicates that the settlement has exceeded 70 to 80% of the probable ultimate settlement at that stage or at the end of 24 hours

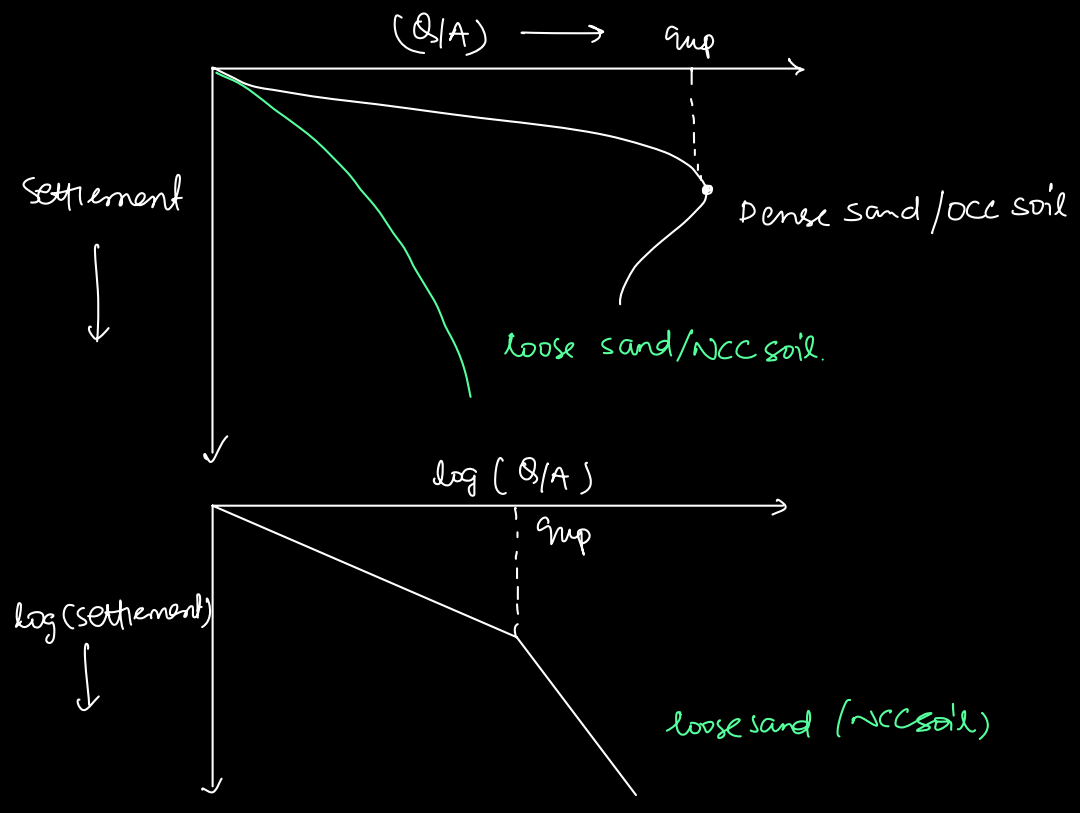
16) For the soils other than clay the load is increased when rate of settlement drops to a value less than 0.02 mm /min.

17) The minimum duration for any load shall however be 60 minutes

18) The loads are applied on the test plate upto the failure (shear failure in soil) or atleast settlement of 25mm has occurred whichever is earlier.

19) Draw the $\left(\frac{\text{Load}}{\text{Area}}\right)$ - settlement plot for results obtained from the test

Pressure : $\frac{\text{load on the plate}}{\text{Area of the plate}}$



20) In case of dense sand or stiff soils the failure point is well defined therefore ultimate bearing capacity for the plate (q_{up}) can be easily calculated from Graph

21) In case of loose or soft soils failure is not well defined due to progressive settlement. In such case the test results are plotted on the log-log scale and then ultimate bearing capacity for the plate (q_{up}) is determined.

Estimate of ultimate bearing capacity v based on shear failure criteria: for foundation

Case I. For clay soils: The ultimate bearing capacity is independent of size of footing therefore,

$$q_{uf} = q_{up}$$

q_{uf}: ultimate bearing capacity for footing

q_{up}: ultimate bearing capacity for plate

Case II: For cohesionless soils: In case of sand UBC is a function of size of footing,

$$\therefore \quad q_{uf} = \frac{B_f}{B_p} \cdot q_{up} \qquad KN/m^2$$

B_f: width of foundation

B_p: width of plate

Estimation of allowable / safe bearing pressure based on settlement failure criterion:

The safe or allowable bearing pressure for the footing can be computed for given permissible settlement of foundation (recommended by IS code) using following equations

$$\frac{S_f}{S_p} = \left(\frac{B_f \ (B_p + 0.3)}{B_p \ (B_f + 0.3)} \right)^2 \qquad \text{(for sands)}$$

[B_f & B_p are in m]

$$\frac{S_f}{S_p} = \left(\frac{B_f}{B_p}\right)^{n+1} \quad (\text{for silts}, \ n = 0.5)$$

$$\left(\frac{S_f}{S_p} = \frac{B_f}{B_p}\right) \quad (\text{for clay soil})$$

S_f : permissible settlement of footing

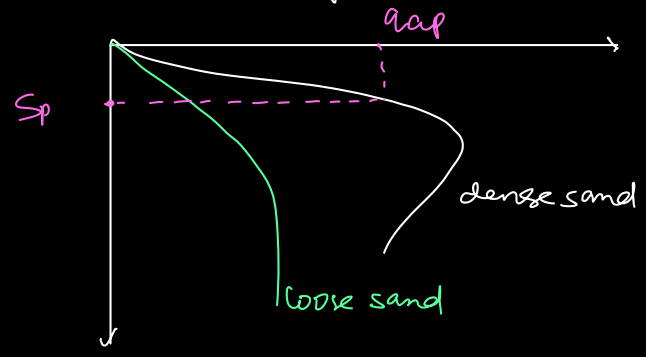
S_p : permissible settlement of plate

B_f : width of footing

B_p : width of plate

↳ from the IS code, we know the value of "S_f". for this permissible settlement of the plate (S_p) using above equations,

↳ Now using pressure settlement curve, allowable bearing pressure can be calculated for the estimated value of permissible settlement of plate (S_p).



↳ Now this q_{ap} is to be converted into the foundation's allowable bearing pressure.

q_{ap} : for plate

for clay, $(q_{af} = q_{ap})$

for sand, $\left(\frac{q_{af}}{q_{ap}} = \frac{B_f}{B_p}\right)$ (∴ Again conversion)

HOUSEL'S ANALYSIS:

According to this analysis, the ultimate load of the plate depends upon the area of plate perimeter of

plate & also influenced by soil characteristics.

In this analysis PLT is carried out on two diff. sizes of plates in the same soil at the same depth from the 1st test:

$$Q_1 = A_1 \cdot m + P_1 n \quad - \text{eq}(i)$$

$$Q_2 = A_2 \cdot m + P_2 n \quad - \text{eq}(ii)$$

Solve eqn (i) and eqn (ii) and estimate soil characteristics m & n

A_1, P_1: for first plate

A_2, P_2: for second plate

So, for foundation the ultimate load can be given as:

$$Q_{uf} = A_f \cdot m + P_f \cdot n$$

So, ultimate bearing capacity for the footing can be given as

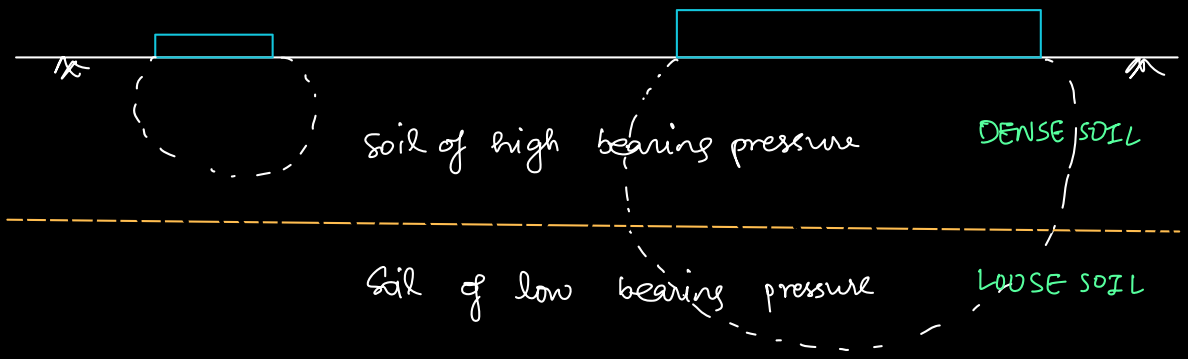
$$q_{uf} = \frac{Q_{uf}}{A_f}$$

A_f: Area of foundation

P_f: Perimeter of foundation

- Some of the major important considerations that should be kept in mind are below:

1. In no case, test plate smaller than 30cm is used
2. The settlement of the foundation cannot exceed about 4 times the size of the plate of 30cm. However larger its width may be. It can be observed in load settlement curve
3. If the soil is not homogeneous upto the large depth relative to the size of foundation, then the PLT can give misleading results.

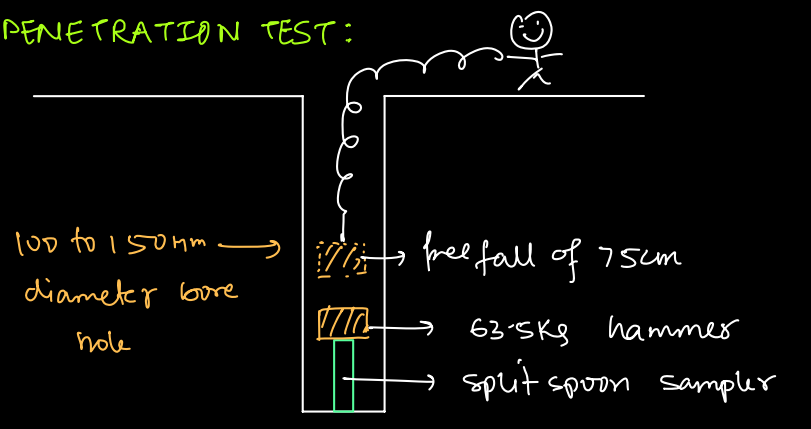


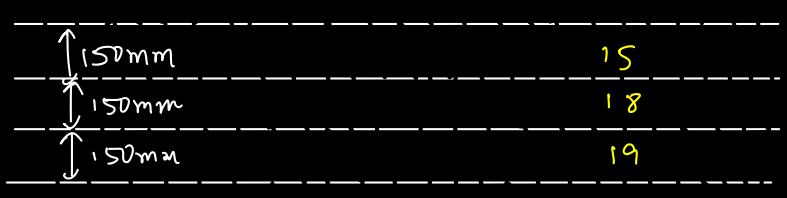
- The foundation being of a much large width its bulb of pressure of pressure of the same load intensity as that on test plate will extend into the weaker strata

4. A plate load test should be performed at the water table level if it is within 1m below the foundation Because due to the capillary the settlement in PLT is underestimated

5. A plate load test is a short duration test. The settlement measured is the only immediate settlement.
In granural soil immediate settlement is taken as total settlement but in case of FGS, immediate settlement cannot be taken as total settlement because Consolidation settlement is a major part of total settlement

2. STANDARD PENETRATION TEST:





1. IS Code 2131 : 1981 can be used for this test
2. A borehole of 100 to 150 mm diameter is prepared
3. To support the soil, casings can be provided in the bore hole
4. The most commonly used sampler is split spoon sample is placed at the base or bottom of the bore hole.
5. Now this sampler is allowed to penetrate into the soil due to self weight
6. Now this split spoon sampler is penetrated by the dynamic action of the hammer of 63.5 kg weight
7. The hammer is dropped from the height of freefall of 75cm
8. The number of blows required to penetrate the sampler can be noted.
9. The number of blows required for the penetration of sampler in first 150 mm thickness of soil can be ignored for security drive
10. The number of blows required for the penetration of sampler in the last 300mm thickness of the soil is termed as SPT number.
11. The entire sample may sometimes sink under its own weight when the soil is very loose or soft in such cases SPT number is considered as zero.
12. The test is performed at every 1.5 m interval or every change in soil strata whichever comes first.
13. The SPT number or SPT value can be empirically correlated to determine the following

- Angle of Internal friction
- Relative density / Density Index

} COHESIONLESS SOILS

- Unconfined compressive strength
- Density

} COHESIVE SOILS.

14. From the above figure, the SPT number can be calculated as

$$N_0 = 18 + 19 = 37$$

N_0: It is termed as SPT NO observed from the field.

15. The observed value of SPT (N_0) is required following sequential Corrections

(i) Overburden correction

(ii) water table / dilatency / fines correction.

A. OVERBURDEN CORRECTION:

When the test is performed at shallow depth, the effective stress is less as a result resistance against the penetration is also less therefore in this test the SPT number observed from the test is underestimated.

Therefore, in this test the SPT number N_0 is corrected for overburden pressure called overburden correction.

That means in this correction, the observed SPT value is increased by introducing the correction factor

$$N_1 = N_0 \times \text{correction factor.}$$

$$N_1 = N_0 \left(\frac{350}{70 + \bar{\sigma}}\right)$$

$\bar{\sigma}$ = overburden pressure (in KN/m^2)

$[\bar{\sigma} > 280\ KN/m^2]$

Note: If overburden pressure ($\bar{\sigma}$) is greater than 280 KN/m^2 then overburden correction is not required. In that case,

$$N_1 = N_0.$$

Overburden correction by other researchers

(i) Gibb's & Holtz: (1957)

$$N_1 = N_0 \left(\frac{500}{1.42\bar{\sigma} + 100} \right) \qquad \bar{\sigma} \geq 282 \text{ KN/m}^2$$

(ii) Peck (1974):

$$N_1 = C_n \cdot N_0$$

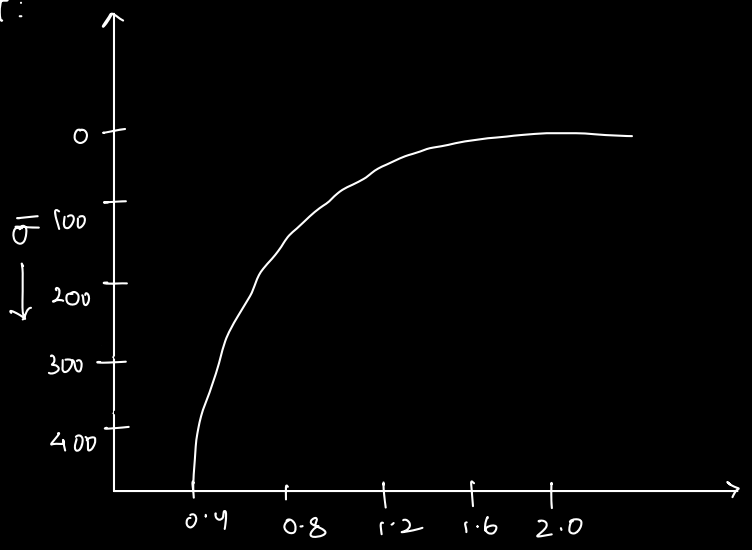
C_n: Normalizing factor

$$N_1 = N_0 \cdot 0.77 \left[\log_{10} \frac{2000}{\bar{\sigma}} \right]$$

if $\bar{\sigma} = 100 \text{ KN/m}^2$, then $C_n = 1$

This equation is useful when $\bar{\sigma} < 100 \text{ KN/m}^2$.

PLOT:



Note: $C_{n\max} = 2.0$

Note: $\frac{N_1}{N_0} > 2.0$ according to peck.

B. WATER TABLE CORRECTION FACTOR / DIGATENCY / FINES CORRECTION:

↳ This correction is applied after the overburden correction

↳ This correction is applied in fine sands & silts

↳ This test is applied when water table is present at or above the test level.

↳ Under the dynamic action of hammer, pore water pressure resistance is developed against the penetration of sampler which

leads to overestimation of SPT number.

↳ In this correction the SPT number obtained after the overburden correction is corrected for water table correction as given below

$$N_2 = 15 + \frac{1}{2}(N_1 - 15)$$

↳ when $N_1 > 15$ then only water table correction is required

If $N_1 < 15$, then water table correction is not required

{ N_2: final corrected SPT number. }

Note: This test is best suitable for cohesionless soils.

This test can also be used for silt & sand. If water table is present in silt & sand pore water resistance is developed

This test is not suitable for clay soil because under the dynamic action of hammer (i) liquefaction may occur (ii) remoulding may occur (iii) pore pressure resistance is developed.

Note: liquefication is generally observed in fine sand & silt during dynamic loading of earthquake.

Note: let N_A, N_B, N_C, N_D are the final corrected values of SPT obtained at different intervals

$$N = N_{avg} = \frac{N_A + N_B + N_C + N_D}{4}$$

N: Average corrected value.

Note: The individual final corrected SPT number should not vary more than 50% of average corrected SPT number.

for ex: $N_A = 30$

$N_B = 32$

$N_C = 62$

$N_D = 36$

$$N_{avg} = \frac{30 + 32 + 62 + 36}{4} = 40$$

$0.5N = 20$

$1.5N = 60$

20 ——— 60

↳ No any value of SPT should lie out of the range 20 to 60 for given problem.

In the given problem, $N_c = 60$ which is greater than $(1.5N = 60)$

In that case this value can be discarded / deleted and the new value of avg SPT(N) shall be calculated with remaining values

$$N = N_{avg} = \frac{N_A + N_B + N_D}{3} = \frac{30 + 32 + 36}{3} = 33$$

$$\boxed{N = 33}$$ # Final

Question:

Q.6 During the subsurface investigations for design of foundations, a standard penetration test was conducted at 4.5 m below the ground surface. The record of number of blows is given below :

Penetration depth (cm)	Number of blows
0 – 7.5	3
7.5 – 15	3
15 – 22.5	6
22.5 – 30	6
30 – 37.5	8
37.5 – 45	7

} 150m का होता है

Assuming the water table at ground level, soil as fine sand and correction factor for overburden as 1.0, the corrected 'N' value for soil would be

(A) 18 (B) 19 (C) 21 (D) 33

that matters.

Solution:

$N_0 = 12 + 15 = 27$

$N_1 = 27 \times 1 = 27$

$N_2 = 15 + \frac{1}{2}(N_1 - 15)$

$= 15 + \frac{1}{2}(27 - 15)$

$\Rightarrow 21$ Ans

Use of SPT number in the estimation of bearing capacity pressure:

Based on shear criteria:

→ Based on shear failure criteria ultimate bearing capacity can be determined using SPT No.

→ SPT No and angle of internal friction ϕ can be empirically correlated.

→ Based on the value of "N", "ϕ" can be determined. Once the value of ϕ is known we can find the bearing capacity factors N_c, N_q, N_γ & then we can estimate ultimate bearing capacity.

Based on settlement criteria:

Based on settlement failure criteria, allowable or safe

bearing pressure can be determined.

1. PECK HAZEN'S EQUATION:

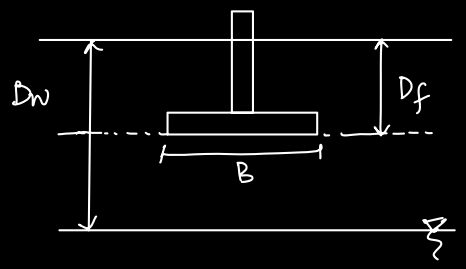
$$q_{a\text{-}net} = 0.44\, N\, S\, C_W \quad KN/m^2$$

N : average corrected SPT number

S : permissible settlement (in mm)

C_W : Water table correction factor.

$$C_W = 0.5\left(1 + \frac{D_W}{D_f + B}\right)$$



D_W: Depth of water table measured from ground surface.

2. TENG'S EQUATION:

The safe bearing pressure can be given as

$$q_{n\text{-}p} = 1.4\,(N-3)\left[\frac{B+0.3}{2B}\right]^2 S\, C_D\, R_W^* \quad KN/m^2$$

N : Average SPT number

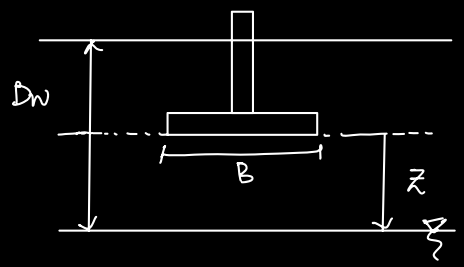
S : permissible settlement

C_D ; Depth correction factor.

R_W^* : Water table correction factor

C_D : $1 + \frac{D_f}{B} \leq 2$

R_W^* : $0.5\left(1 + \frac{Z}{B}\right)$



Z : depth of water table measured from base of the foundation.

if $Z = 0$, $R_W^* = 0.5$ &

$Z = B$, $R_W^* = 1$.

3 IS CODE METHOD:

use of Terzaghi's equation with some modifications.

$$q_{a\text{-}net} = 1.38\,(N-3)\left[\frac{B+0.3}{2B}\right]^2 S \cdot C_W$$

↳ used for footings.

Peck, Hanson & Thornburn have given correlation for the allowable bearing pressure for the allowable bearing pressure for the Raft foundation

$$q_{a\text{-}net} = 0.88 \cdot N \cdot S \cdot C_W \quad KN/m^2$$

for permissible settlement of 25mm

$$q_{a\text{-}net} = 22 N C_W \quad KN/m^2$$

$$C_W = 0.5\left(1 + \frac{D_W}{D_f + B}\right)$$

D_W: Depth of water table measured from the ground surface.

let water table is at the Ground surface

$$\{q_{a\text{-}net} = 11 \cdot N \quad KN/m^2\}$$

Question

Q.12 **A strip footing is resting on the ground surface of a pure clay bed having an undrained cohesion c_u. The ultimate bearing capacity of the footing is equal to**

(A) $2\pi c_u$ **(B)** πc_u **(C)** $(\pi+1)c_u$ **(D)** $(\pi+2)c_u$

[GATE 2017 : IIT Roorkee]

Solution

Strip footing

$\phi = 0$

cohesion = C_u

ultimate bearing capacity ?

$$q = CN_c + qN_q^{\,0} + \frac{1}{2}B\gamma N_\gamma^{\,0}$$

$$(q = cN_c)$$

$$q_u \Rightarrow (\pi+2)C_u$$

Question

Q.13 A plate load test was conducted in sand on a 300mm diameter plate. If the plate settlement was 5mm at a pressure of 100kPa, the settlement (in mm) of a 5m×8m rectangular footing at the same pressure will be

(A) 9.4 (B) 18.6

(C) 12.7 (D) 17.8

[GATE 2001 : IIT Kanpur]

Solution

300mm dia plate

5mm at a pressure of 100kPa

(5m x 8m rectangular footing)

$$\frac{S_f}{S_p} = \left[\frac{B_f\,(B_p + 0.3)}{B_p\,(B_f + 0.3)}\right]^2$$

$$S_f = 5mm \cdot \left[\frac{5 \times .6}{.3}\,\frac{}{5.3}\right]^2 \times 3.56$$

$$\boxed{S_f = 17.8mm}$$ Ans

Question

Q.14 The plate load test was conducted on a clayey strata by using a plate of 0.3m×0.3m dimension, and the ultimate load per unit area for the plate was found to be 180 kPa. The ultimate bearing capacity (in kPa) of a 2 m wide square footing would be

(A) 27 (B) 180

(C) 1200 (D) 2000

[GATE 2017 : IIT Roorkee]

Solution

clayey strata

$B_p = 0.3m$

$q_{up} = 180$ kPa

$\approx$ 180 only

ultimate load is same for clay.

Question

Q.2 A strip footing of 2.5 m width with its base at a depth of 1.5 m below ground surface, rests on a saturated clay. The clay properties are.

$C_u = 40 kN/m^2$ $\phi_u = 0^0$, $C' = 10 kN/m^2$ $\phi' = 20^0$

The water table is at 3m below ground surface and $\gamma_{bulk} = 19\ kN/m^3$.

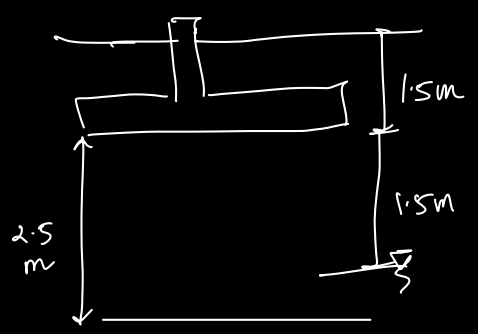
The bearing capacity factors may be taken from the following table :

ϕ	N_c	N_q	N_γ
0^0	5.7	1	0
20^0	17.7	7.4	5

The ultimate bearing capacity of the soil for short term stability condition is

(A) 256.50 kN/m² (B) 412.34kN/m²

(C) 506.65kN/m² (D) 280.50 kN/m²



Solution :

Strip

$B = 2.5m$

$D_f = 1.5m$

Saturate clay

$S = 1$

C_u, 40 KN/m²

$\phi_u = 0°$, $C' = 10 KN/m^2$

$\phi' = 20°$

1.5+2.5 = 4.

Short term / undrained condition.

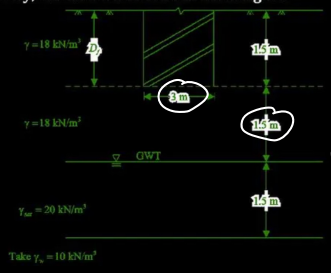
$q_u = C_u N_c + q N_q + \frac{1}{2} B\gamma N_\gamma R_\gamma$*

$= 40 \times 5.7 + 19 \times 1.5 \times 1$

$= 256.5 kN.$

Question:

Q.3 Using Peck – Hansen's equation find net safe bearing pressure (KN/ m²) below the foundation for the data shown in fig. Total permissible settlement is 40 mm. The SPT is conducted at 1.5 m interval as shown in figure and observed value of SPT are 10, 15, 20 at a depth of 1.5 m, 3 m, 4.5 m respectively, the unit wt. of soil shown in figure



10 N_{01} 20

15 N_{02} 22.5

20 N_{03} 27.5

Solution:

Net safe bearing capacity

Peck · Hansen.

$S = 40$ mm

$N_0 = 35$

$q = 18 \times 1.5 = 27$ KN/m²

@ 1.5m depth, $N_1 = 10\left[\frac{350}{70+27}\right]$

$= 3.6 \times 10$

$\because \frac{N_1}{N_0} \geq 2$ true

$N_1 = 20$, $N_2 = N_1 = 20$

@ 3m depth $N_1 = N_0\left(\frac{350}{70+54}\right) \Rightarrow N_1 = N_0 \times 2.82$

$\because \frac{N_1}{N_0} \geq 2$ true

$N_1 = N_0 \times 2 = 2 \times 15 = 30$

NT Correction,

$N_2 = 15 + \frac{N_1 - 15}{2}$

$= 15 + 7.5 = 22.5$

@ 4.5 depth,

$N_1 = N_0\left(\frac{350}{70+18\times3+10\times1.5}\right)$

$N_1 = N_0\left(\frac{350}{139}\right)$

$N_1 = N_0 \times 2.52$

$\because \frac{N_1}{N_0} \geq 2$ true, $N_1 = N_0 \times 2 = 40$

WT correction: $N_2 = 15 + \frac{1}{2}(40-15) = 27.5$

$$N = N_{avg} = \frac{20 + 22.5 + 27.5}{3} = 23.33 \approx 23$$

$$\boxed{N = 23}$$

$$0.5 \times 23 \leq N \leq 1.5 \times 23$$

$(11.5 \leq N \leq 34.5)$ This all approve

Peck Hazen equation:

$$q_{a\text{-}net} = 0.44\, N\, C_w\, S$$

$$= 0.44 \times 23 \times 40 \times \left[0.5\left(1 + \frac{D_w}{D_f + B}\right)\right]$$

$$= 0.44 \times 23 \times 40 \times \left[0.5\left(1 + \frac{3}{3+1.5}\right)\right]$$

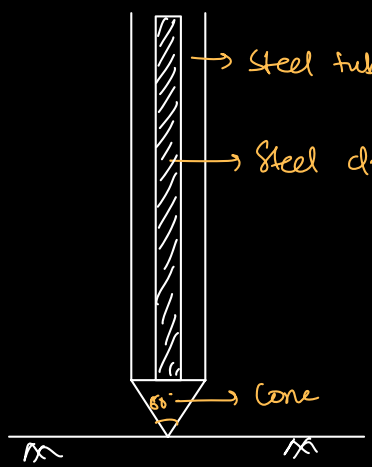
$$= 337.33 \text{ KN/m}^3.$$

3. STATIC CONE PENETRATION TEST (CPT Test):

(i) It is also termed as Dutch static cone penetration test

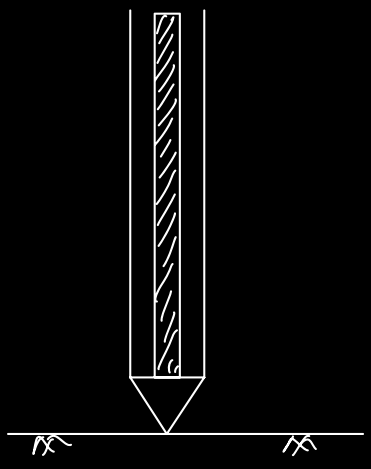
(ii) For this test IS code 4968 (Part 3) can be used

(iii) This test is performed to measure the continuous cone resistance (q_c) which can be expressed as CPT value in KN/m^2 or in kg/cm^2.

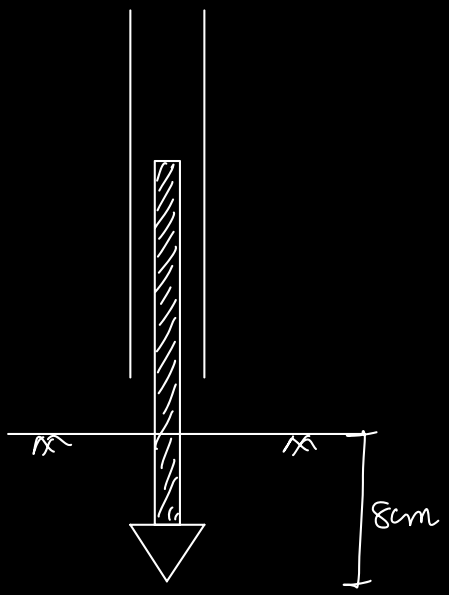


(iv) A cone of base area of $10\ cm^2$ (3.6cm in dia) and vertex angle 60° is forced into the ground.

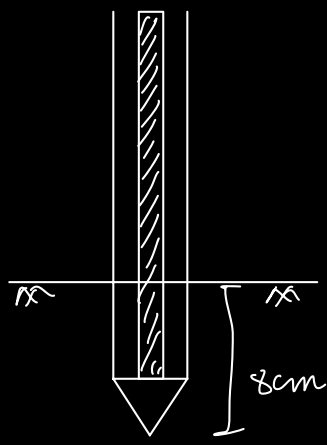
(v) The cone is carried at the lower end of steel driving rod which passes through the steel tube with external diameter equal to the dia of base of the cone.



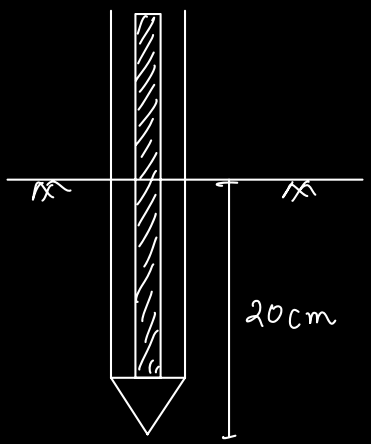
1st step.



2nd step



3rd Step



4th step

(vi) To determine the cone tip resistance the cone alone is first forced down for a distance of 8 cm and the maximum value of cone resistance is measured

$$q_c = \frac{Q_c}{\text{Area}}$$

Q_c : load required to force down the cone alone.

q_c : cone resistance or tip resistance

vii) The steel tube is then pushed down upto the cone

viii) Both steel tube & cone together are further penetrated through a depth of 20 cm to determine the total resistance which includes both cone resistance & side friction resistance along tube

$$Q_t = Q_c + Q_f$$

$$Q_{sf} = Q_t - Q_c$$

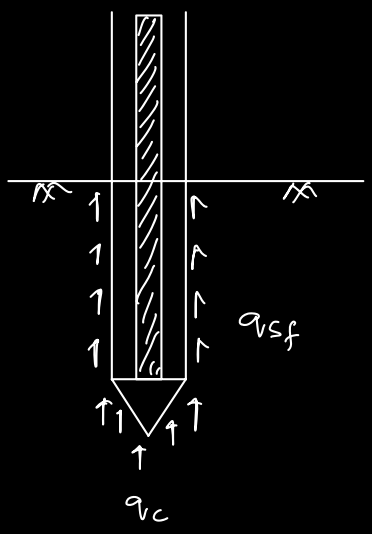
Q_t : Total force required

Q_c : force required for cone

Q_{sf} : force required for steel tube

$$q_{sf} = \frac{Q_{sf}}{\text{Surface area of steel tube}} \quad KN/m^2$$

q_{sf} : side friction / skin friction.



Friction Ration (R_f) : It can be defined as the ratio of sleeve friction to the tip / cone resistance expressed as percentage.

$$R_f = \frac{\text{Sleeve friction}}{\text{tip resistance}} = \frac{q_{sf}}{q_c}$$

Note : R_f decrease with increase in mean size of particles

Note : As the SPT is performed at different intervals, in the same way CPT test is also performed at different intervals & avg value of CPT number is considered.

Note : This avg CPT value can be correlated to determine the safe bearing pressure using different empirical equation.

Question :

At a site, static CPT was carried out, the measured point (tip) resistance (q_c) was 1000 Kpa at a certain depth, The friction ratio (f_R) was estimated as 1% @ the same depth. The value of skin side friction (in Kpa) at that depth was?

$$\left\{ 1000 \times \frac{1}{100} = 10 \text{ Kpa} \right\}$$

Settlement in footing :

$$S_t = S_i + S_p + S_s$$

S_i : Intermediate settlement

S_p : Primary consolidation settlement

S_s : Secondary consolidation settlement

1. Immediate settlement :

→ Immediate or elastic settlement which takes place immediately or short time (7 days) after application of loads.

→ In a clay soil it is also known as distortion settlement and it occurs due to change in the shape of the soil structure / pattern of arrangement of soil without a change in volume or water content.

→ In clay soil, the immediate settlement is very less as compared to granular soil therefore it can be ignored in clay soil

→ Granular soil is predominent in coarse grain soil

→ In granular soil Immediate settlement accounts virtually for the entire settlement.

The immediate settlement according to theory of elasticity shall be given as

$$S_i = \frac{q\, I_f\, B\,(1-\mu^2)}{E}$$

q : net pressure at the base of the foundation

B : width of the footing ($B < L$)

μ : Poisson's ratio

E_s : Modulus of elasticity of soil

I_f : Influence factor which depends upon the shape & rigidity of foundation.

Shape	(I_f) Flexible foundation			I_f (Rigid)	I_f (Rigid) IS8009 : 1976
	Centre	Corner	Avg		
Circular	1	0.64	0.85	0.86	$1 \times 0.8 = 0.8$
Square	1.12	0.56	0.95	0.82	$1 \times 1.12 = 0.9$
Rectangular					
$\frac{L}{B} = 1.5$	1.36	0.68	1.20	1.06	$1.36 \times 0.8 = 1.09$
$\frac{L}{B} = 2$	1.52	0.76	1.30	1.20	$1.52 \times 0.8 = 1.22$

Note: As suggested by IS code 8009 : 1976 Part I the total settlement of rigid foundation is taken as 0.8 times the settlement at the centre of the flexible foundation.

Question:

A footing 4m x 2m in plan, transmits a pressure of 150 KN/m^2 on a cohesive soil having $E = 6 \times 10^4$ KN/m^2 and $\mu = 0.50$. Determine the immediate settlement of the footing at the center assuming it to be (i) flexible (ii) Rigid.

Solution: Flexible. $B/L = 2$

$I_f = 1.52$

$$S_i = \frac{q_u \cdot I_f \cdot B (1-\mu^2)}{E}$$

$$= \frac{150 \times 1.52 \times 2 (1 - 0.5^2)}{6 \times 10^4} = 5.7 \text{ mm @ centre}$$

Rigid, $I_f = 1.2$

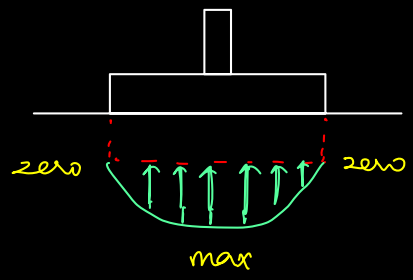
$$S_i = 5.7 \times \frac{1.2}{1.52} = 4.5 \text{ mm} \quad \text{or} \quad 0.8 \times 5.7 = 4.56 \text{ m}$$

Contact pressure distribution:

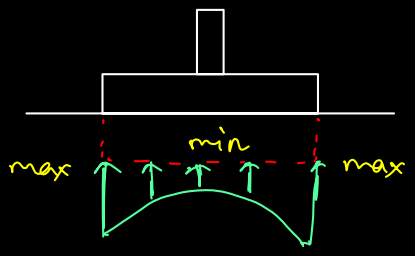
Contact pressure distribution is the response of the soil when footings are placed over its surface. It depends upon the flexibility or rigidity of the foundation and type of soils.

NOTE: The foundation of the earth embankment can be treated as flexible foundation & raft/mat foundation can be treated as Rigid.

(i) Rigid footing: In case of rigid footing the settlement is uniform but contact pressure distribution depends upon the types of soil.

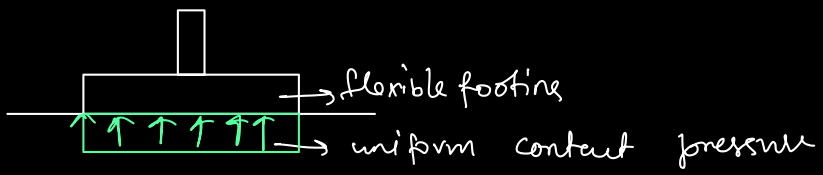


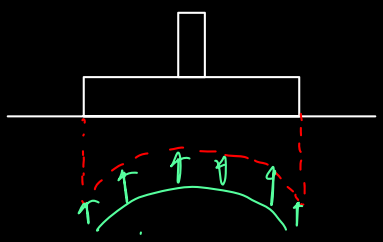
(SANDS)



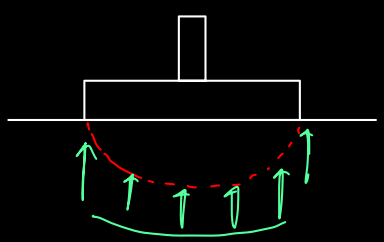
(CLAY)

(ii) Flexible footing: In case of flexible footing the contact pressure is uniform but the settlement depends upon type of soil.



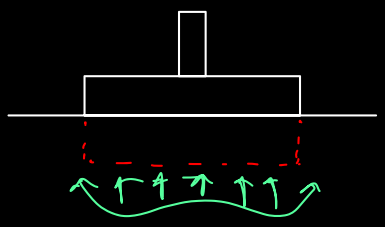


SAND.



CLAY

Note: Contact pressure distribution in silty soils in case of rigid footing can be shown as.



SILTY SOILS

Note: In case of rigid footing in sandy soil, the CP distribution is maximum at the centre & zero at the edges if the rigid footing is very deep then CP distribution in sandy soil is more like rigid footing in clay soil. That is max at edges and minimum at the centre.

Question:

width of square footing = dia of circular footing both placed in surface of sandy soil. the ratio of qult of both will be?

Solution:

SANDY SURFACE

$$q_{u(sq)} = 1.3 \cancel{cN_c^{0}} + \cancel{qN_q^{0}} + 0.4 N_\gamma B \gamma$$

$$q_{u(cir)} = 1.3 \cancel{cN_c^{0}} + \cancel{qN_q^{0}} + 0.3 N_\gamma D \gamma$$

$$\frac{q_{cir}}{q_{sq}} = \frac{0.3}{0.4} = \frac{3}{4} \quad [\because D = B]$$

DEEP FOUNDATION

CLASSIFICATION OF PILES:

(i) Based on material:

a) Timber

b) Steel

c) Concrete

(d) Composite

(ii) Based on cross section:

a) Circular

b) Square

c) Hexagonal

d) I- section

e) H- section

(iii) Based on shape:

a) cylindrical

b) Tapered

c) Under-reamed pile

(iv) Based on mode of load transfer

(a) Endbearing pile

(b) Friction pile

(c) Bearing friction pile

(v) Based on method of forming:

a) Cast in situ pile

b) Precast pile

c) Prestressed pile

(vi) Method of installation:

a) displacement / driven pile:

Displacement piles are essentially precast piles. which are driven in the ground by action of Hammer. In case of displacement piles bearing & friction resistance are effectively developed.

(b) BORED PILE: In this case a bore hole is prepared & either precast or cast in situ pile is placed in the borehole. Bored piles are less efficient

vii) Based on function of pile:

a. Compaction file: Compaction piles are used to compact the

loose granular soil, thus increasing their bearing capacity the compaction pile themselves do not carry any load. Hence they may be of weaker material sometime of sand only. The pile tube driven to compact the soil is gradually taken out and sand is filled in its place to form SAND PILE.

b. Tension pile or uplift pile: when the structure is subjected to uplift pressure due to hydrostatic pressure & overturning the tension piles are provided.

c. Anchor piles: Anchor pile provide the anchorage against the horizontal pull.

d. Fender Piles: Fender piles and dolphines are used to protect the water front structures against the impact from ships and floating objects.

e. Sheet piles: Sheet piles are commonly used as bulk heads they are also used to reduce the seepage and uplift pressure at downstream side of the hydraulic structures

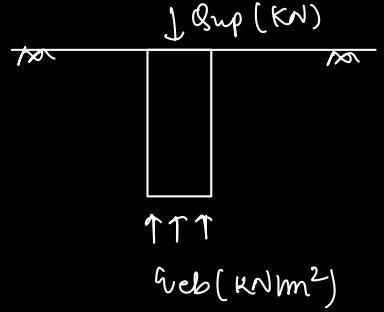
f. Batter piles: To resist the large lateral / inclined loads piles are placed at some inclination known as Batter piles.

TYPES OF PILE BASED ON ACTION OR MODE OF LOAD TRANSFER:

(i) END BEARING PILE: a. This pile rest over hard or stiff soil strata below the ground surface

(b) This pile derives it's capacity by End bearing resistance ocuring at the base of the pile

(c) The depth of this pile depends upon the depth of hard strata.



$Q_{up} = Q_{eb} = q_{eb} \cdot A_b$

A_b: bearing Area

: $\frac{\pi}{4} D^2$ or B^2

Q_{eb}: End bearing force in KN

q_{eb} = End bearing resistance in KN/m²

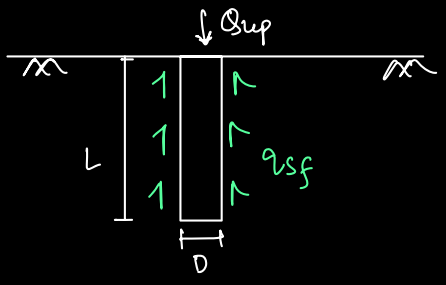
Q_{up}: ultimate load carrying capacity

(ii) FRICTION PILE: a. The pile which derives its load carrying capacity by skin friction resistance acting over its surface is termed as floating pile or friction pile.

It is also termed as hanging pile.

b. It is generally provided in loose sand and soft clay soil with larger depth.

c. It's length may vary 10m to 20m



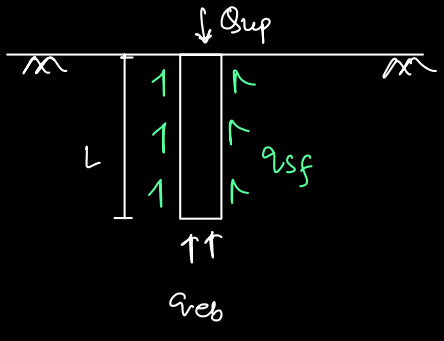
q_{sf}: skin friction resistance (in KN/m²)

$Q_{up} = Q_{sf} = q_{sf} \cdot A_s$

$A_s = \pi DL$ (for circular pile)

$= 4BL$ (for square pile)

(iii) Bearing friction pile: a. The ultimate load carrying capacity of the pile is derived due to both end bearing resistance and skin friction resistance.



b. It is generally provided in medium to dense / stiff soil.

$Q_{up} = Q_{eb} + Q_{sf}$

$(Q_{up} = q_{eb} \cdot A_b + q_{sf} \cdot A_s)$: KN.

METHODS TO CALCULATE ULTIMATE LOAD CAPACITY OF PILE:

1. Static pile load method (Analytical Method)
2. Dynamic method (Pile driving formula)

3. Field Methods

a. Pile load test

b. Correlation with SPT-value

c. Correlation with CPT-value

STATIC PILE LOAD METHOD:

→ when a compressive load Q is applied at the top of pile the pile will tend to move vertically downward relative to the surrounding pile

→ This will cause the development of shear stresses over the surface of the pile shaft.

→ As a result the applied load is distributed as friction load along certain length of the pile. measured from the top.

→ As the load is increased till certain load level, the entire length of the pile is involved in generating the frictional resistance.

→ If the load at the top of pile is further increased then it is to be transferred to the soil at the base of the pile. This load is known as point load (end bearing load) goes on increasing till the soil at the base of the pile fails by punching shear failure.
This time ultimate load capacity of pile Q_{up} is the sum of Q_{sf} & Q_{eb}

$$Q_{up} = Q_{sf} + Q_{eb}$$

→ If $Q_{sf} >> Q_{eb}$; pile is termed as friction pile

→ If $Q_{eb} >> Q_{sf}$: pile is termed as end bearing pile.

→ It has been observed that

a. when the ultimate skin friction is mobilized that time only a fraction of the ultimate point load is mobilized

b. when the ultimate point load is mobilized the skin friction resistance has decreased to a lower value then its peak.

→ It has been observed that

a. The relative movement required to fully mobilise the friction resistance is approximately equal to 0.5 to 1% of the dia of pile.

b. The vertical movement required to fully mobilise the base resistance is about 10-20% of the base diameter.

Estimation of ultimate and allowable load on pile:

$$Q_{up} = Q_{eb} + Q_{sf}$$

$$Q_{up} = q_{eb} \cdot A_b + q_{sf} \cdot A_s$$

$$Q_{ap} = \frac{Q_{up}}{FOS}$$

$$Q_{ap} = \frac{Q_{eb} + Q_{sf}}{FOS}$$

Q_{up}: ultimate load capacity of pile

Q_{ap}: Allowable load on pile

FOS: Factor of safety (minimum 2.5 should be taken)

Note: In some cases two different values of FOS can be taken for point resistance and skin friction resistance.

$$Q_{ap} = \frac{Q_{eb}}{FOS_1} + \frac{Q_{sf}}{FOS_2}$$

Methods for Estimation of Q_{eb} & Q_{sf}:

Methods for Q_{eb}.

(i) Meyerhoff

(ii) Vesic's method

(iii) Janbu method

(iv) Skempton's method

Methods for Q_{sf}

(i) α-method [clay soil]

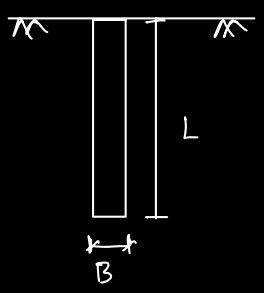
(ii) λ-method [clay soil]

(iii) δ-method [sandy soil]

Estimation of Q_{eb} :

The general equation of end bearing or point resistance can be given as

$$q_{eb} = cN_c + \bar{\sigma} N_q + \frac{1}{2} B\gamma N_\gamma$$



In case of deep foundation (pile)

$$B << L$$

$$q_{eb} = cN_c + \gamma L N_q + \frac{1}{2} B\gamma N_\gamma$$

from the above equation it is clear that third term is much smaller than 2nd term ($\frac{1}{2} B\gamma N_\gamma << \gamma L N_q$)

Therefore third term can be ignored.

$$\therefore q_{eb} = cN_c + \bar{\sigma} N_q \quad KN/m^2$$

(i) For C-ϕ soil :

$$Q_{eb} = q_{eb} \cdot A_b$$

$$Q_{eb} = [cN_c + \bar{\sigma} N_q] \cdot A_b$$

(ii) SANDY SOIL (Cohesionless) :

$$Q_{eb} = q_{eb} \cdot A_b$$

$$Q_{eb} = (\bar{\sigma} N_q) \cdot A_b$$

$\bar{\sigma}$: Effective stress at the base of the pile.

(iii) Clay soil ($\phi = 0$)

$$Q_{eb} = q_{eb} \cdot A_b$$

$$Q_{eb} = cN_c \cdot A_b \quad (KN)$$

C : undrain cohesion. of the soil at the base of the pile

N_c : Bearing capacity factor, it can be taken as 9 as suggested by Skempton & Meyerhoff also

$$Q_{eb} = 9 \cdot C \cdot A_b$$

A_b : Bearing Area

$A_b : \frac{\pi}{4} \cdot D^2$ (CIRCLE) & $A_b : B^2$ (SQUARE)

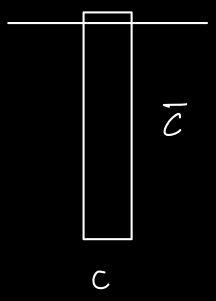
Note: In case of cohesive soils, end bearing resistance is much smaller than skin friction resistance therefore in case of cohesive soils. end bearing force can be neglected

Estimation of skin friction force (Q_{sf}):

(i) α-method: It is used for clay soil

$$Q_{sf} = q_{sf} \cdot A_s$$

$$Q_{sf} = \alpha \cdot \bar{C} \cdot A_s$$



$\bar{C}$: undrain cohesion over the depth of pile (above the base of pile.)

α: termed as adhesion factor or reduction factor. The value of α depends upon the type of soil which can be determined by pile load test

The value of α can be taken as 0.4 if length of pile not exceeding the 20D. For the deeper piles the value of α varies from 0.4 to 1.

ultimate load capacity for clay soil can be given as

$$Q_{up} = Q_{eb} + Q_{sf}$$

$$= 9 \cdot C \cdot A_b + \alpha \cdot \bar{C} \cdot A_s$$

for circular pile

$$A_b = \frac{\pi}{4} \cdot D^2$$

$$A_s = \pi D L$$

For square pile.

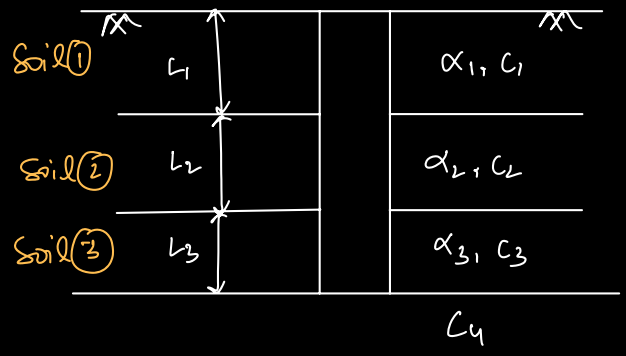
$$A_b = B^2$$

$$A_s = 4BL$$

C: uniform cohesion at base

$\bar{C}$: uniform cohesion over the depth.

SPECIAL CASE : Stratified soil deposites



$$Q_{up} = Q_{eb} + Q_{fs}$$

$$Q_{up} = 9 C A_b + \alpha \bar{C} A_s$$

$$\Rightarrow 9 \cdot C_4 \cdot \frac{\pi}{4} \cdot D^2 + \alpha_1 C_1 \pi L_1 D + \alpha_2 C_2 \pi L_2 D + \alpha_3 C_3 \pi L_3 D$$

$$\Rightarrow 9 \cdot C_4 \cdot \frac{\pi}{4} \cdot D^2 + D (\alpha_1 C_1 L_1 + \alpha_2 C_2 L_2 + \alpha_3 C_3 L_3)$$

(iii) λ-method : It is used for clay soils

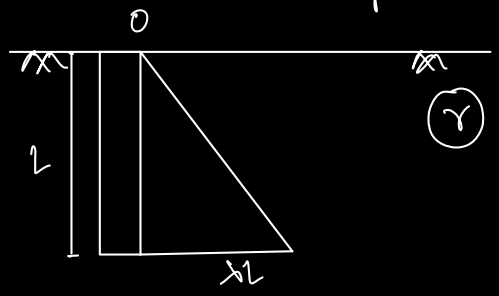
$$Q_{sf} = q_{sf} \cdot A_s$$

$$Q_{sf} = \lambda [\bar{\sigma}_m + 2\bar{C}_m] \cdot A_s \quad (KN)$$

λ : Dimensionless parameter / constant

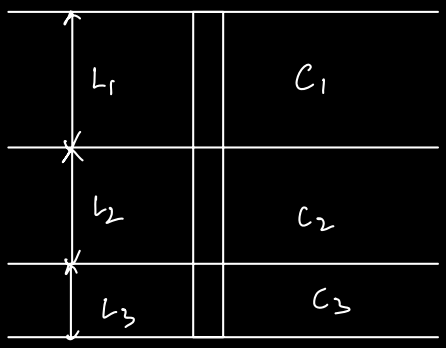
A_s : Surface Area

$\bar{\sigma}_m$: mean or Average effective overburden pressure over the depth of pile.



$$\bar{\sigma}_m = \frac{0 + \gamma L}{2} = \frac{\gamma L}{2}$$

$\bar{C}_m$ = mean or average undrain cohesion over the depth of pile.



$$\bar{C}_m = \frac{(C_1 L_1 + C_2 L_2 + C_3 L_3)}{(L_1 + L_2 + L_3)}$$

Note : ultimate load on the bearing frichion pile in clay soil can be given as

$$Q_{up} = Q_{eb} + Q_{sf}$$

$$Q_{up} = q_{cb} \cdot A_b + q_{sf} \cdot A_s$$

$$Q_{up} = q_c \cdot A_b + \lambda [\bar{\sigma}_m + 2\bar{c}_m] \cdot A_s$$

Finally for clay soil we have two equation to find Q_{up}

$$Q_{up} = q_c \cdot A_b + \alpha \bar{c} \cdot A_s \quad - \text{eq}(1)$$

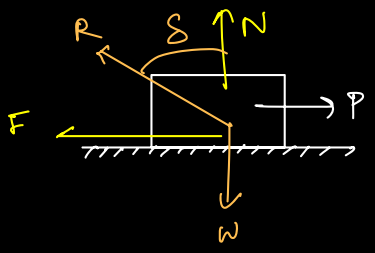
$$Q_{up} = q_c A_b + \lambda [\bar{\sigma}_m + 2\bar{c}_m] \cdot A_s. \rightarrow \text{eq}(11)$$

Note: Based on the given data in the question either eq(1) or eqn (2) is used.

(iii) This method is used for sands (cohesionless soils):

$$Q_{sf} = q_{sf} \cdot A_s$$

$$\left(Q_{sf} = [K \cdot \bar{\sigma}_m \cdot \tan\delta] \cdot A_s\right) \text{ KN}$$



$$\tan\delta = \frac{F}{N}$$

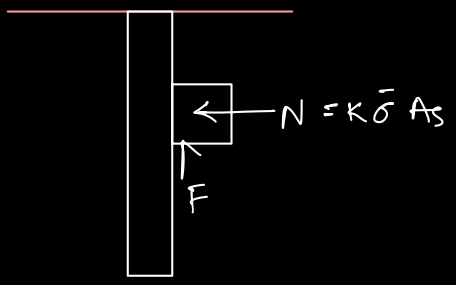
$$F = N \cdot \tan\delta$$

$$F = \mu N$$

$$\therefore \boxed{\mu = \tan\delta}$$

μ : coefficient of friction.

δ : friction angle.



$$F = N \tan\delta$$

$$F = K \cdot \bar{\sigma} \cdot A_s \cdot \tan\delta$$

$$Q_{sf} = [K \cdot \bar{\sigma}_m \cdot \tan\delta] \cdot A_s \quad (\text{KN})$$

K : lateral Earth pressure coefficient

δ : friction angle between soil & pile

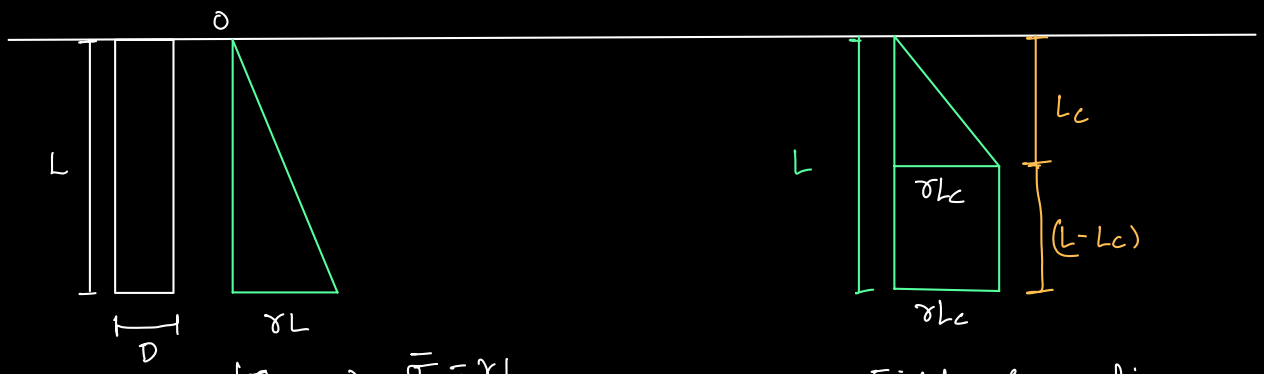
Pile material	δ	Value of K	
		Loose Sand	Dense Sand
Steel	2ϕ	0.5	1.0
Concrete	$\frac{3}{4}\phi$	1.0	2.0
Timber	$\frac{2}{3}\phi$	1.5	4.0

ultimate load on the bearing friction pile in sands:

$$Q_{up} = Q_{eb} + Q_{sf}$$

$$Q_{up} = (\bar{\sigma} N_q)(A_b) + (K \cdot \bar{\sigma}_m \tan\delta)\, A_s.$$

Effect of Arching in Granular soil: [SPECIAL CASE].



(Theory) (without considering Arching)

$\bar{\sigma} = \gamma L$

$\bar{\sigma}_m = \frac{\gamma L + 0}{2}$

Field observation [considering arching effect]

$\bar{\sigma} = \gamma L_c$, $\bar{\sigma}_m = \frac{0 + \gamma L_c}{2} + \gamma L_c$

$\bar{\sigma}_m = \frac{3}{2}\gamma L_c$.

↳ when the pile is provided in granular soil the point resistance (end bearing resistance) is considered as $\bar{\sigma} N_q$

↳ This $\bar{\sigma}$ goes on increasing as the embedded length of pile increases but many field observation suggested that this effective stress increases upto a certain depth then it remains constant.

↳ The depth upto which effective stress increases is termed as critical depth or height of the pile

↳ This critical length of pile, L_c can be taken as,

$L_c = 15 \cdot D$ [loose to medium dense sand]

$L_c = 20 \cdot D$ [Dense Sand]

where D is the diameter of pile.

↳ It is recommended that the point resistance or end bearing resistance (q_{eb}) shall be limited to 11000 KN/m² in normal silica sand and 5000 KN/m² in calcerous sand.

↳ The arching effect and the concept of critical length of pile shall not be considered in clay soil.

Without considering arching:

$$Q_{up} = \bar{\sigma} N_q A_b + (K \cdot \bar{\sigma}_m \tan\delta) \cdot A_s$$

$$= \gamma L N_q A_b + K \cdot \frac{\gamma L}{2} \cdot \tan\delta \cdot A_s.$$

$$Q_{up} = (\gamma L) N_q \cdot \frac{\pi}{4} D^2 + K \cdot \left(\frac{0+\gamma L}{2}\right) \cdot \tan\delta \cdot \pi D L.$$

With considering arching effect:

$$Q_{up} = (\bar{\sigma} N_q) \cdot A_b + (K \cdot \bar{\sigma}_m \cdot \tan\delta) \cdot A_s$$

$$Q_{up} = \gamma L_c N_q \cdot \frac{\pi}{4} D^2 + K \cdot (\pi D) \tan\delta [\bar{\sigma}_m \cdot L]$$

$$Q_{up} = \gamma L_c N_q \cdot \frac{\pi}{4} D^2 + K \cdot \pi D \tan\delta \left[\frac{\gamma L_c}{2} \cdot L_c + \gamma L_c (L - L_c)\right]$$

Ultimate bearing capacity of Silty soil [C-ϕ soil]:

$$Q_{up} = Q_{eb} + Q_{sf}$$

$$Q_{up} = (9C + \bar{\sigma} N_q) \cdot A_b + [\alpha \bar{C} + K \cdot \bar{\sigma}_m \cdot \tan\delta] \cdot A_s$$

or

$$Q_{up} = (9C + \bar{\sigma} N_q) \cdot A_b + \left[\lambda (\bar{\sigma}_m + 2\bar{C}_m) + K \cdot \bar{\sigma}_m \tan\delta\right] \cdot A_s$$

Question

L = 10m long concrete pile

500 x 500 mm

C = 40 kPa

clay में

ULC = 700 kN

cross section = 250 x 250 mm

length = 20m

$$Q_u = 9 \cdot C \cdot A_b + \alpha \cdot C \cdot A_s$$

$$700 = 9 \cdot 40 \cdot (500)(500) + \alpha \cdot 40 \cdot (4 \times .5 \times 10)$$

$$\alpha = 0.7625$$

$$Q_{unew} = 9 \times 40 \times .250 \times .250 + 0.7625 \times 40 \times (4 \times .25 \times 20)$$

$$= 632.5 \text{ kN}$$

Question2:

$D = .5\ m$

$L = 20m$

(SANDY)

Friction pile.

$\phi = 30°$

$\gamma_d = 20\ KN/m^3$

$\delta = 20°$

$K = 2.7$

$N_q = 25$

$20 \times 20 \times 25 = 10000$ thus OK

$$Q_u = \bar{\sigma} N_q \cdot A_b + (K \bar{\sigma}_m \tan\delta) \cdot A_s$$

$$= 10000 \times \frac{\pi}{4} \times .5^2 + 2.7 \times \left[\frac{20 \times 20 + 0}{2}\right] \tan 20 \times \pi \times .5 \times 20$$

$$= 6174.61\ KN$$

Question3 :

$D = 0.4m$

CLAY

5 layers

Neglecting end bearing.

$\{\lambda = 0.15\}$

$$Q_u = \lambda \cdot (\bar{\sigma}_m + 2c_u) A_s$$

$$Q_u = 0.15\left[\left[18 \times \frac{5}{2} + 2 \times 40\right] \times \pi \times 0.4 \times 5 \quad \cdots\right]$$

$$Q_u = 0.15 \times \pi \times 0.4 \times 5 \left[\frac{18 \times 5}{2} + 2 \times 40 + 18 \times \frac{15}{2} + 18 \times \frac{25}{2} + \right.$$

$$\left. + 18 \times \frac{35}{2} + 18 \times \frac{45}{2} + 2 \times (50 + 60 + 70 + 80)\right]$$

$$= 0.94 \times [1125 + 600] = 1621.5\ KN.$$

simply bhi कर सकते हैं avg lekar.

Question

Q.4 A 0.5×0.5m square concrete pile is to be driven in a homogeneous clayey soil having undrained shear strength, = 50 kPa and unit weight, γ=18kN/m³. The design capacity of the pile is 500 kN. The adhesion factor is given as 0.75. The length of the pile required for the above design load with a factor of safety of 2.0 is

(A) 5.2 m (B) 5.8 m

(C) 11.8 m (D) 12.5 m

[GATE 2018 : IIT Guwahati]

Solution:

$B = 0.5m$, $\gamma = 18KN/m^3$

$c_u = 50\ Kpa$, $Q = 500KN$

$\alpha = 0.75$, $FOS = 2.0$

$$1000 = 9 c A_b + \alpha \bar{c} A_s$$

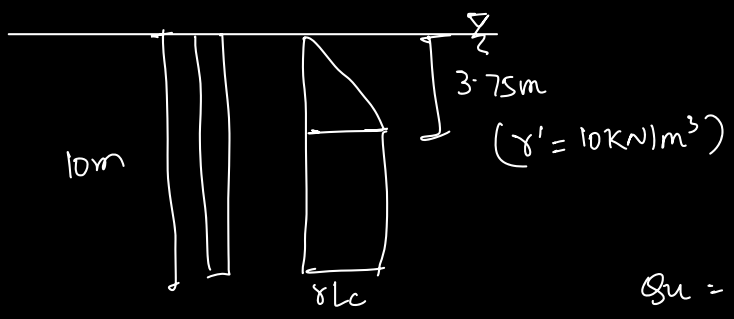
$$1000 = 9 \times 50 \times 0.5 \times 0.5 + 0.75 \times 50 \times 4 \times .5 \times L$$

$(L = 11.83m)$

Question

Q.5 A 10 m long, 250 mm diameter pile is provided in a uniform deposit of sand (having $\phi = 30^0$). The water table is at ground level. The average saturated unit weight of sand is $20 kN/m^3$. The critical depth occurs at a depth of 15 times of diameter of pile. Take coefficient of earth pressure (k) as 2.2 and friction angle between pile and soil as $\frac{2}{3}\phi$. The Bearing capacity factor, N_q is 23. Using factor of safety 3, the safe load carrying capacity of the pile is ____. $(\text{Take}\gamma_w = 10 kN/m^3)$

(A) 389.89 kN (B) 77.98 kN
(C) 233.96 kN (D) 155.95 kN



Solution

$L = 10m$

$D = .25m$

$\gamma = 20 KN/m^3$

Sand, $\phi = 30^\circ$

$\delta = 20^\circ$

$L_c = 15 \times .25 = 3.75m$

$K = 2.2$

$N_q = 23$, FOS = 3

$$Q_u = q N_q \cdot A_b + K \bar{\sigma}_m \tan\delta A_s$$

$$Q_u = 37.5 \times 23 \times \frac{\pi}{4} \times .25^2 + 2.2 \times \tan 20^\circ \times 3.14 \times .25 \times \left[\frac{10 \times 3.75}{2} \times 3.75 + 10 \times 3.75 \times 6.25\right]$$

$$Q_u = 42.33 + .6285 \times [304.68]$$

$$= 42.33 + 191.49 = 233.82$$

$$Q_s = 77.94 KN \approx \text{option C}$$

DYNAMIC PILE LOAD TEST:

The pile is driven by the dynamic action of hammer, the total driven energy is equal to the weight of hammer times the height of drop.

Following formula can be used to determine pile load capacity

(1) Engineering new record formula: [ENR formula]

The allowable load on the pile can be given as

$$Q_{ap} = \frac{WH}{FOS(S+C)}$$

W: Weight of the Hammer

H: Height of drop hammer.

FOS: Factor of safety (FOS = 6)

S: Finalised set (Penetration) per blow, Generally taken as average penetration per blow for last 5 blows of a drop hammer or 20

blows of a steam hammer.

C: Empirical constant

= 2.5 cm for drop hammer

= 0.25 cm for steam hammer (single & double acting hammer)

(i) DROP HAMMER: The hammer which is lifted manually and dropped freely by gravity is termed as drop hammer.

$$q_{ap} = \frac{WH}{FOS\,(S+C)} = \frac{WH \to cm}{6(S+2.5)}$$

(H: cm, S: cm)

∵ C is taken in cm, therefore H and S should be kept in cm.

(ii) SINGLE ACTING STEAM HAMMER: The Hammer which is lifted by the compressed air or the steam pressure & allowed to drop by gravity on the top of the pile is termed as single acting steam hammer. [last 20 readings]

$$q_{ap} = \frac{WH}{6(S+C)} = \frac{WH \to cm}{6(S+0.25)}$$

(H: cm, S: cm)

(iii) DOUBLE ACTING STEAM HAMMER: The hammer which is lifted & dropped by the steam pressure to accelerate the stroke is termed as double acting steam hammer

$$q_{ap} = \frac{(W+ap)\cdot H}{FOS\,(S+C)} = \frac{(W+ap)\,H \to cm}{6(S+0.25)}$$

(H: cm, S: cm)

a: Effective area of piston

P: Mean effective steam pressure.

(ii) Hiley's formula: IS 2911 Part 1 : 1964 has given the following expression based on original expression of Hiley.

$$Q_{up} = \frac{\eta_h \eta_b W H}{S + \frac{C}{2}}$$

Q_{up}: ultimate load on the pile

H: Height of drop

S: Final set or penetration

C: total elastic compression

$$C = C_1 + C_2 + C_3$$

C_1, C_2, C_3: are the temporary elastic compression of dolly & packing, pile, and soil respectively.

η_h: Efficiency of hammer that depends upon the type of hammer.

η_h = 65% for some steam hammers [Double] (variable)

η_h = 100% for drop hammer

η_h = 65% to 100% [Single Acting]

η_b: Efficiency of blow - It can be defined as the ratio of energy after impact to the shrinkage energy of ram.

$$\eta_b = \frac{W + e^2 P}{W + P} \quad \text{------ (when } W > eP)$$

$$\eta_b = \frac{W + e^2 P}{W + P} - \left(\frac{W - eP}{W + P}\right)^2 \quad \text{.... (when } W \leq eP)$$

W: weight of Hammer

P: weight of pile, helment, follower

e: coefficient of restitution [varies from 0 to 0.5]

Allowable load can be calculated using Hiley's formula

$$Q_{ap} = \frac{Q_{np}}{FOS} = \frac{\eta_h \cdot \eta_b \cdot W \cdot H}{FOS\,(S + C/2)}$$

Is FOS may vary from 2 to 2.5.

Important points of dynamic formula.

(1) In Hiley's formula number of constants are involved which are difficult to determine.

(2) Energy losses due to vibrations, heat and damage to dolly or packing are not accounted for.

(3) The formula do not take into account the reduced bearing capacity of pile when in a group.

(4) Dynamic formula give no indication about probable future settlement or temporary changes in soil structure

(5) For clays, the dynamic formulae are valueless because the skin friction developed in clay very much less (due to change in soil structure from flocculated to dispersed or due to thixotropic effect) then which occurs after a period of time also the point resistance is much more at the time of driving because of pore pressure developed in clay which reduces later on when the pore pressure dissipate

(6.) In case of submerged uniform fine sand impact of driving may cause liquefaction of soil thus showing much less resistance than that which will occur under a static load.

(7) There is no correlation between static & dynamic loading.

(8) Dynamic formula are best suited to coarse grain soils for which the shear strength is independent of rate of loading because they allow no development of excess pwp around the pile during driving in saturated or dry soils.

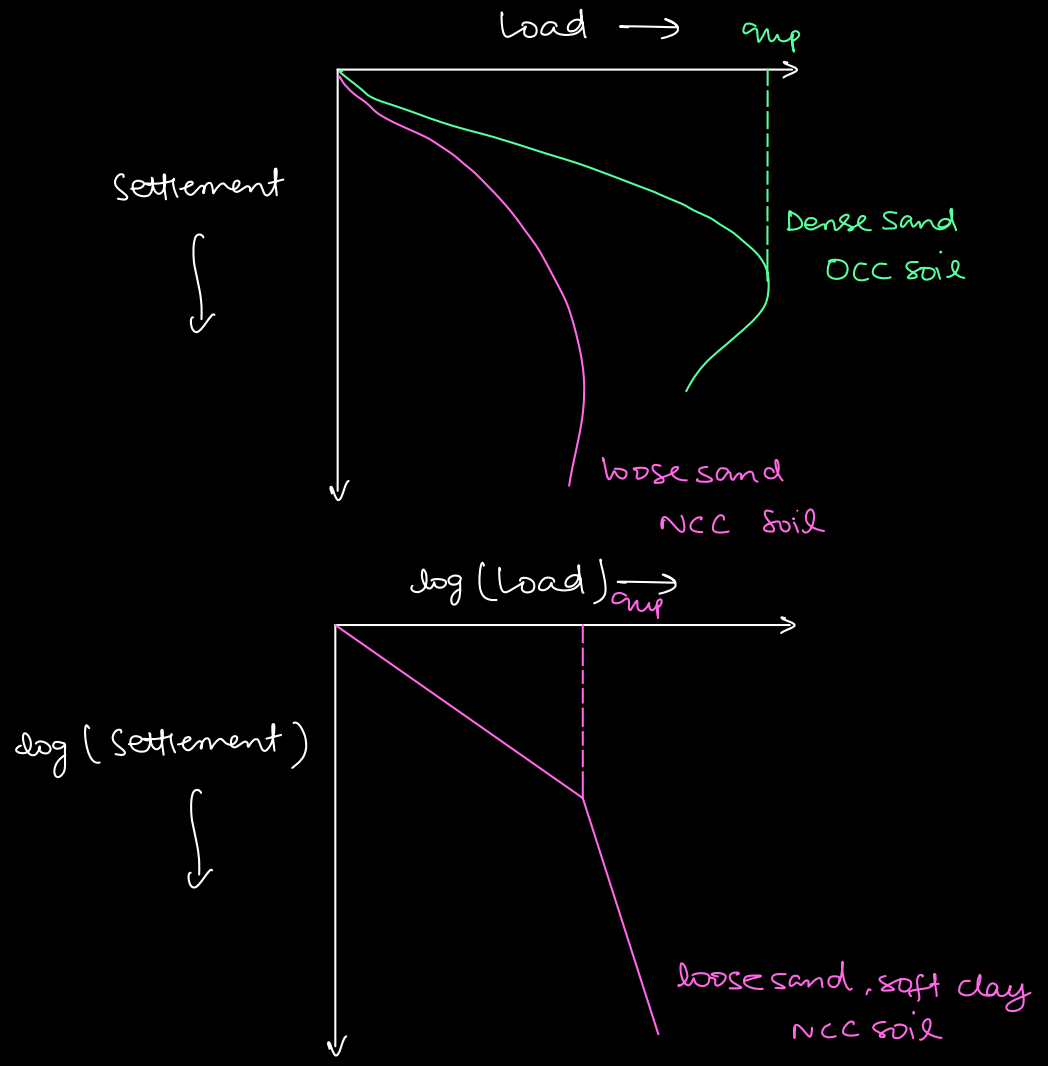
FIELD METHODS:

A. PILE LOAD TEST:

1. The pile load test can be performed either on the test pile or an actual pile.
2. The test load is applied with the help of calibrated jack placed over a rigid circular or square plate
3. This plate is placed over the head of the test pile projecting above the ground level.
4. The reaction of the jack is borne by a truss or platform which may have gravity loading (in the form of sandbags
5. The load is applied in equal increments of about $\frac{1}{5^{th}}$ of the estimated allowable load
6. The settlements are recorded with the help of three dial gauges of sensitivity 0.02mm arranged symmetrically over the test plate.
7. Each load increment is kept for the sufficient time till rate of settlement becomes 0.02mm / hr.
8. The test load is increased until the ultimate load is reached. The test load is increased to a value 2.5 times the estimated allowable load or to a load which causes settlement equal to $\frac{1}{10^{th}}$ of the

diameter of pile whichever occurs earlier.

Draw the load settlement curve and ultimate load capacity of pile is determined.



9. For loose / soft soils failure is progressive which is not well defined on Arithmetic scale, therefore it is drawn on log - log - scale.

To calculate the safe load of the pile or allowable load on the pile as per IS2911 (Part IV) : 1985, lesser value of the followings can be taken.

(i) $\frac{2}{3}^{rd}$ of the final load at which settlement attends value of 12mm

(ii) 50% of final load at which settlement attains a value of 10% of dia of pile or 7.5% of dia of bulb in case of underreamed pile.

(iii) $\frac{2}{3}$rd of final load at which net plastic settlement attains a value of 6mm

(Net plastic settlement means after removal of load)

To calculate the safe or allowable load on the pile group as per IS2911, lesser value of the following can be taken

(i) The final load at which settlement attains a value of 25mm

(ii) $\frac{2}{3}$rd of the final load at which settlement attains the value of 40mm

Question:

Determine safe load on pile of length 10m and dia 200mm In the load settlement curve, settlement attains a value of 12mm at the load of 2500 KN and 20mm at the load of 3500 KN?

50% of 3500 = 1750 KN &

$\frac{2}{3} \times 2500 = 1667$ KN ⟶ Ans

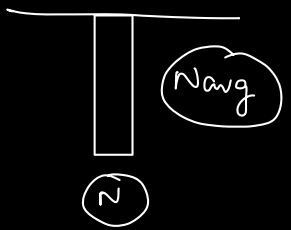
Correlation with SPT test data:

Correlation with SPT numbers: [No corrections Applied in N values directly from field]

$$Q_{up} = Q_{eb} + Q_{sf}$$

$$Q_{up} = q_{eb} \cdot A_b + q_{sf} \cdot A_s$$

1. For driven piles:



$$Q_{up} = 400 N \cdot A_b + 2 N_{avg} \cdot A_s$$

2. For driven piles: (Small / little displacement)

$$Q_{up} = 400N \cdot A_b + N_{avg} \cdot A_s$$

3. For Bored piles:

$$(q_{eb})_{bored} = \frac{1}{3} (q_{eb})_{displacement\ pile}$$

$$(q_{sf})_{bored} = \frac{1}{2} (q_{sf})_{displacement\ pile}$$

$$\left(Q_{up} = \frac{400}{3} N \cdot A_b + N_{avg} \cdot A_s \right)$$

Correlation with CPT-value (q_c)

$$Q_{up} = Q_{eb} + Q_{sf}$$

$$Q_{up} = q_{eb} A_b + q_{sf} \cdot A_s$$

1. For driven piles:

$$Q_{up} = q_c \cdot A_b + \frac{q_{c_{avg}}}{2} \cdot A_s.$$

2. For driven piles: (small / little displacement) (H-pile)

$$Q_{up} = q_c \cdot A_b + \frac{q_{c_{avg}}}{4} \cdot A_s$$

3. For Bored piles:

$$(q_{eb})_{bored} = \frac{1}{3} (q_{eb})_{displacement\ pile}$$

$$(q_{sf})_{bored} = \frac{1}{2} (q_{sf})_{displacement\ pile}$$

$$\left(Q_{up} = \frac{q_c}{3} \cdot A_b + \frac{q_{c_{avg}}}{4} \cdot A_s \right)$$

KN/m^2 units

q_c: cone resistance at the base of the pile

$q_{c_{avg}}$: Average cone resistance (in kg/cm^2) over the length of the pile

$|||$

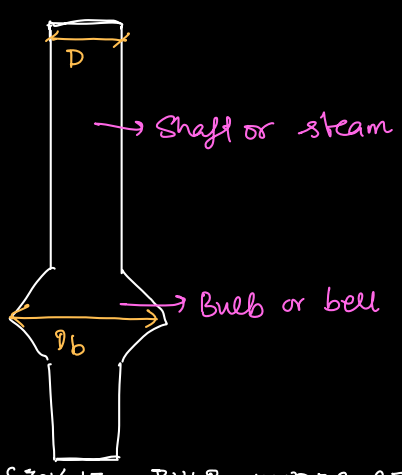
(KN/m^2)

Note: H-pile is a driven pile with very small displacement. Thus it can be treated as nondriven pile.

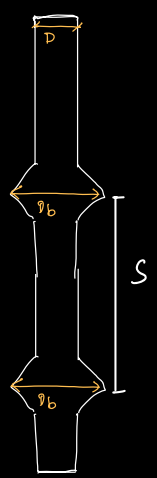
$$(q_{sf})_{H\text{-}pile} = \frac{1}{2} (q_{sf})_{disp\ pile}.$$

Note: To determine the point resistance and skin friction resistance seperately, cyclic pile load test can be performed.

Under-Reamed pile foundation:



SINGLE BULB UNDER REAMED



$S \not> 1.5 D_b$

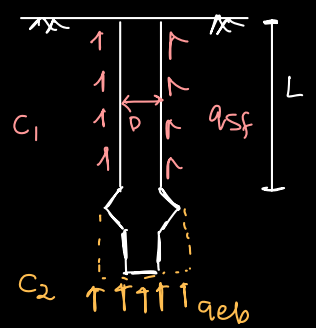
MULTI-BULB UNDER REAMED PILE.

- Under-reamed piles are bored cast in-situ concrete piles with one or more number of bulbs.
- The bulb is formed by enlarging the stem of the pile near the bottom in a double conical shape.
- The purpose of providing the bulb is as follows.
  a. To provide anchorage to the foundation for preventing its movement in vertical directions due to alternate swelling & shrinkage in expansive soils due to seasonal changes in water content.
  b. To provide additional bearing area through enlarged bulb in firm strata, underlying the top weak or filled up ground.
  c. To increase ultimate load carrying capacity for downward, upward and lateral loads & movements
  d. To take the foundation below the scour level.

Note: The anchorage provided by the under-reamed pile prevents cracks in floors & walls of the building in expansive soil areas

Note: The provision of an additional bulb increases the load carrying capacity by about 50%.

CASE 1: SINGLE BULB UNDERREAMED PILE WHEN BULB IS SLIGHTLY ABOVE THE TIP OF PILE OR NEAR THE TIP OF PILE:

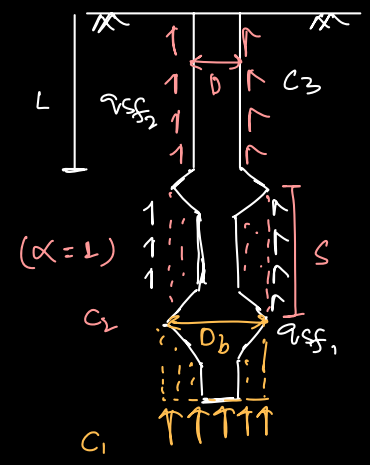


$$Q_{up} = Q_{eb} + Q_{sf}$$
$$= qC \cdot A_b + \alpha \bar{C} A_s$$
$$Q_{up} = qC_1 \frac{\pi}{4} \cdot D_b^2 + \alpha C_2 \pi D L_{eff}$$

(L_{eff}: $L - 2D$)

(sometimes L_{eff} can be taken as L)

CASE 2 : MULTI BULB UNDERREAMED PILE WHEN BOTTOM BULB IS SLIGHTLY ABOVE THE TIP OF PILE OR NEAR THE TIP OF PILE :



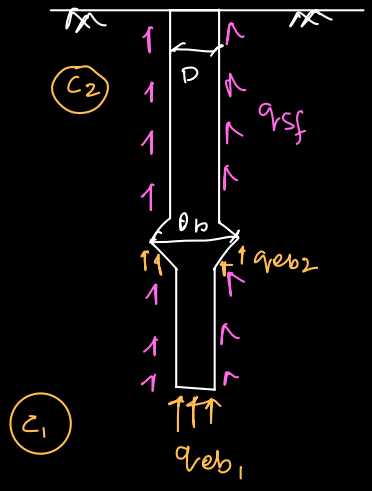
$$Q_{up} = q \cdot C_1 \cdot A_b + \alpha C_2 A_{s_1} + \alpha C_3 A_{s_2}$$

$$= q \cdot C_1 \cdot \frac{\pi}{4} \cdot D_b^2 + C_2 \pi D_b \cdot S + \alpha C_3 \pi D \, l_{eff}$$

l_{eff} : $L - 2D$

(sometimes l_{eff} can be taken as L)

CASE 3 : SINGLE BULB UNDERREAMED PILE WHEN BULB IS QUITE ABOVE THE TIP OF PILE OR FAR FROM TIP OF PILE :



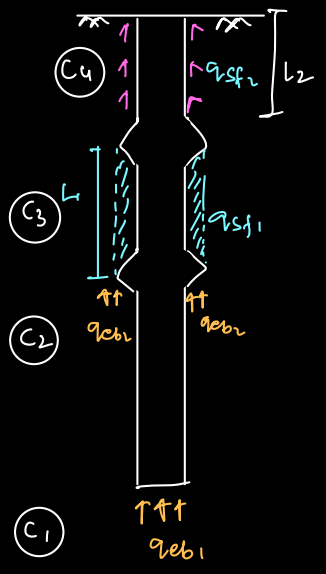
$$Q_{up} = Q_{eb} + Q_{sf}$$

$$Q_{up} = q_{eb_1} A_{b_1} + q_{eb_2} A_{b_2} + q_{sf} \cdot A_s$$

$$Q_{up} = q \cdot C_1 \cdot \left(\frac{\pi}{4} D^2\right) + q \cdot C_2 \cdot \frac{\pi}{4}(D_b - D^2) + \alpha C_3 (\pi D \, l_{eff})$$

l_{eff} : $L - 2D$

CASE 4 : MULTI BULB UNDERREAMED PILE WHEN BOTTOM BULB IS QUITE ABOVE THE TIP OF PILE OR FAR FROM TIP OF PILE :



$$Q_{up} = q_{eb_1} \cdot A_{b_1} + q_{eb_2} \cdot A_{b_2} + q_{sf_1} A_{sf_1} + q_{sf_2} A_{sf_2}$$

$$Q_{up} = q C_1 \frac{\pi}{4} D^2 + q C_2 \frac{\pi}{4}(D_b^2 - D^2) + C_3 \cdot \pi D_b L_1 + \alpha C_4 (\pi D \, l_{eff})$$

$l_{eff} = L - 2D$ (Sometimes take L only)

Note: C/c spacing of bulb is generally taken as 1.5 times the diameter of bulb.

(This C/c dia may vary b/w 1.25 Db to 1.5 Db)

Note: Dia of bulb is taken as 2.5 times the diameter of steam of pile.

$$D_b = 2.5D$$

$$(2D \leq D_b \leq 3D)$$

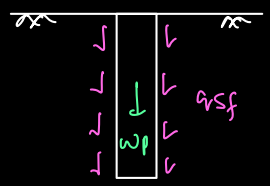
PILE SUBJECTED TO UPLIFT PRESSURE:

→ The tall structure chimneys, Silos & hydraulic structures are provided with the pile foundation

→ when the structure is subjected to the uplift pressure due to overturning effect of the structure or due to heavy seepage then the piles are called uplift piles or tension piles.

→ In calculation of ultimate capacity of tensile or uplift pile the end bearing component is neglected.

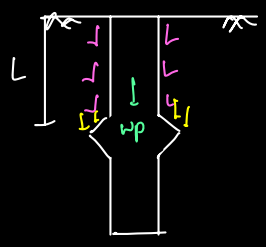
CASE I: A NORMAL PILE (without bulb):



$$Q_{up} = \alpha \bar{c} (\pi D L) + W_p$$

W_p: Weight of pile

CASE 2: The pile with BULB



$$Q_{ut} = \alpha c_i \pi D L + 9c \frac{\pi}{4}(D_b^2 - D^2) + W_p$$

HURRAY!! TIME FOR QUESTIONS.

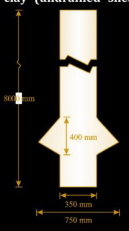
Question

Q.6 A singly under-reamed, 8 m long, RCC pile (shown in the adjoining figure) weighing 20kNwith 350mm shaft diameter and 750 mm under-ream diameter is installed within stiff, saturated silty clay (undrained shear strength is 50 kPa, adhesion factor is 0.3, and the applicable bearing capacity factor is 9) to counteract the impact of soil swelling on a structure constructed above. Neglecting suction and the contribution of the under-ream to the adhesive shaft capacity, what would be the estimated ultimate tensile capacity (rounded off to the nearest integer value of kN) of the pile?

(A) 132kN (B) 156kN

(C) 287kN (D) 301kN

[GATE 2011 : IIT Madras]



Solution

DATA

L : 8m

SINGLE

Wp = 20KN

D = 350m

Db = 750mm

(SILTY CLAY)

Cu = 50KPa

$\alpha = 0.3$

$N_c = 9$

$$Q_u = \alpha C_u \cdot A_s + W_p + 9 \cdot C \cdot \frac{\pi}{4}(D_b^2 - D^2)$$

$$= 0.3 \times 50 \times \pi \times .35 \times 7.6 + 20 + 9 \cdot 50 \cdot \frac{\pi}{4} \times (.75^2 - .35^2)$$

$$= 125.34 + 20 + 155.50 = 300.85 \text{ KN}.$$

GROUP OF PILES:

a. If a single driven pile is used as the foundation then there can be structural failure of the pile due to bending stresses developed due to eccentric loading on the pile. This eccentricity in loading may occur due to tilting of pile during installation. Therefore a group of minimum 3 piles can be used in case of driven piles.

b. With these 3-pile a group of triangular pattern can be prepared.

c. If more than 3 piles are used to form a group then it should be symmetrical w.r.t load

d. A group can be of square, circular, triangular, hexagonal shape. The most preferred shape is square pattern.

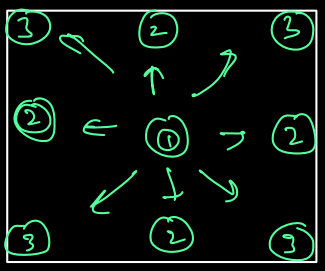
e. The requirement of driven piles is that they be arranged in a group of at least threepiles is not necessary applicable for bored piles. Bored piles can be placed in vertical position fairly accurately, hence even one pile may be sufficient where the loads are light

f. The load is transferred to the piles in the group through a reinforced slab known as pile cap

g. Pile cap may be either stand clearly above the ground level, on the ground level or clearly buried below ground level

h. When driving piles in sand it is advisable to begin at the centre of group & then proceed outwards to avoid problems of ground tightening.

[That means centre pile shall be installed first & then outer piles are installed]



i. As per IS 2911 Part I. The minimum spacing b/w the piles can be given as:

For end bearing piles : 2.5 D

For frichon piles : 3 D

Note: If the piles are provided in loose sand or in the fill deposits the minimum spacing between the piles can be taken as 2D.

D: Diameter of the pile.

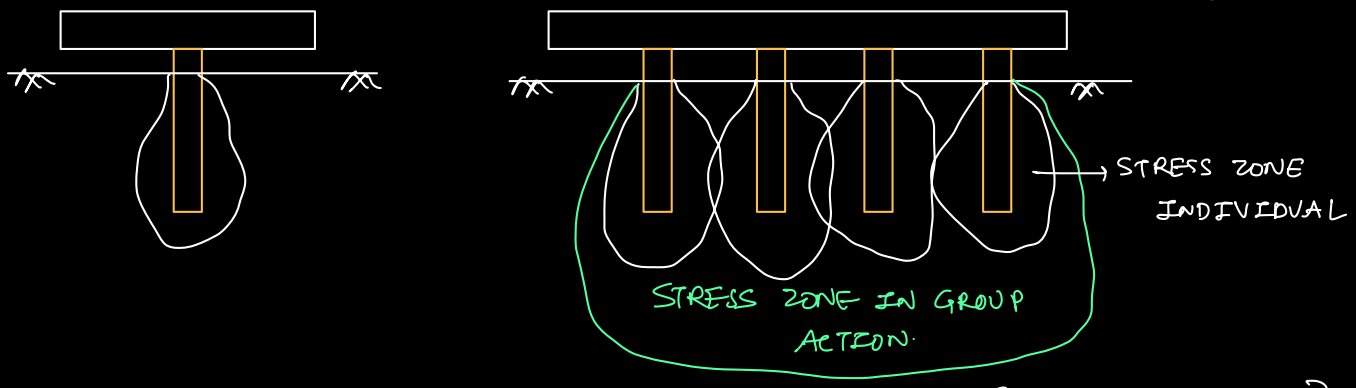
j. When the piles are closely spaced their stress zone overlaps each other which may reduce efficiency of the group.

h. When the piles are provided in the group two type of action are observed

1. Individual Action [Individual failure]

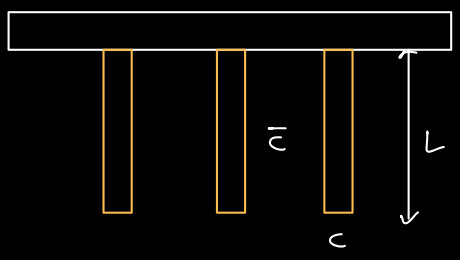
2. Group Action [Block failure]

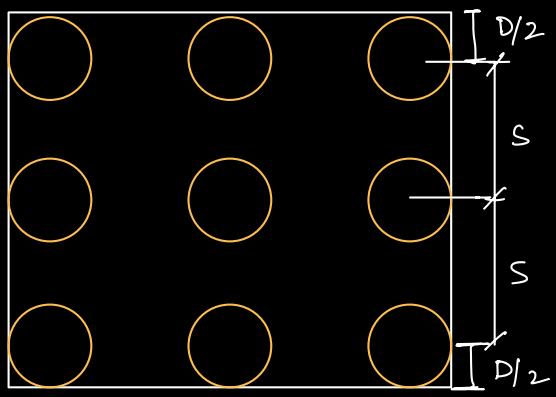
i. When the piles are provided in the group the bearing area & the surface area increases. The depth of stress zone is larger in case of group of pile which results the large settlement of the group as compared to individual pile.
single.



{Figures: NOT TO SCALE}

ESTIMATION OF ULTIMATE LOAD CAPACITY:





{B = 2S + D} (3×3 Pattern)

$\boxed{B = (M-1)S + D}$ (m×m Pattern)

Condition 1: CLAY SOIL:

Individual pile:

$$Q_{up} = 9CA_b + \alpha \bar{C} A_s$$

$$(Q_{up})_g = N[CA_b + \alpha \bar{c} A_s]$$

$$(Q_{up})_g = N\left[C \cdot \frac{\pi}{4} \cdot D^2 + \alpha \bar{C} \pi D L\right]$$

Group Action:

$$Q_{ug} = 9CA_b + \alpha \bar{C} A_s$$

$$Q_{ug} = 9 \cdot C \cdot B^2 + \bar{C} \cdot 4B \cdot L$$

$\therefore \alpha = 1$ (soil, soil)

2. Condition 2: When pile group is provided in SANDS:

Individual Action:

$$Q_{up} = \bar{\sigma} N_q \cdot A_b + K \cdot \bar{\sigma}_m \tan\delta \cdot A_s$$

$$N \cdot Q_{up} = N \cdot \left[\bar{\sigma} N_q \cdot \frac{\pi}{4} D^2 + K \cdot \bar{\sigma}_m \tan\delta \cdot \pi D L \right]$$

Group Action:

$$Q_{ug} = \bar{\sigma} N_q \cdot A_b + K \cdot \bar{\sigma}_m \tan\delta \cdot A_s$$

$$Q_{ug} = \bar{\sigma} N_q \cdot B^2 + K \cdot \bar{\sigma}_m \tan\delta \cdot 4BL$$

$$\left\{ Q_{ap} = \min \left(Q_{up} \cdot N, Q_{ug} \right) \right\}$$

$$\left\{ Q_{SP} = Q_{ap} / FOS \right\}$$

→ ye bhi ho sakta hai!!

SAFE LOAD can be taken as minimum of following

(i) $\dfrac{\text{ultimate load from individual action}}{FOS}$

(ii) $\dfrac{\text{ultimate load from group Action}}{FOS}$

PILE GROUP EFFICIENCY:

It can be defined as the ratio of ultimate load capacity in group action (Q_{ug}) to the ultimate load capacity in the individual action ($N \cdot Q_{up}$)

$$\left\{ \eta_g = \frac{Q_{ug}}{Q_{up} \cdot N} \times 100 \right\}$$

if $\eta_g < 100\%$, $Q_{ug} < N \cdot Q_{up}$

$\left\{ Q_{safe} = \dfrac{N \cdot Q_{up}}{FOS} \right\}$ → Group banana not beneficial.

if $\eta_g > 100\%$, $Q_{ug} > N \cdot Q_{up}$

$$\left\{ Q_{safe} = \frac{Q_{ug}}{FOS} \right\}$$

→ when the piles are closely spaced, due to interaction between the piles and due to overlapping of their stress zones, the group efficiency is reported less than 100% ($\eta_g < 100\%$)

→ when piles are provided with sufficient spacing the group efficiency reaches unity ($\eta_g = 100\%$)

→ In case of driven piles soil goes towards densification as a result group efficiency becomes greater than 100%. ($\eta_g > 100\%$)

NOTE: Due to the densification of soil in case of driven piles, the soil moves upward therefore driven piles should be sufficiently spaced

NOTE: In case of bored piles, densification doesnot occur therefore spacing b/w two pile can be smaller than driven piles

The group efficiency can be empirically expressed as

(i) Converse Labarre formula:

$$\eta_g = 1 - \frac{\theta}{90^\circ}\left[\frac{m(n-1) + n(m-1)}{mn}\right]$$

η_g: Group efficiency

m: No of rows in the group

n: No of piles in a row.

θ: $\tan^{-1}\left(\frac{D}{S}\right)$

D: Diameter of pile

S: Spacing b/w pile group

(ii) Seiler Keeney formula:

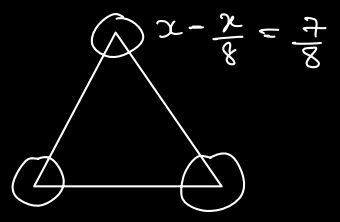
$$\eta_g = \left[1 - 0.479\left(\frac{S}{S^2 - 0.093}\right)\left(\frac{m+n-2}{m+n-1}\right)\right] + \frac{0.3}{m+n}$$

S: avg or c/c spacing of pile (in meter)

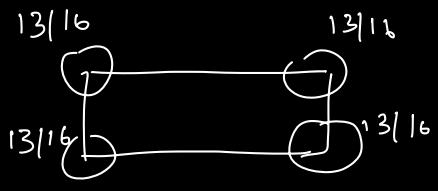
D: Diameter of pipe.

FELD'S RULE: According to this rule, the value of each pile is reduced by $\frac{1}{16}^{th}$ on account of the effect of the nearest pile in each diagonal or straight row of which the pile in question is a member.

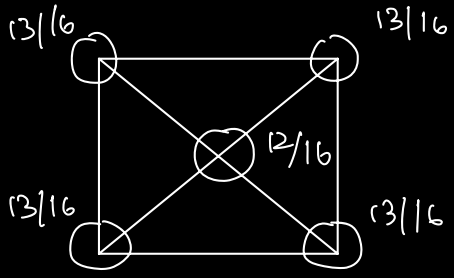
$$\eta_g = \frac{\frac{30}{16}}{\frac{32}{16}} \Rightarrow \frac{30}{32} \times 100 = 93.75\%$$



$$n_g = \frac{3 \times \frac{7}{8}}{3} = 87.5\%$$



$$\{ n_g = 13/16 = 81.25\% \}$$



$$\frac{4 \times \frac{13}{16} + \frac{12}{16}}{5} = 80\%$$

Question.

A group of 16 piles of 10m length & 0.5 diameter is installed in 10m thick stiff clay layer underlying by rock. The pile-soil adhesion factor is 0.4, average shear strength of soil on the sides is 100kPa on the base is also 100kPa.

Solution

$$Q_{eb} = 9C \cdot A_b = 9 \times 100 \times \frac{\pi}{4} \times .5^2 = 176.71 \text{ kN}$$

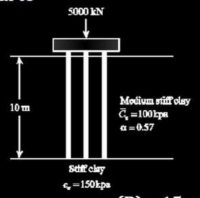
$$Q_{sf} = \alpha \bar{C} \cdot A_s = 0.4 \times 100 \times \pi \times .5 \times 10 = 628.31$$

Assuming $n_g = 100\%$,

$$(Q_{sf})_g = 16 \times 628.31 = 10053.1 \text{ kN}.$$

Question:

Q. For the soil profile shown in figure below, the minimum number of precast concrete piles of 300 mm diameter required to safely carry the load for a given factor of safety of 2.5 (assuming 100% efficiency for the pile group) are equal to



(A) 10 (B) 15
(C) 20 (D) 25

[GATE 2006 : IIT Kharagpur]

FOS = 2.5

$$Q_{up} = 5000 \times 2.5 = 12500 \text{ kN}$$

$$Q_{up} = 9 \cdot C \cdot A_b + \alpha \bar{C} \cdot \pi D L$$

$$= 9 \times 150 \times \frac{\pi}{4} \times .3^2 + 0.57 \times 100 \times \pi \times .3 \times 10.$$

$$= x \text{ kN}$$

$$\text{No of piles} = \frac{12500}{x} = 19.75 \approx 20 \text{ (ultale liya tha)}$$

Question

Q. A group of 16 piles (diameter = 500 cm, length = 14 m, centre to centre spacing = 1 m) arranged in a square pattern passes through a recent fill (thickness = 3 m) overlying a soft clay deposit (thickness = 5 m) which is consolidating under the fill load and rests in a stiff clay strata. All the strata are saturated. The soil properties of different strata are :

Type of soil	Unit weight (γ) kN/m³	Strength parameters c_u (kPa)	φ_u	Adhesion parameter (α)
Fill	16	50	0	0.60
Soft clay	17	20	0	0.40
Stiff clay	21	70	0	0.45

C_1
C_2
C_3

Estimate the ultimate load capacity (Qu) of the pile group.

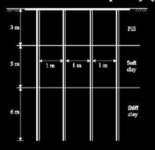


Fig. (a)

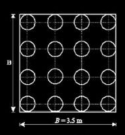


Fig.(b)

B=3.5m

Solution

B = 3.5m

16 piles

D = .5m

L = 14m

S = 1m

$$Q_{up} = 9 \cdot C_3 \cdot A_b + \pi \cdot D \left[\alpha_1 C_1 L_1 + \alpha_2 C_2 L_2 + \alpha_3 C_3 L_3 \right]$$

$$= 9 \cdot 70 \times \frac{\pi}{4} \times .52 + \pi \times .5 \left[0.60 \times 50 \times 3 + 0.40 \times 20 \times 5 + 0.45 \times 70 \times 6 \right]$$

$$= 123.70 + 1.57 \left[319 \right]$$

$$= 624.53$$

[N · Qup = 999248 KN] → (Qup) ✓

$$Q_{ug} = 9 \times 70 \times 3.5^2 + 4 \times 3.5 \left[50 \times 3 + 20 \times 5 + 70 \times 6 \right]$$

$$= 7717.5 + 9380 = 17097.5 \text{ KN}$$

NEGATIVE SKIN FRICTION :

→ When the pile is provided in loose sand or unconsolidated strata in then the settlement of surrounding soil is more than the settlement of pile which results the downward skin frichon over the surface of pile, known as negative skin frichon.

→ Due to negative skin frichon the load carrying capacity of the pile reduces

→ This negative skin frichon may occur due to :

(i) Recently filled deposits (use of filler material)

(ii) Relative movement b/w soil & pile (i.e settlement of surrounding soil is more than settlement of soil)

(iii) Excessive surcharge on surrounding soil

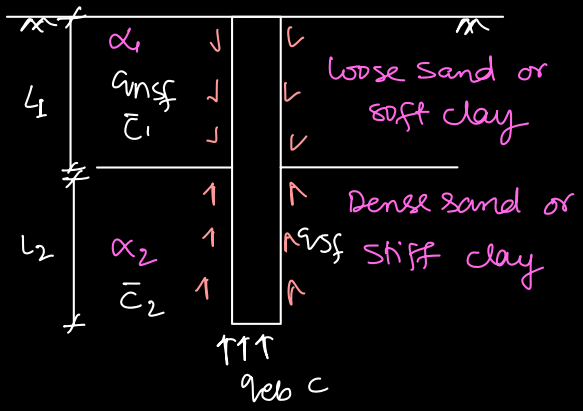
(iv) Vibration & Earthquakes

(v) Drawdown of water table may cause the increase in effective stress which enhance the process of consolidation & negative skin friction occurs.

NOTE: Relative movement of about 10 mm b/w soil and pile is sufficient to develope the negative skin friction.

Estimation of negative skin friction:

CASE I: In a single pile:



(i) In clay soil:

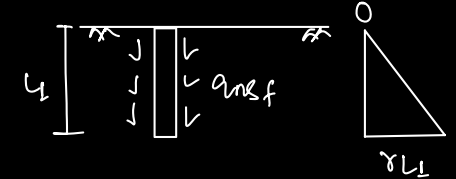
$$Q_{up} = \underbrace{q_c \cdot A_b}_{\text{end bearing force}} + \underbrace{\alpha \bar{C} A_{sf}}_{\text{positive skin friction force}} - \underbrace{\alpha \bar{C} A_{nsf}}_{Q_{nsf}}$$

$$Q_{up} = q_c \cdot \frac{\pi}{4} D^2 + \alpha_2 \bar{C}_2 \pi D L_2 - \alpha_1 \bar{C}_1 \pi D L_1$$

(ii) In cohesionless soils:

$$Q_{NSf} = q_{NSf} \cdot A_s$$

$$Q_{NSf} = (K \cdot \bar{\sigma}_m \tan\delta) \cdot A_s \quad (KN)$$



$$Q_{NSf} = K \cdot \left(\frac{0 + \gamma L_1}{2}\right) \tan\delta \cdot \pi D L_1$$

$$Q_{NSf} = \frac{1}{2} K \cdot \gamma L_1^2 \tan\delta \cdot \pi D$$

$$\left\{ Q_{NSf} = \frac{1}{2} P L_1^2 \gamma K \tan\delta \right\}$$

(P: Perimeter of pile)

CASE II: In pile Group:

Condition 1: In clay soil.

Note: Negative skin friction is calculated in individual action and in group Action & maximum of these two shall be considered

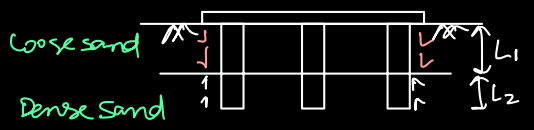
In Individual action:

$$N \cdot Q_{NSf} = N \cdot [\alpha \cdot \bar{C} \cdot A_S]$$

$$N \cdot Q_{NSf} = N \cdot [\alpha \bar{C} (\pi D L_1)]$$

L_1: thickness of zone of negative skin friction.

N: Number of pile in the group.



b) Group Action:

$$(Q_{NSf})_g = q_{nsf} \cdot A_{sn} + \text{weight of the soil of the negative skin friction zone}$$

$$(Q_{NSf})_g = \alpha \bar{C} \cdot 4BL_1 + (B^2 \times L_1)\gamma$$

$\alpha \to 1$: Adhesion b/w soil & soil

$$(Q_{NSf})_g = \bar{C}(4BL_1) + (B^2 L_1 \gamma)$$

$$\text{Final negative skin friction force} = \max \begin{cases} N\, Q_{NSf} \\ (Q_{NSf})_g \end{cases}$$

Condition 2: Cohesionless soil:

a. Individual Action:

$$N \cdot Q_{NSf} = (K \cdot \bar{\sigma}_m \tan\delta) \cdot A_{sn} N$$

$$N \cdot Q_{NSf} = N \cdot K \cdot \bar{\sigma}_m \cdot \tan\delta \cdot \pi D \cdot L_1$$

b. Group Action

$$(Q_{NSf})_g = (q_{NSf})_g \cdot A_{sn} + \text{weight of soil in that zone}$$

$$(Q_{NSf})_g = (K \cdot \bar{\sigma}_m \cdot \tan\delta) \cdot 4BL_1 + \gamma B^2 L_1$$

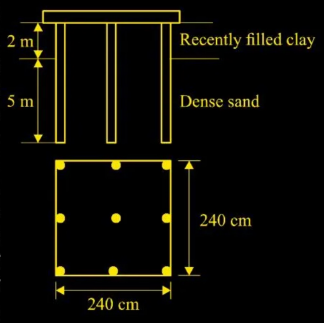
Effect of negative skin friction on the factor of safety with respect to the ultimate load capacity:

$$FOS = \frac{\text{ultimate load capacity}}{\text{working load} + \text{Negative skin friction load}}$$

Note: In the field, negative skin friction can be reduced in precast piles by painting the pile surface with bitumen.

Question

Q.9 A group of nine piles in a 3×3 square pattern is embedded in a soil strata comprising dense sand underlying recently filled clay layer, as shown in the figure. The perimeter of an individual pile is 126 cm. The size of pile group is 240cm×240cm. The recently filled clay has undrained shear strength of 15 kPa unit weight of 16kN/m^3. The negative frictional load (in kN, up to two decimal places) acting on the pile group is _____.



[GATE 2018 : IIT Guwahati]

Solution:

$\pi D = 126\text{cm} = 1.26\text{m}$

$B = 2.4\text{m}$

$C_u = 15\text{ KPa}$, $\gamma = 16\text{ KN/m}^3$

$\alpha_{assumed} = 0.5$ (for clay soil)

* In individual action

$$N \cdot Q_{NSF} = 9 \times 0.5 \times 15 \times 1.26 \times 2$$

$$= 170.1\text{ KN}$$

* group Action

$$(Q_{NSF})_g = CA_S + B^2\gamma \times 2$$

$$= 15 \times 4 \times 2.4 \times 2 + 2.4^2 \times 16 \times 2$$

$$= 472.32\text{ KN}.$$

Question

A group of 9 friction piles. square grid

$D = .3\text{m}$

$L = 2\text{m}$

cohesionless : $\phi = 32°$

c/c spacing (in m) for $\eta_g = 60\%$?

$$\delta = \frac{2}{3}\phi = 21.33°$$

$$\eta_g = 1 - \frac{\theta}{90°}\left[\frac{m(n-1) + n(m-1)}{mn}\right]$$

$$\eta_g = 1 - \frac{\theta}{90}\left[\frac{3\times 2 + 3\times 2}{9}\right]$$

$$.6 = 1 - \frac{\theta}{90}\left[\frac{12}{9}\right]$$

$$\left\{\theta = \frac{0.4 \times 90 \times 9}{12} = 270\right\}$$

$$\tan^{-1}\left(\frac{D}{S}\right) = 27° \Rightarrow \frac{D}{S} = 0.50$$

$$\boxed{S = .6\text{m}}$$ Ans

Settlement of Pile group

a. The depth of influence (or stress zone) is much larger in compared to a single pile therefore, the settlement of the pile group is more than the settlement of a single pile even when the load on the single pile and load on each pile of the pile group is same.

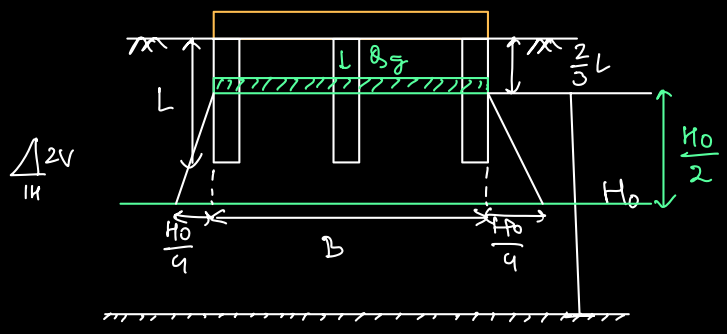
b. The settlement of a pile group in a clay soil can not be estimated from the data of a load test on a single pile because the time effect, the effect of remoulding of soil due to pile driving and the scale effect are quite different for a single test pile & pile group.

c. The widely used approach for estimation of settlement in a pile group is "Equivalent Raft approach"

d. In this method the pile group is assumed to act as a single, large raft assumed to be placed at some arbitary depth inside the soil

e. The piles are assumed to transfer the vertical load on the equivalent raft.

CASE I: When displacement piles or frictionpiles are provided in the homogeneous clay deposits.



$$\Delta \bar{\sigma}_0 = \frac{Q_g}{\left(B + \frac{H_0}{2}\right)^2}$$

$\bar{\sigma}_0$ = Effective overburden pressure at middepth of the compressible soil.

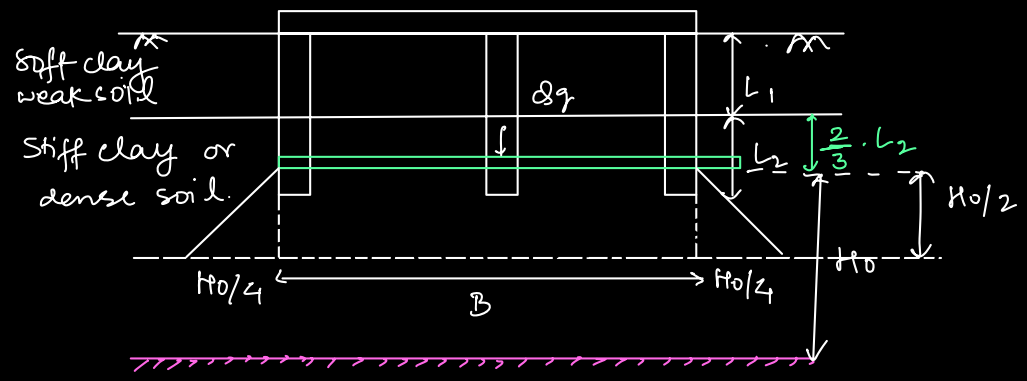
Settlement can be computed as

$$\Delta H = \frac{C_c H_0 \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)}{1 + e_0}$$

→ In the above case an equivalent raft is assumed to place at a depth of $\frac{1}{3}$rd of the length of the pile

CASE 2: when the piles are driven through two different soil deposits. The top layer is of soft clay and bottom layer is of stiff clay.

In such cases an equivalent raft is assumed to place at a depth of $\frac{2}{3}$rd of the thickness of stiff or strong clay layer from top of the stiff/strong clay layer.



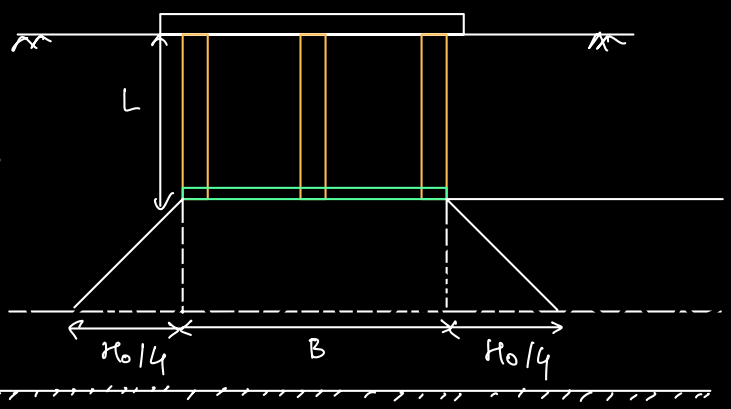
$$\Delta\bar{\sigma} = \frac{Q_g}{\left(B + \frac{H_0}{2}\right)^2}, \qquad \bar{\sigma}_0 = \text{at middepth from stiff layer.}$$

$$\left\{ \Delta H = \frac{C_c H_0}{1+e} \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right) \right\}$$ THUS, here we will get this

CASE 3: FOR BORED PILES or END BEARING PILE GROUP IS PROVIDED ON FIRM STRATA IN CLAY SOIL:

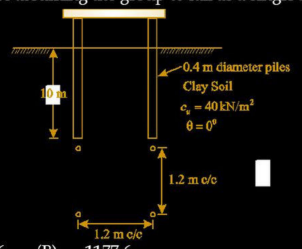
→ The equivalent raft is assumed to be placed at the base of the pile group.

SAME FORMULA AND ALL.



Question

Q.10 What is the ultimate capacity in KN of the pile group shown in the figure assuming the group to fail as a single block?



(A) 921.6 (B) 1177.6
(C) 2438.6 (D) 3481.6

$L = 10m$
$D = 0.4m$
$C_u = 40 KN/m^2$
$\alpha = 0.4$

$B = 1.2 + 0.4$
$B = 1.6m$

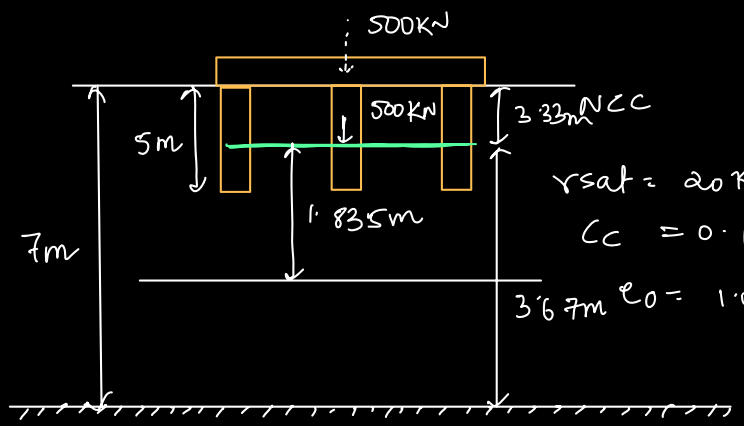
$$Q_{net} = 9 \cdot C \cdot A_b + \alpha \bar{C} A_s$$

$$= 9 \cdot 40 \times 1.6^2 + 1 \times 40 \times 4 \times 1.6 \times 10$$

$$= 3481.6 \text{ kN}$$

Question

Calculate settlement of pile group shown in figure.



c/c spacing 0.5
D = .2mm
B = 2.5 + D
= 1.2

$\gamma_{sat} = 20 KN/m^3$
$C_c = 0.027$
$e_0 = 1.05$

$(H_0 = 3.67m)$

$$\Delta \bar{\sigma}_0 = \frac{500}{(1.2 + 1.835)^2} = 54.28 \text{ KN/m}^2$$

$$\Delta \bar{\sigma}_0 = 20 \times (1.835 + 3.33) = 103.3 \text{ KN/m}^2$$

$$\Delta H = \frac{C_c H_0}{1 + e_0} \log \left[\frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}_0} \right]$$

$$= \frac{0.027 \times 3.67}{1 + 1.05} \log \left[\frac{103.3 + 54.28}{103.3} \right]$$

$$= 8.86 \text{ mm}$$

Settlement of pile group provided in sand:

→ For estimating the settlement of a pile group in sand, the

common practice is to extrapolate this from the settlement of an individual test pile measured in pile load test

→ Skempton compared the settlement of a number of pile group, consisting of driven piles with the same load per pile

→ let Sg : settlement of pile group
Si : settlement of individual pile.

where $\frac{Sg}{Si}$: group settlement ratio (or settlement ratio) for the same average log per pile

$$\left\{ \frac{Sg}{Si} = \left(\frac{4B + 2.7}{B + 3.6} \right)^2 \right\}$$

NOTE: It is observed that group settlement ratio can have maximum value of 16.

$$[\![Sg = 16 \cdot Si]\!]$$

Effect of Arching:

1. Effect of Arching is considered when the pile is provided in the granular / cohesionless soil. NOT observed in case of clay
2. Due to effect of Arching the effective stress does not increases linearly throughout the depth. It increases linearly upto the depth of 15D to 20D, then remains constant.
3. Hence, end bearing and skin friction resistance also does not increase after reaching a specified value.
4. The maximum end bearing resistance can be taken as 11000 KN/m² in case of normal silica sand and 5000 KN/m² in case of calcerous sand.
5. The maximum value of q_{sf} can be taken as 100 KN/m² in case of silica sand. & 20 KN/m² in case of calcerous sand.

6. For normal sands (silica sand)

$q_{eb} \neq 11000\ KN/m^2$

$q_{sf} \geq 100\ KN/m^2$

for calcerous sand

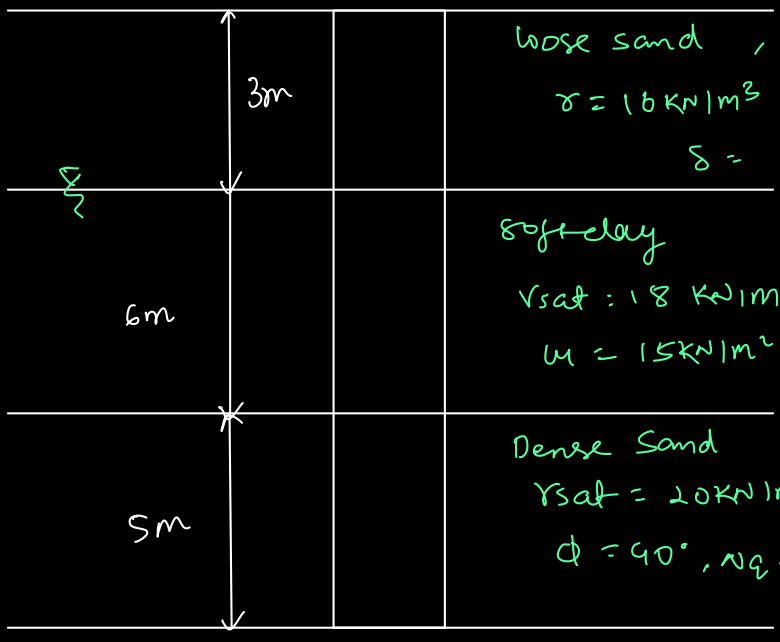
$q_{eb} \neq 5000\ KN/m^2$

$q_{sf} \neq 20\ KN/m^2$

Question:

Determine the allowable load capacity of 40cm dia pile driven in a pile as shown in figure

(GOPAL RANJAN)



$15D = 15 \times 0.4 = 6m$

$\therefore$ In loose sand:

$$\frac{L}{D} = \frac{3m}{0.4} = 7.5 < 15 \quad \text{(NO ARCHING)}$$

$$\bar{\sigma}_m = \frac{48}{2} = 24\ KN/m^2$$

$$q_{sf} = K \bar{\sigma}_m \tan\delta$$

$$= 1 \cdot 24 \cdot \tan(22.5)$$

$(q_{sf})_{loose} = 9.94\ KPa < 100\ KN/m^2 = 9.94\ KN/m^2$

$(Q_f) = 9.94 \times \pi \times 0.4 \times 3 = 37.47\ KN$

$\therefore$ In clay, (NO ARCHING).

$$q_{sf} = \alpha \bar{C} A_s = \alpha \times 15 \times \pi \times 0.4 \times 6$$

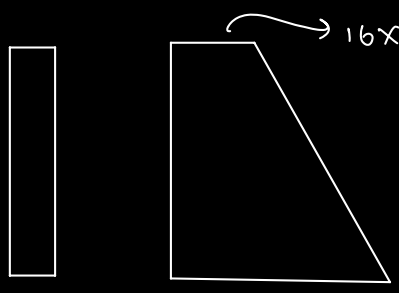
$$= 15 \times \pi \times 0.4 \times 6$$

$$= 36\pi = 113.09\ KN$$

∴ In dense sand :

$$\frac{L}{D} = \frac{5}{0.4} = 12.5 < 20 \text{ (NO ARCHING)}$$

$$(q_{sf})_{dense} = K \bar{\sigma}_m \tan\delta$$



$$\bar{\sigma}_m = \frac{146 + 96}{2} = 121 \text{ KN/m}^2$$

$$(q_{sf}) = K \cdot \bar{\sigma}_m \tan\delta$$

$$= 2 \times 121 \times \tan 30 = 139.71 \text{ KN/m}^2 > 100$$

thus take as 100 thus

$$(Q_{sf}) = 100 \times \pi \times .4 \times 5 = 628.32 \text{ KN}$$

Net skin friction = 628.32 + 113.09 + 37.47

= 778.88 KN

end bearing, $q_{eb} = \bar{\sigma} N_q = 146 \times 160$

$= 23360 \text{ KN/m}^2 > 11000 \text{ KN/m}^2$

thus, $q_{eb} = 11000 \text{ KN/m}^2$

$$Q_{eb} = 11000 \times \frac{\pi}{4} \times .4^2 = 1382.3 \text{ KN}$$

thus $Q_{up} = 1382.3 + 778.88 = 2161.18 \text{ KN}$

$$\left\{ Q_{allowable} = \frac{2161.18}{2.5} = 864.472 \text{ KN} \right\}$$

SOIL EXPLORATION AND SITE INVESTIGATION

Site investigation and subsoil-exploration:

1. An investigation of site is essential for Judging its suitable for proposed ensineering projects.
2. The methods of site investigation are largely dependent upon the nature of engineering projects and site.
3. In general site investigation should be started with the collection and examination of existing data about the soil & geological conditions.
4. If existing data (information) is not sufficient then site should be explored in detail.
5. The exploration of soil should be proceeded by site reconaissence.
6. Site reconnaissance is required for the study of topographical features.

SITE EXPLORATION:

The objective of site exploration is to determine the complete information that provides reliable, specific and detailed Analysis about the soil and ground water condition.

It should yield the information about the following:

(i) the order of occurance and extent of soil & rock strata. (soil profile) / (geological profile).

(ii) Nature & engineering properties of the soil.

(iii) location of ground water table and its variation.

Depth of Exploration:

In general, the depth of exploration can be taken as the depth upto which the increase in pressure due to structural

loading is likely to cause perceptible settlement or shear failure, such depth is called depth of exploration. It is generally safe to assume the significant depth upto a level at which net increase in vertical pressure becomes less than 10% of the initial overburden pressure.

→ Alternatively the pressure bulb of $\frac{1}{5}$th to $\frac{1}{10}$th of surface load intensity is sometimes assumed to defined the minimum depth of exploration

The depth of exploration at the start of the work may be decided according to the following guide rules.

1. Isolated spread footing: 1.5B (B: width of footing)
2. Adjacent footing with clear spacing less than 2B: 1.5B
3. Pile foundation: 10 to 30 meters or atleast 1.5 times the width of structure.
4. Retaining wall: 1.5 times the base width or 1.5 times the height of wall whichever the greater exposed
5. Floating Basement: Depth of construction
6. DAMS: 0.5 times the bottom width of earth dam
   2 times the height from bed to crest of concrete Dam.
7. Roads, cuts and fills: 1m where little cut or fill is required
   → for fill section, 2m below the ground level or equal to height of fill whichever is greater
8. From the consideration of weathering:
   (i) 1.5m in general
   (ii) 3.5m in Black cotton soil.

There are two types of exploration:

(i) General exploration

(ii) Detailed exploration

(i) General exploration: General exploration is done for small project. It gives information about.

1. Order of ocurance of soil strata
2. depth of soil strata
3. depth of water table & its variation
4. Preliminary selection of foundation type.
5. Engineering properties requiring disturbed or partially undisturbed samples

(ii) Detailed exploration: It is carried out in case of heavy or costly construction.

soil properties such as permeability, compressibility, sheer strength are determined

The field test may be conducted for determining the in-situ values of density and bearing capacity.

Methods of soil exploration:

1. open excavation
2. Borings
3. Subsurface soundings
4. Geophysical methods

OPEN EXCAVATION:

a. Test pits & trenches can be prepared in this methods
b. It is suitable for all the soils
c. The cost of open excavation increases with increase in the depth of excavation.
d. This method is suitable for shallow foundations upto 3m depth it can be used upto the 6m also, with the help of machines
e. If ground water table is encountered then lowering

of ground water table is required.

2. BORING AND DRILLINGS:

(i) Auger boring
(ii) Auger & Shell Boring
(iii) Wash Boring
(iv) Percussion drilling
(v) rotary boring

(i) Auger boring:

a. Auger boring is suitable for cohesive & other types of soft soils above water table
b. Hand Auger upto 6m depth and mechanically operated Auger also known as power driven auger upto greater depth can be used
c. Power driven Auger can be used in gravally soils
d. Sample obtained from the Augers are badly disturbed which can be used for identification purpose only
e. Auger boring is best suitable for highway exploration at shallow depth and for exploring pits and shallow foundation

2. Auger Shell boring:

a. cyeindrical Auger & shell with cutting edges on teeth at the lower end can be used for making deep borings
b. Augers are suitable for soft to stiff clays and shells are suitable for very stiff & hard clays.

3. Wash Boring:

a. wash boring is a fast & simple method for advancing tools in all types of soil except boulders & rocks
b. In this method, a casing is driven first.

c. A hollow drill rod with sharp chisel or chopping bit at the lower end is inserted

d. The water is forced under pressure through drilling rod which is alternatively raised and dropped and also rotated

e. The soil cuttings are forced up to the ground surface in the form of soil water slurry b/w casing and drilling rod.

f. The soil sample obtained is almost useless

4. Percussion boring:

a. This method is suitable for advancing the boreholes in all types of soils especially boulders & rocks

b. In this method boulders or rocks are broken by repeated blows of heavy chisels

5. Rotary Borings:

a. Rotary boring or Rotary drilling is a very fast method of advancing the bore holes in rocks & soil both

b. A big diameter bore hole can be prepared with continuously rotating drills (rotary boring).

c. In this method a drilling mud usually water solution of bentonite is continuously forced down the hollow drill rods. The mud returns upwards brings the cuttings to the surface. The method is known as mud rotary drilling. NO CASZNG REQUIRED.

TYPES OF SOIL SAMPLES:

Soil samples can be classified into two categories

(i) Disturbed sample

(ii) Undisturbed sample

(i) Disturbed Sample: When natural soil structure is modified or destroyed during the process of sampling the obtained soil sample is called as disturbed sample. There are two types of disturbed sample (i) Representative sample. and (ii) Non representative samples

a. Representative Samples:

If the natural soil structure gets modified but natural water content & mineral proportions remains preserved or unchanged then it is termed as representative sample.

b. Non-representative sample:

When natural soil structure, water content and mineral proportion (soils from other layer mixed up) all are modified then it is termes as Non-representative sample.

Note: These are not used for testing purpose of soil.

(ii) Undisturbed Sample:

when natural soil structure, water content and mineral proportion all remain same or unchanged during the process of sampling then it is termed as undisturbed sample.

Note: Practically it is impossible to get a undisturbed samples, some extent of disturbance is always present

Types of Soil Sample required for laboratory test:

Type of tests	Types of Samples required
(1) Natural water content	UNDISTURBED
(2) Unit weight (density)	UNDISTURBED
(3) Permeability (K)	UNDISTURBED

④ Compressibility (C_c, C_r, a_v, C_v)

⑤ Shear strength parameter

⑥ Specific gravity

⑦ Grain size distribution

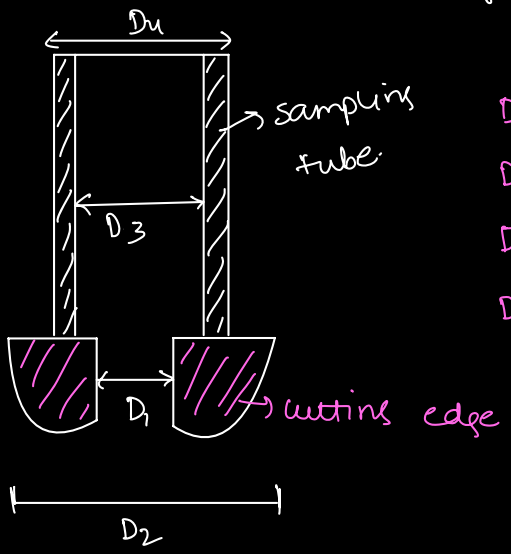
⑧ Consistency limits

} Representative or undisturbed.

The degree of disturbance depends upon:

1. Cutting edges
2. Inside wall friction
3. Non-return valve.

Design of Sampler: The following ratios related to the dimensions of cutting edge and the sampler are useful in the design of sampler so that degree of disturbance can be reduced.



D_1 : Inside diameter of cutting edge

D_2 : Outside diameter of cutting edge

D_3 : Inside diameter of sampling tube

D_4 : Outside diameter of sampling tube

1. INSIDE CLEARANCE: Inside clearance helps in reducing the friction between soil & sampler when the soil enters the elastic lateral expansion of soil when soil enters the Sampling tube.

$$\left\{ C_i = \frac{D_3 - D_1}{D_1} \times 100 \right\}$$

→ If inside clearance is too large then there will be too much lateral expansion of the soil which may result the disturbance of soil.

→ It should be between 1-3%.

2. OUTSIDE CLEARANCE: It helps in reducing the friction while sampler is being driven & being withdrawn after sample is collected.

$$C_o = \frac{D_2 - D_4}{D_4} \times 100.$$

→ The outside clearance should not be much greater than outside clearance

The value of outside clearance varies from 0-2%.

3. AREA RATIO:

$$A_r = \frac{A_o - A_i}{A_i} \times 100$$

$$A_r = \frac{\frac{\pi}{4} \times D_o^2 - \frac{\pi}{4} \times D_i^2}{\frac{\pi}{4} \times D_i^2} \times 100 = \frac{D_o^2 - D_i^2}{D_i^2}$$

→ Area ratio should be as less as possible

→ The area ratio should be less than 20% for stiff foundation & should be less than 10% for soft sensitive clays. [10% is general value]

If area ratio ↑ → thickness ↑ → sharpness ↓ → friction ↑

If area ratio ↓ → thickness ↓ → sharpness ↑ → friction ↓

Note: Thicker samplers are used to obtain disturbed (Representative) samples

Note: Thinner samplers are used to obtain undisturbed samples

4. RECOVERY RATIO (Lr):

It is defined as the ratio of recovered length of sample to the penetration length of sampler.

$$Lr = \frac{\text{Recovered length of sample}}{\text{Penetration length of sampler}}$$

if Lr = 1 (Good recovery)

Lr < 1 (Compression / shrinkage of soil during sampling)

Lr > 1 (swelling of soil sample)

Types of Sampler:

The commonly used samplers can be classified into three categories:

1. Open end sampler or open drive sampler:

→ This sampler essentially consists of a seamless open end steel tube with a cutting edge

→ The tube is connected through a bend to the drill rod

→ The sampler head is provided with vents to vermits water & air to escape during sampling.

→ A ball check valve to retain the sample during the with-drawl is also provided.

→ It is also known as split spoon sampler which is the most commonly used sampler in SPT test.

2. Stationary Piston Sampler:

→ It consists of two seperate parts:

a. Sampler cylindrical

b. Piston System.

→ The piston attached to a long piston rod extending upto the ground surface through the drill rod.

→ During lowering the sampler through the hole, the lower end of the sampler is kept closed with the piston

→ when the desired sampling depth is reached the piston rod is clamped, thereby keeping the

→ The sample is the lifted up, with piston rod in the clamped position.

→ The piston prevents the entry of water and soil into the tube when it is being lowered and then greately helps to retain the sample during lifting operations.

Note: This sampler is more suitable for sampling soft soils.

3. Rotary sampler:

1. A Rotary sampler is a double walled tube sampler with a inner removable liner.
2. The outer tube or the rotating barrel is provided with a cutting bit
3. The bit cuts the annular ring when the barrel is rotated
4. The inner tube which is stationary slides above the cylindrical sample cut by the outer rotating barrel
5. The sample is collected in the inner liner
6. Rotary samplers are useful for sampling in firm to hard cohesive soils and particulary in rocks

NOTE: Block or chunk samples can be obtained from open excavations like test pits, shafts etc. For chunk sampling, it is neeeessary that the soil has a trace of cohesion.

3rd method of exploration: Sub Surface soundings.

For subsurface soundings following penetration test can be performed:

(i) Standard Penetration test

(ii) Static cone penetration test

(iii) Plate load test

(iv) Dynamic cone penetration test.

(v) Pressure meter test

* Dynamic cone penetration test:

→ In this test, a cone with 60° angle of vertex attached to a drill rod is driven into the soil by blows of hammer of 65Kg, falling from a height of 75cm.

→ The blows count for every 100mm penetration of cone is

→ The cone is driven till refusal or upto the required depth

→ The number of blows required for 300mm penetration is noted as dynamic cone penetration resistance, Ncd.

→ The test gives a continuous record of Ncd with depth

→ No sample can be obtained in this test.

→ 50mm diameter cone is used without bentonite slurry.
65mm diameter cone is used with bentonite slurry.

→ To reduce the friction on drilling rod, bentonite slurry is used.

→ It is less expensive test & much quicker than the SPT.

→ The correlations can be used are:

Applicable for medium to fine sand:

(when 50mm dia cone is used)

$N_{cd} = 1.5N$ for depths upto 3m

$N_{cd} = 1.75N$ for depths 3m to 6m

$N_{cd} = 2.0N$ for depths > 6m

NOTE: CPT gives a continuous record of variation of both cone resistance & side friction resistance with depth In

CPT test also sample is not collected.

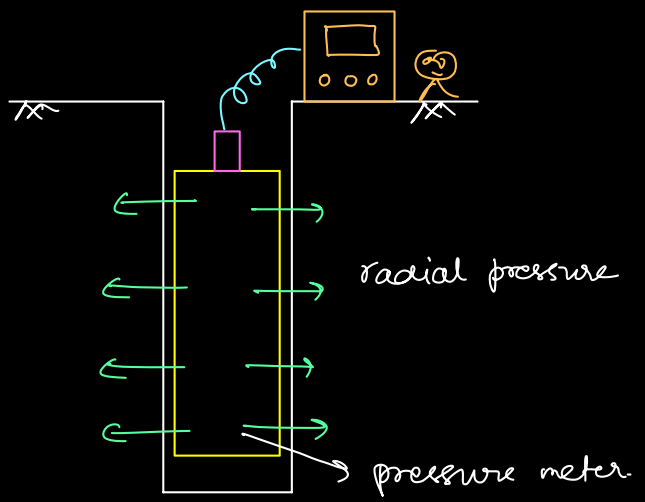
Note: SPT Test → N (SPT No.)

CPT test → q_c (CPT value)

DCPT test → N_d (DCPT value)

* Pressure Meter test:

- Pressure meter test is a form of load test.
- The load is applied by a uniform radial pressure to the sides of borehole in which a pressuremeter is placed.



There are two basic type of pressure meter:

a. The Menard pressure meter (MPM), which is used into a performed borehole

b. The self boring pressuremeter (SBP) which forms its own borehole thus cost much less disturbance to the soil prior to testing.

In both cases, the pressuremeter test consist of applying known stresses to the soil and measuring the resulting soil deformation

Note: In this test modulus of elasticity of soil can be determined.

(Menard (1975) gave a relationship between Young's modulus (E) and menard elastic modulus (E_m) as follows:

$$E = \frac{E_m}{\alpha}$$

where α : Rheological factor.

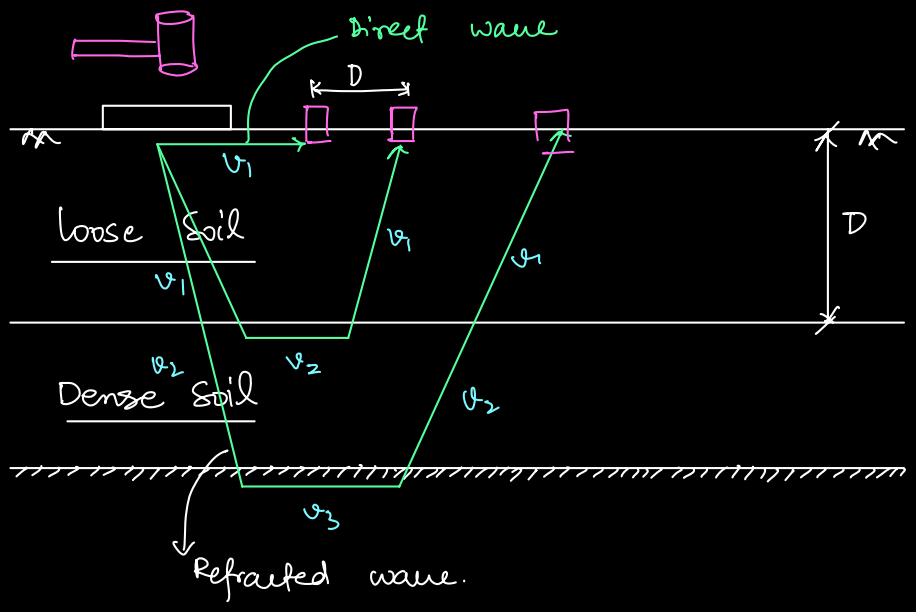
4th Method of soil exploration: Geophysical methods

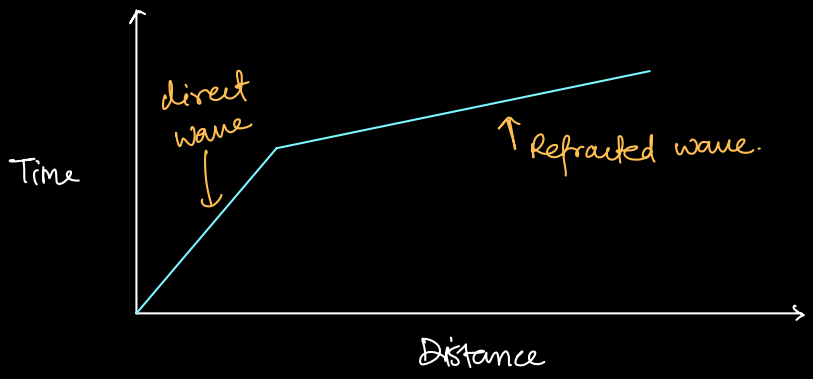
The two commonly used methods in civil engineering are

(i) Seismic refraction method

(ii) Electrical resistivity method

(i) Seismic Refraction method:





- The depth of boundary between two soil strata can be estimated as

$$D = \frac{d}{2} \cdot \sqrt{\left(\frac{V_2 - V_1}{V_2 + V_1}\right)}$$

1. The method is based on the fact that seismic waves

have different velocities in different soils.

2. The waves are refracted when the cross the boundary between different types of soil.

3. In this method we determine the general type of soil & its approximate depth of boundaries.

4. The method consists of including impact (by striking a plate on the soil with a hammer) & generating shocks.

5. These waves / shocks are recorded / detected by the geophones installed on the ground surface

6. If the soil is dense then the refracted waves travel much faster but if the soils are loose seismic waves moves slowly.

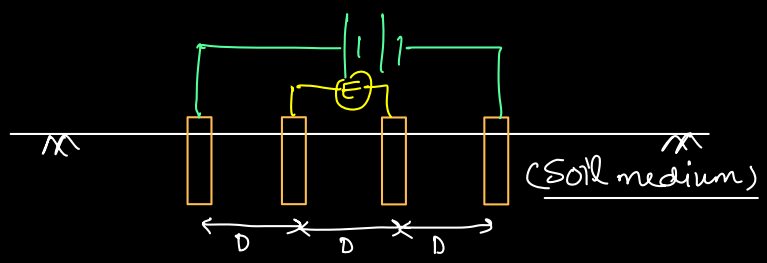
7. If the distance between source & geophones is increased then refracted waves may reach to geophones earlier than direct waves.

d: distance between geophones

V_1: velocity of waves in loose soil

V_2: velocity of waves in dense sand

(ii) Electrical resistivity Test:



The apparent resistivity can be given as:

$$\rho = \frac{2\pi DE}{I} = 2\pi DR$$

→ The electrical resistivity method is based on the measurement of changes in mean resistivity or apparent specific resistance of various soils

→ The electrical resistivity (ρ) is usually defined as the resistance b/w the opposite faces of a unit cube of material.

→ The electrical resistivity is different for different soil

→ The electrical resistivity of any soil depends upon the types and proportion of minerals, water content, DOS & density / packing of soil.

→ The test is carried out by driving 4 metal electrodes into the ground at straight line at equal spacing.

→ Current (I) from a battery flows through the soil b/w the two outer electrodes, producing an electrical field within the soil

→ The potential difference (E) b/w two inner electrodes is then measured.

→ Conductivity of the soil $\propto \frac{1}{\text{Resistivity of soil}}$

→ if water content ↑ : conductivity ↑ : Resistivity ↓

→ if DOS ↑ : conductivity ↑ : Resistivity ↓

→ if γ_d ↑ : compactness ↑ : conductivity ↑ : Resistivity ↓

Note: loose soil is more resistive in comparison to compacted soil.

→ Rocky soil has very high resistivity

→ Bentonite clay soil has very low resistivity & high conductivity

→ A Dry soil has high resistivity if it contains no soluble salts

→ $\rho_{clay} < \rho_{silt} < \rho_{sand} < \rho_{gravel} < \rho_{rocks}$

FOUNDATION IN EXPANSIVE SOILS

1. Expansive soils are those soil which represent large change in volume on changing the water content
2. If water content is increased then it shows increases in volume of soil known as swelling.
3. If water content is decreased then it shows decrease in volume of soil known as shrinkage
4. Such soil contains Montmorillonite mineral in large proportion
   Example: Black cotton soil, bentonite soil
5. this expansive soil is also termed as swelling soil.
7. In india they are called Black cotton soil. Because they are found black in color and good for cotton growth
8. Expansive soil has considerable strength in dry state but the strength goes on reducing on absorption of water.
9. Expansive soil has
   1. large liquid limit
   2. large plastic limit
   3. large plastic index
   4. lower shrinkage limit
   5. Higher shrinkage ratio
   6. Larger activity
   7. Higher % of clay particles

Problems associated with expansive soils:

1. If water content remains constant they won't create any problem. But when there are significant changes in water content then large damages are reported in the structure

2. Expansive soils usually won't have problems with heavy structures but with light structures & pavements, significant damages (cracks) are observed due to swelling & shrinkage of soil.

3. The damages occur due to the swelling & shrinkage behaviours of such soils. Since the structures built on such soils gets lifted up during rainy seasons (due to heave of foundation soil) and settle down during summer season (due to shrinkage).

4. Due to uplift (due to swelling) and settlement (due to shrinkge) light structure gets damaged & cracks

Note: Expansive soils have lower shrinkage limit therefore they show change in volume of soil on adding small quantities of water.

Note: In India, Black cotton soil has following properties.

$$\left\{ \begin{array}{ll} w_L: & 50\% \text{ to } 100\% \\ I_P: & 20\% \text{ to } 65\% \end{array} \right\}$$

% clay ($< 2\mu$) = 50% to 60%

w_S = 9% - 14%.

SWELLING PRESSURE: Defined as the pressure required to prevent any volume expansion in a soil contact with the water. It can be determined in one dimensional consolidometer test.

Identification of expansive soil:

(1) Advanced Techniques:

a) Differential Thermal Analysis (DTA)

b) X-ray diffraction method

c) Electron microscopy

(ii) Simple test:

a) Free swell test

b) Differential free swell test

c) Swelling pressure test

FREE SWELL TEST:

$$\% \text{ free swell} = \frac{V_f - V_i}{V_i} \times 100$$

V_f: final volume of powred soil after 24 hours contact with water.

V_i: initial volume of powred soil.

(i) Take 10 cm^3 volume of dry soil passing 425μ sieve

(ii) Pour this soil into 100 cm^3 graduated cylindrical container containing water

(iii) The volume of expansive soil will expands when it comes in the contact of water

Note: If free swell value is less than 50% then it's effect has no much significance.

Note: For bentonite soil (Montmorillonite) free soil value varies from 1200 % to 2000%. whereas for kaolinite it is about 80% and for illite it varies from 30% to 80%.

b. DIFFERENTIAL FREE SWELL TEST:

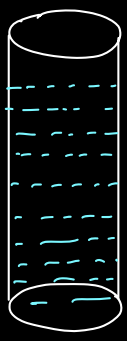
(i) IS 2720 (Part 40) : 1977

(ii) Apparatus required:

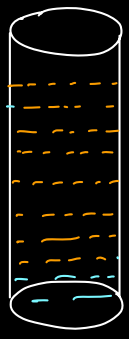
↳ 425μ IS Sieve

↳ Graduate glass cylinder of 100ml (or 100 cm^3)

↳ Glass rod



DISTILLED WATER.



KEROSENE

$$\text{Differential free swell index (\%)} = \frac{V_D - V_K}{V_K} \times 100$$

V_D: Volume of soil in distilled water

V_K: Volume of soil in kerosene

(iii) Take two representative oven dry samples each of 10g passing through 425 μ sieve

(iv) pour each soil sample into each of the two glass cylinder of 100 ml capacity

v) Fiel one cylinder with distilled water and other with Kerosene upto 100ml mark

vi) Remove the air by stirring with a glass rod

vii) Allow the sample to settle in both the cylinders

viii) After 24 hours record the final volume of soils in each of the cylinders.

∴ Differential free swell Index	Degree of expansiveness.
0 - 20%	Low
20 - 35%	Moderate
35 - 50%	High
> 50%	Very high

NOTE: If % differential free swell index is greater than 35%

then conventional shallow footings will not be suitable

NOTE: The degree of expansiveness will be more if I_p of soil is more, w_s is less, I_s of soil is more, linear shrinkage is more, colloid content is more

NOTE: If plasticity of soil is more that means that the soil can invite more water which results high degree of expansiveness.

NOTE: To control swelling good surface and subsurface facility can be provided to remove rainy water.

Use impervious materials in pavements

NOTE: If proper drainage is not available then rainy water will infiltrate through the expansive soil that may cause the problem of swelling

C. SWELLING PRESSURE TEST:

→ When an expansive soils imbibes water from outside, pressure builds up inside the soil

→ If free swelling is restrained by the placement of a structure over the soil, this pressure called swelling pressure, is exerted by the soil on the overlying structure

→ A swelling pressure of less than about 20 KN/m² may not be regarded as of much consequence.

→ The swelling pressure can be computed from the one-dimensional consolidation test.

NOTE: Designing the structure which is strong & rigid enough to withstand the effects of swelling may prove to be highly uneconomical except in the case of very small structure

NOTE: The cost of providing the structure with extra strength & rigidity would be much more as compared to the cost of underreamed pile foundation

NOTE: Swelling can be controlled but cannot be eliminated by providing an impervious apron around the structure